CS 558: Computer Vision
5th Set of Notes

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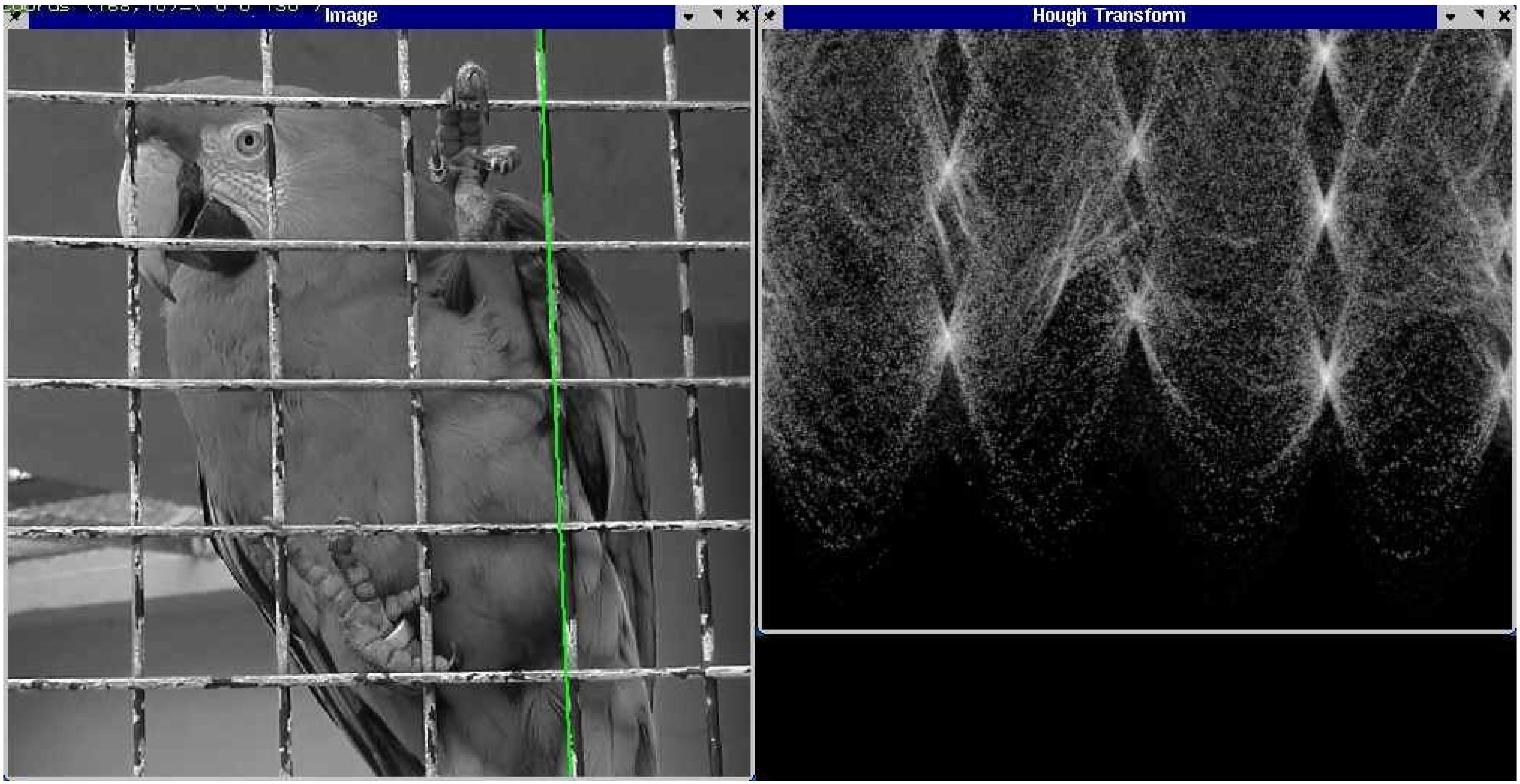
Overview

- Hough Transform
- Template Matching
- Image Alignment
  - Based on slides by S. Lazebnik, K. Grauman and D. Hoiem

Fitting: The Hough transform



Slides based on S. Lazebnik's and K. Grauman's slides

Voting schemes

- Let each feature vote for all the models that are compatible with it
- Hopefully the noise features will not vote consistently for any single model

Hough transform

- An early type of voting scheme
- General outline:
  - Discretize *parameter space* into bins
  - For each feature point in the image, put a vote in every bin in the parameter space that could have generated this point
  - Find bins that have the most votes

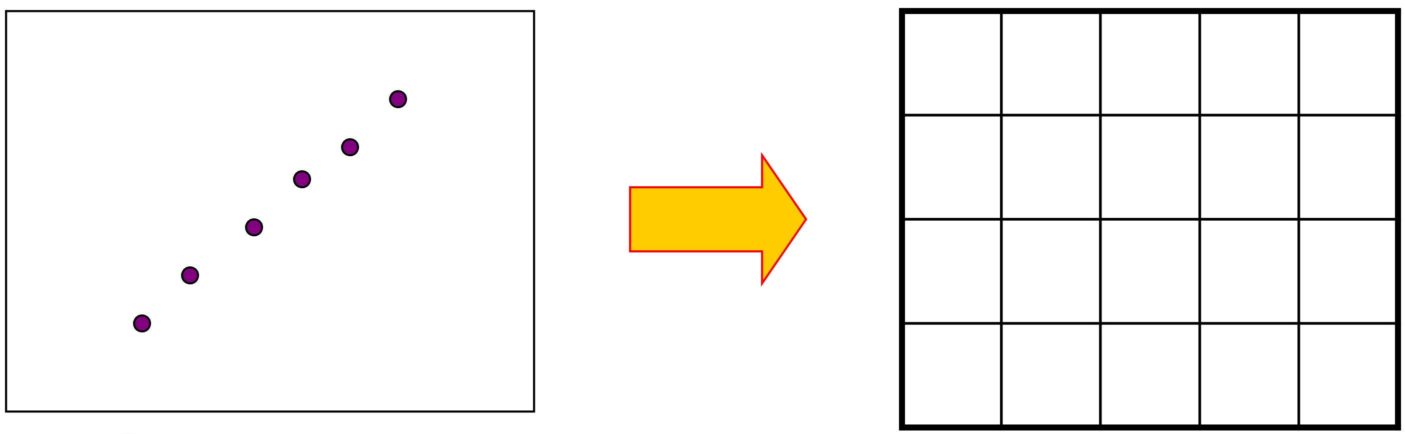


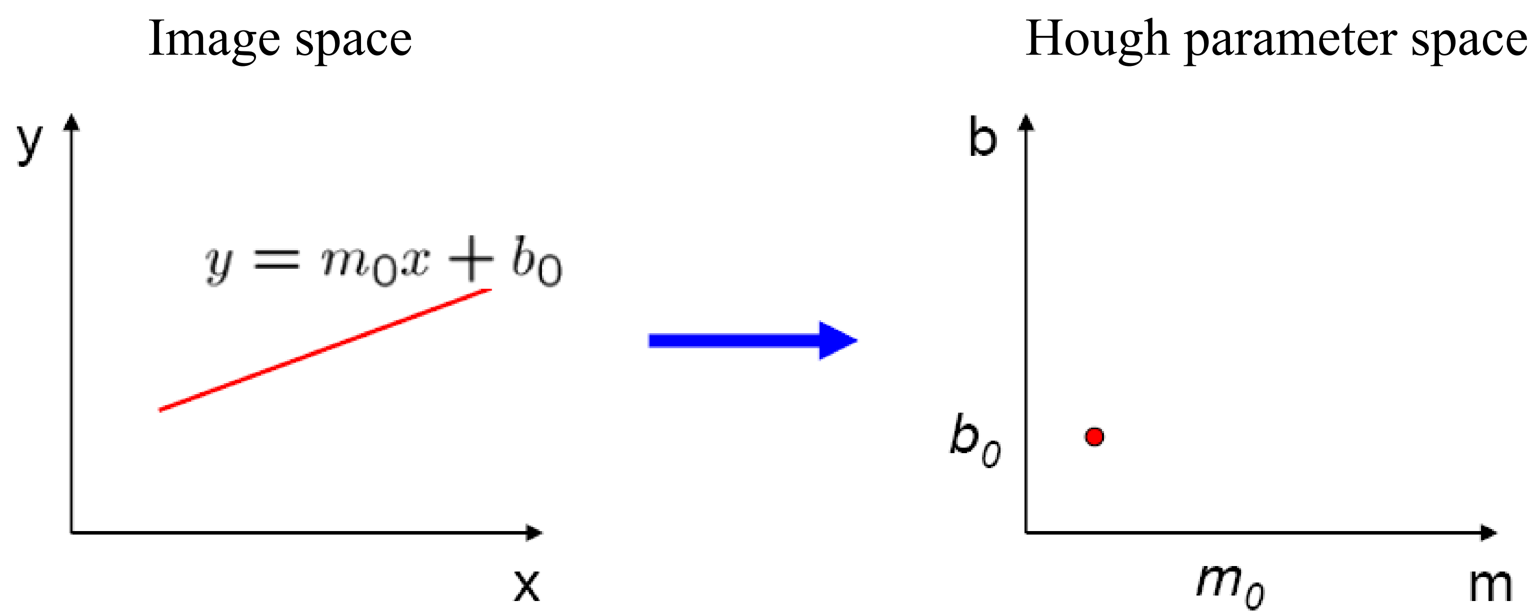
Image space

Hough parameter space

P.V.C. Hough, *Machine Analysis of Bubble Chamber Pictures*, Proc. Int. Conf. High Energy Accelerators and Instrumentation, 1959

Parameter space representation

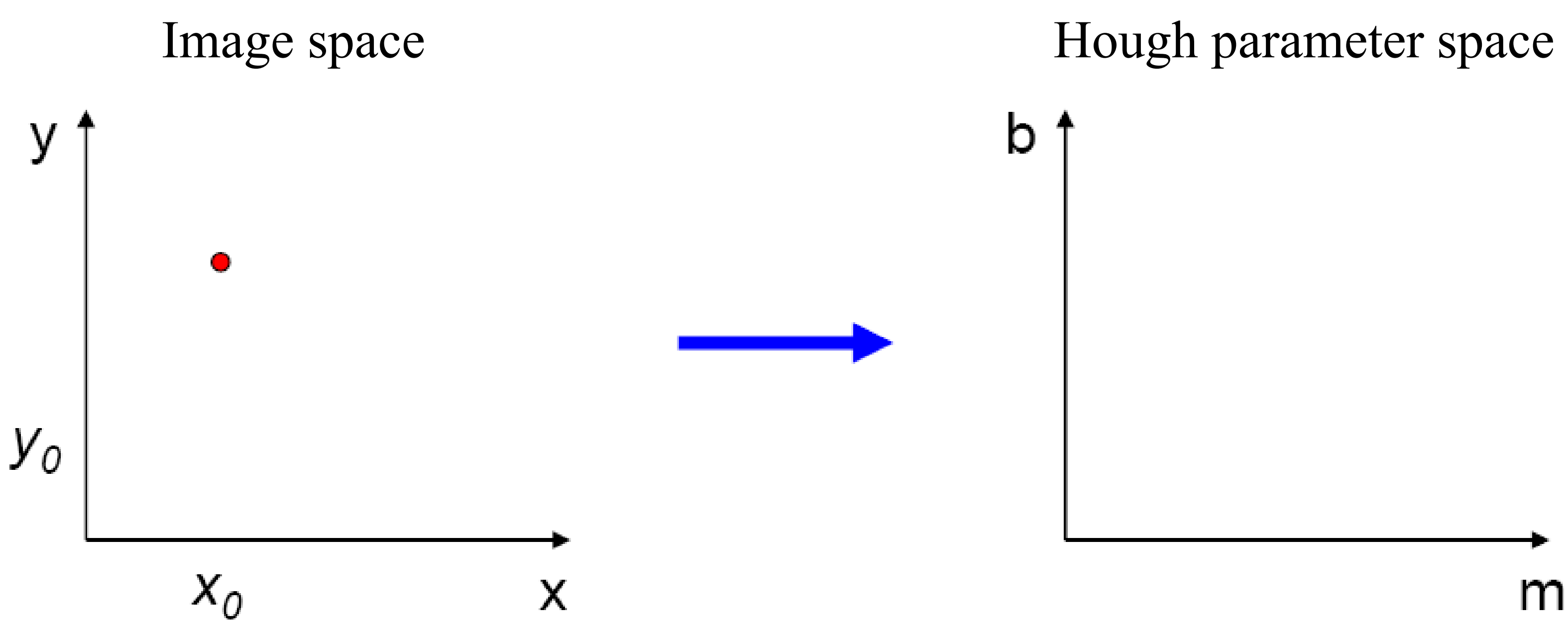
- A line in the image corresponds to a point in Hough space



Source: S. Seitz

Parameter space representation

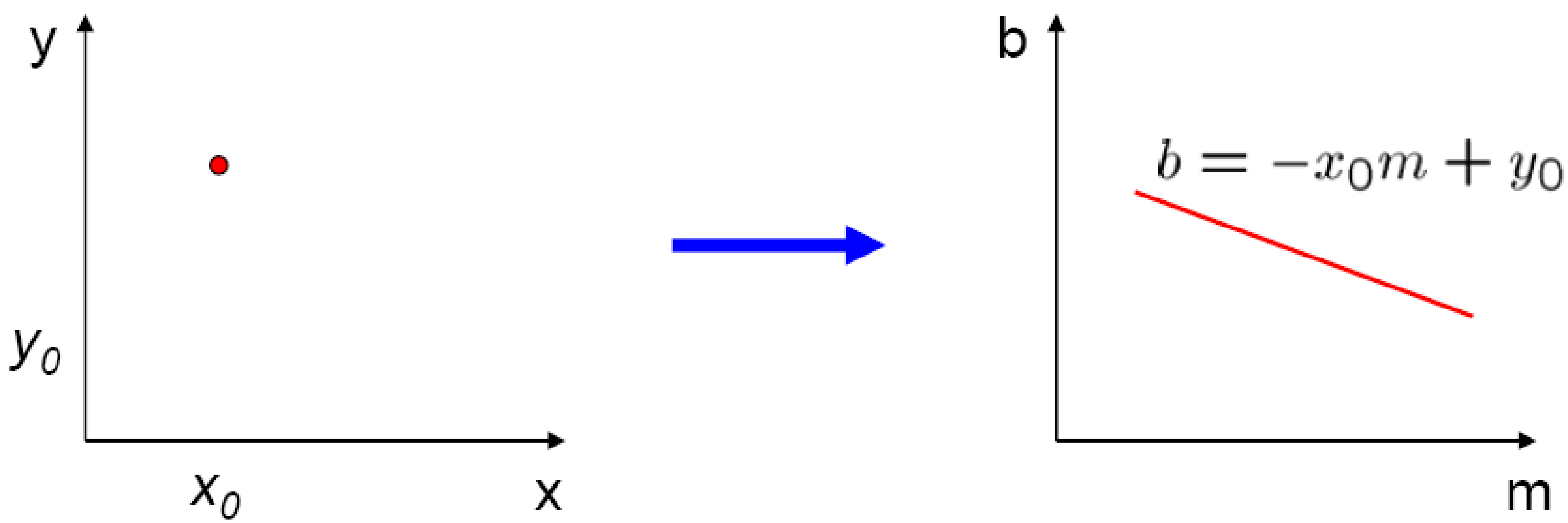
- What does a point (x_0, y_0) in the image space map to in the Hough space?



Parameter space representation

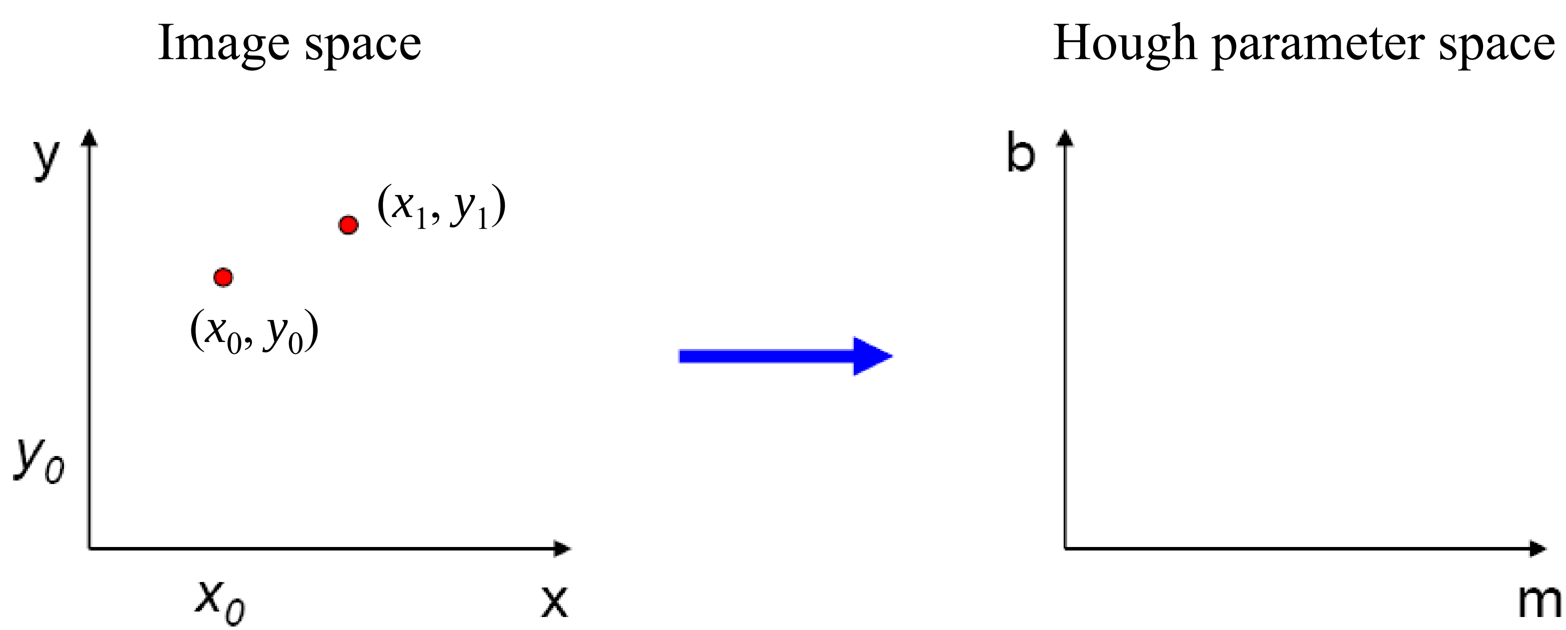
- What does a point (x_0, y_0) in the image space map to in the Hough space?
  - Answer: the solutions of $b = -x_0 m + y_0$
  - This is a line in Hough space

Image space　　　　　　Hough parameter space



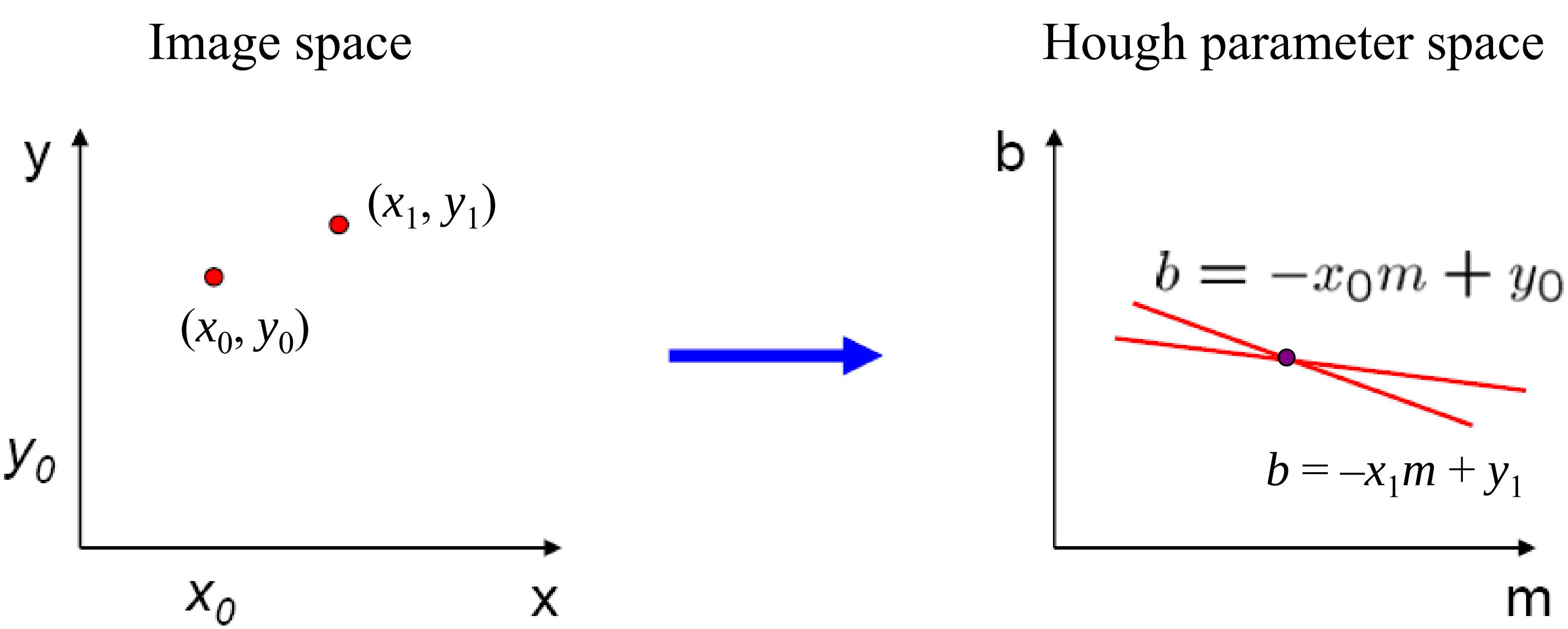
Parameter space representation

- Where is the line that contains both (x_0, y_0) and (x_1, y_1)?



Parameter space representation

- Where is the line that contains both (x_0, y_0) and (x_1, y_1)?
  - It is the intersection of the lines $b = -x_0m + y_0$ and $b = -x_1m + y_1$



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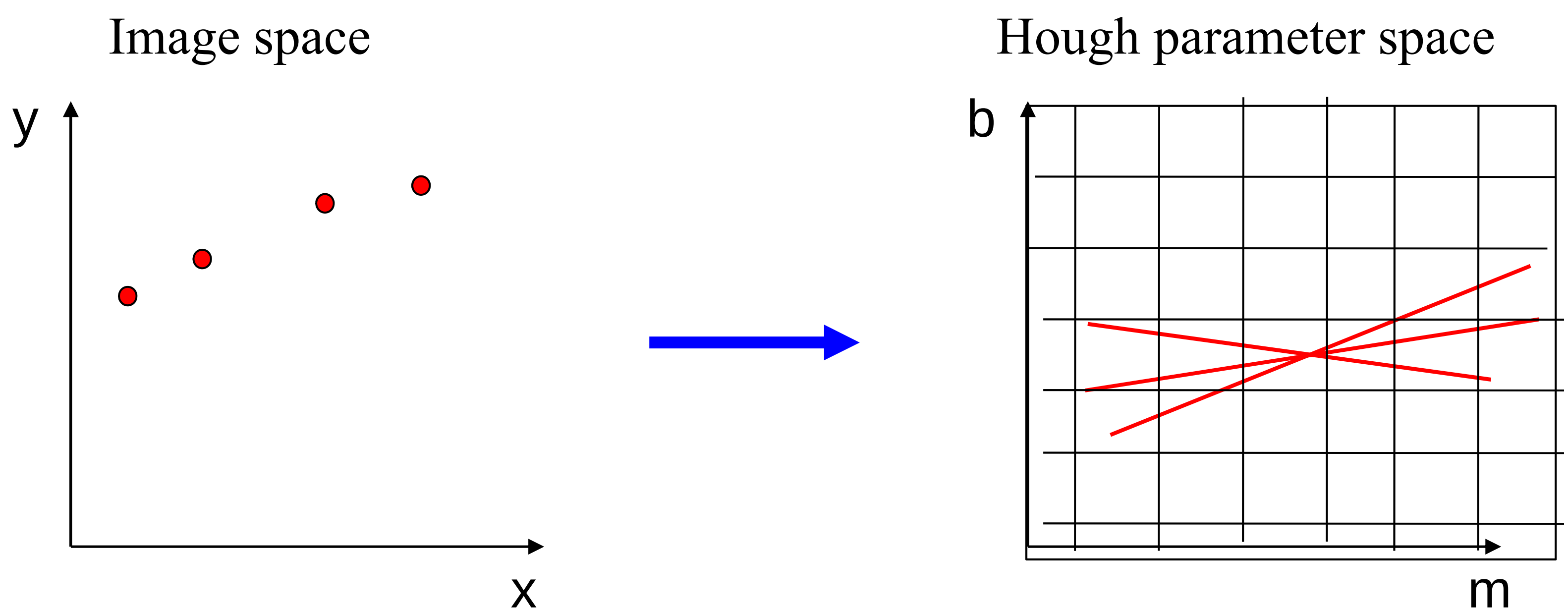
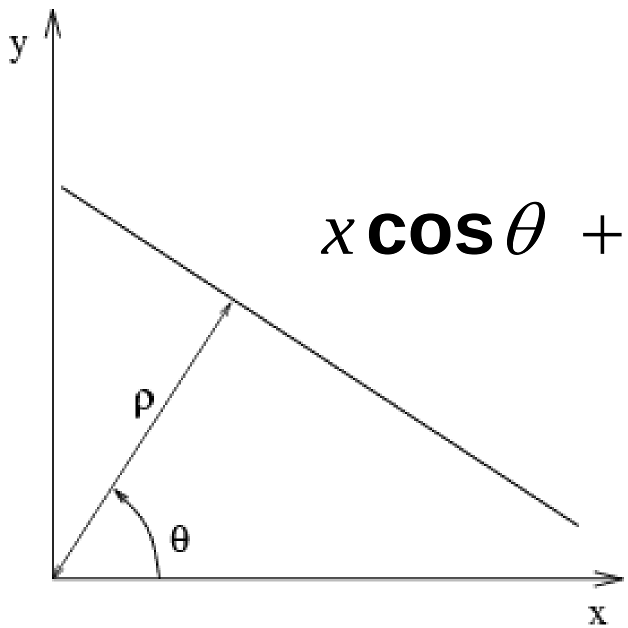


Parameter space representation

- Problems with the (m,b) space:
  - Unbounded parameter domains
  - Vertical lines require infinite m
- Alternative: *polar representation*

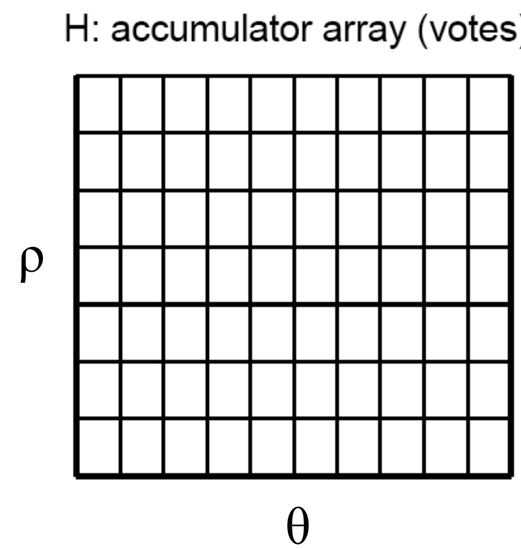


$$x\,\mathbf{cos}\,\theta + y\,\mathbf{sin}\,\theta = \rho$$

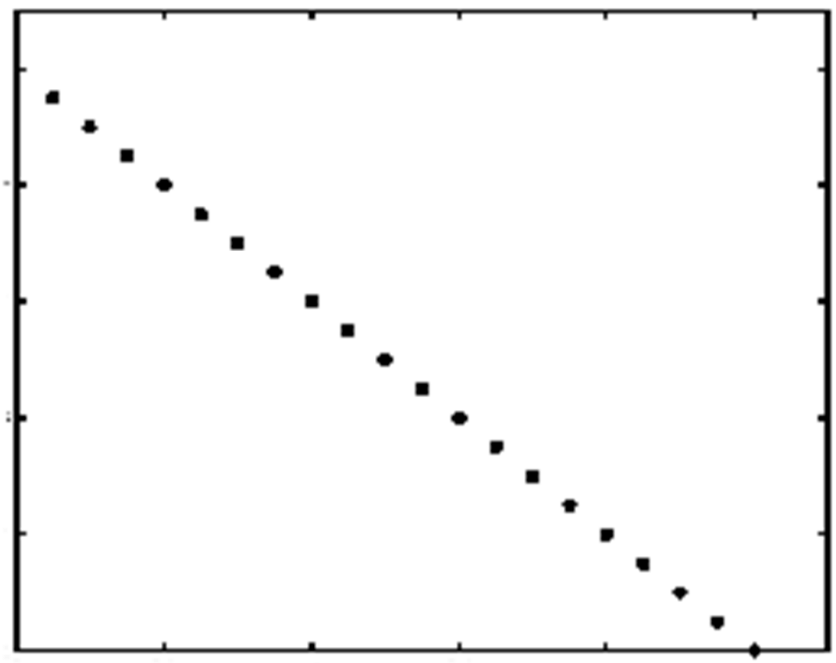
Each point (x,y) will add a sinusoid in the (θ,ρ) parameter space

Algorithm outline

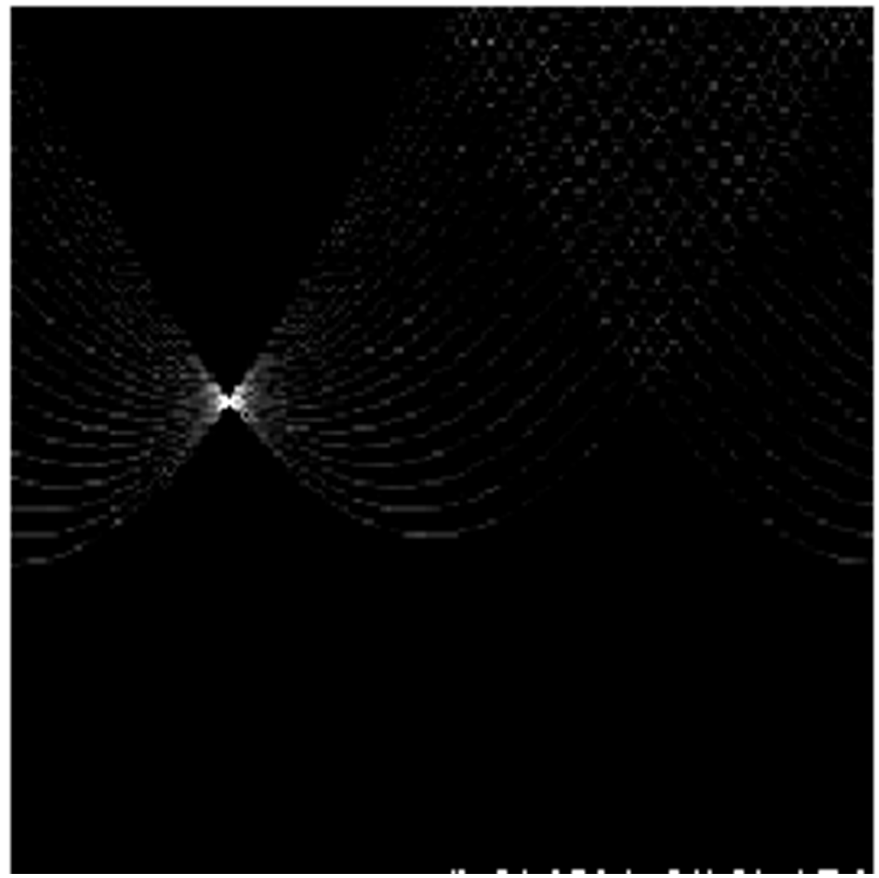
- Initialize accumulator H to all zeros
- For each feature point (x,y) in the image
  For θ = 0 to 180
  $\rho = x \cos\theta + y \sin\theta$
  $H(\theta, \rho) = H(\theta, \rho) + 1$
  end
  end
- Find the value(s) of (θ, ρ) where $H(\theta, \rho)$ is a local maximum
  – The detected line in the image is given by $\rho = x \cos\theta + y \sin\theta$



Basic illustration

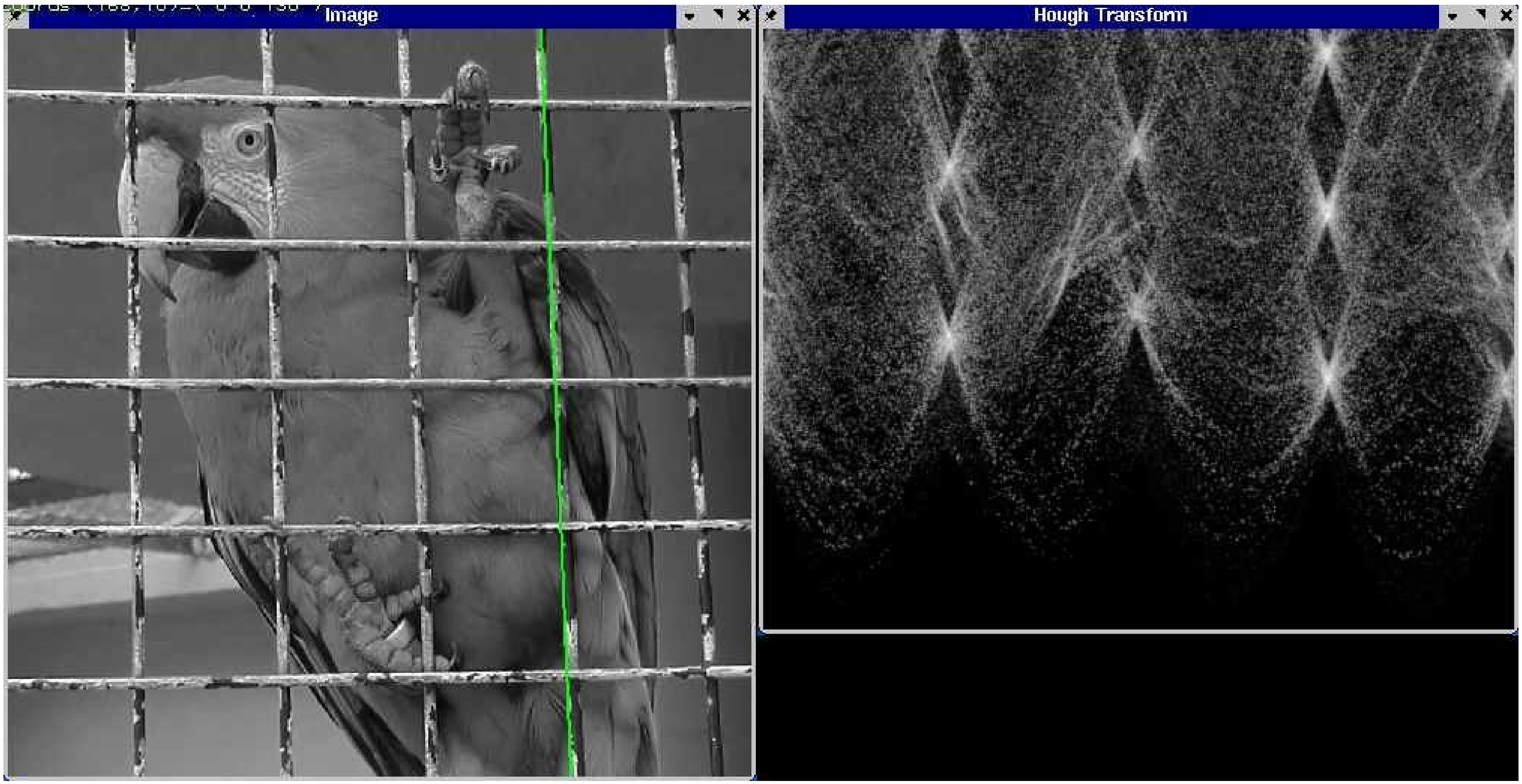


features



votes

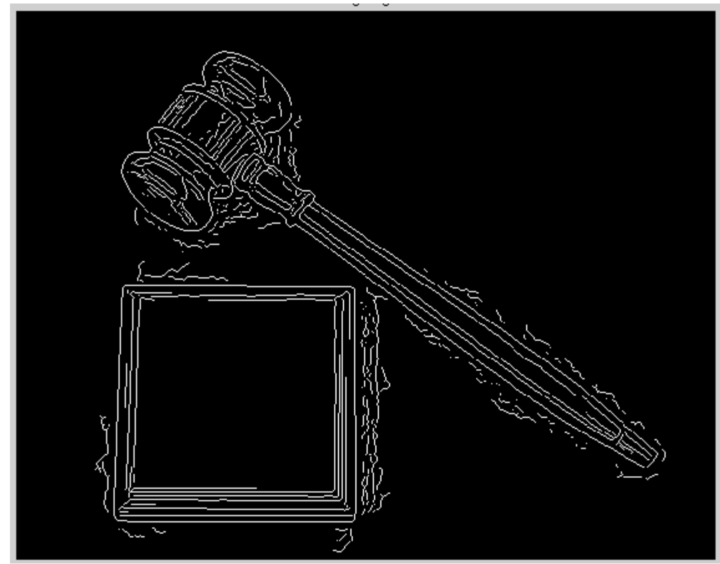
A more complicated image



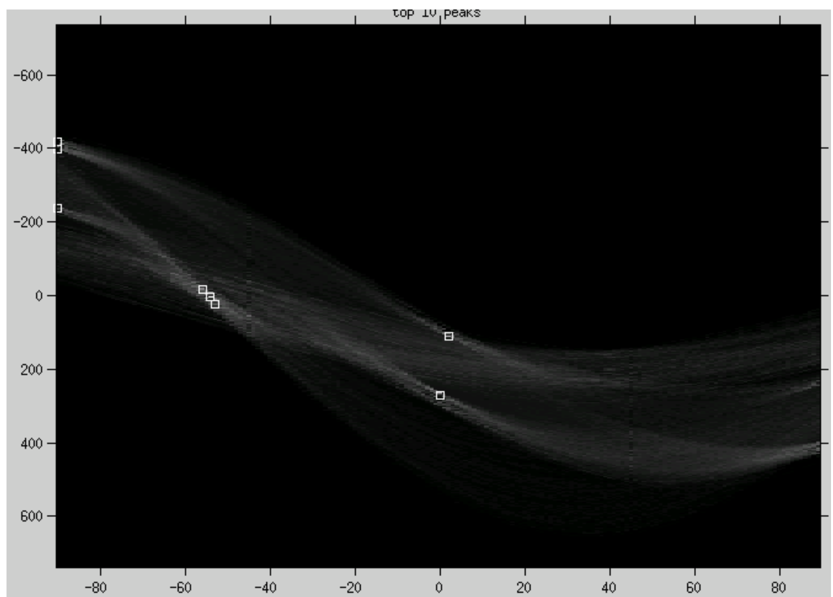
http://ostatic.com/files/images/ss_hough.jpg

Original image

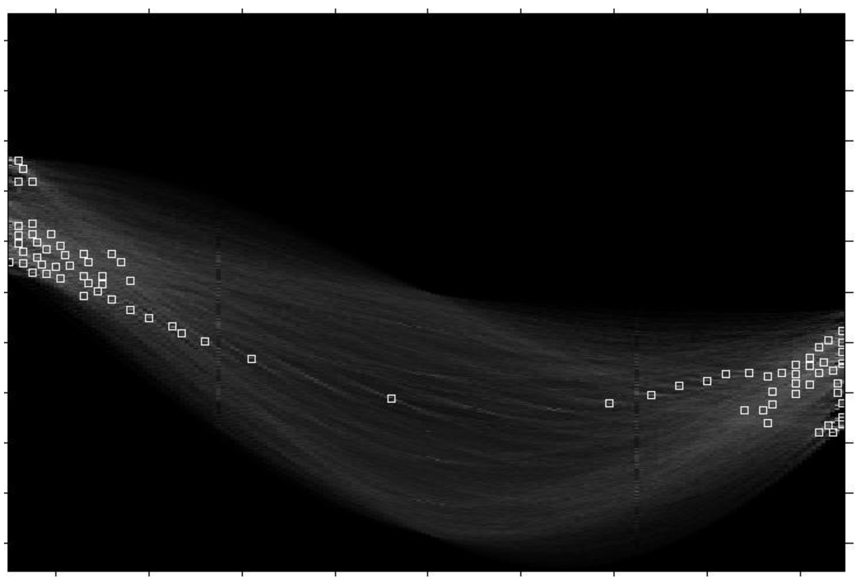
Canny edges

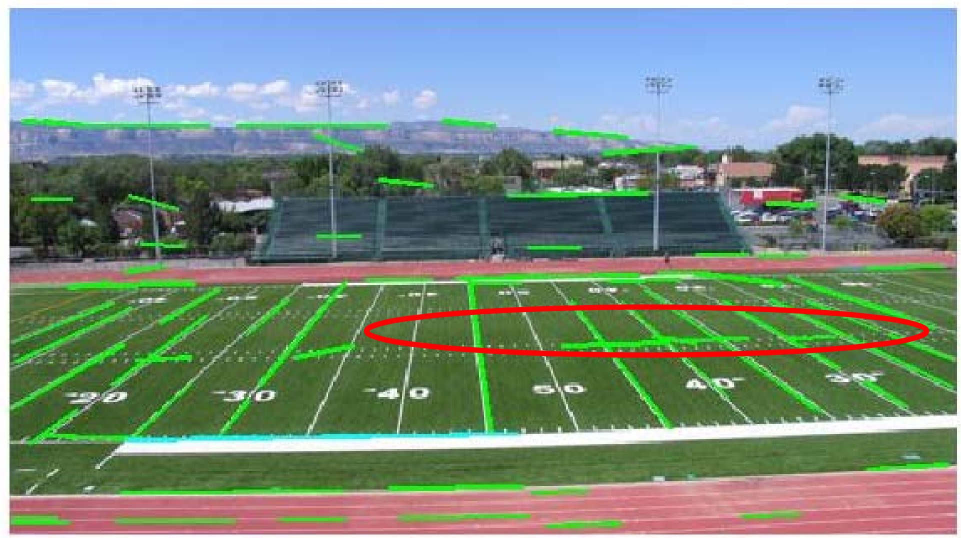


Vote space and top peaks



Kristen Grauman





Showing longest segments found

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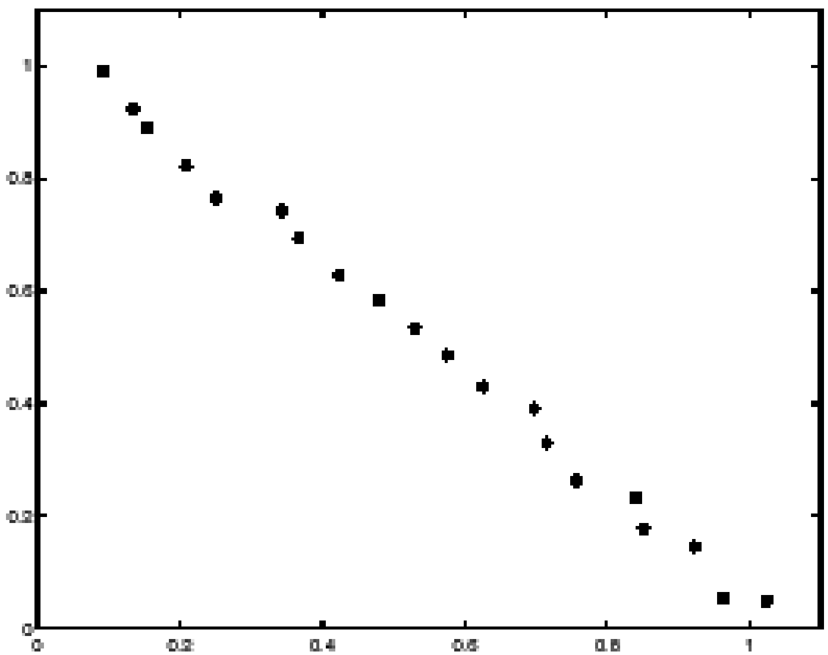
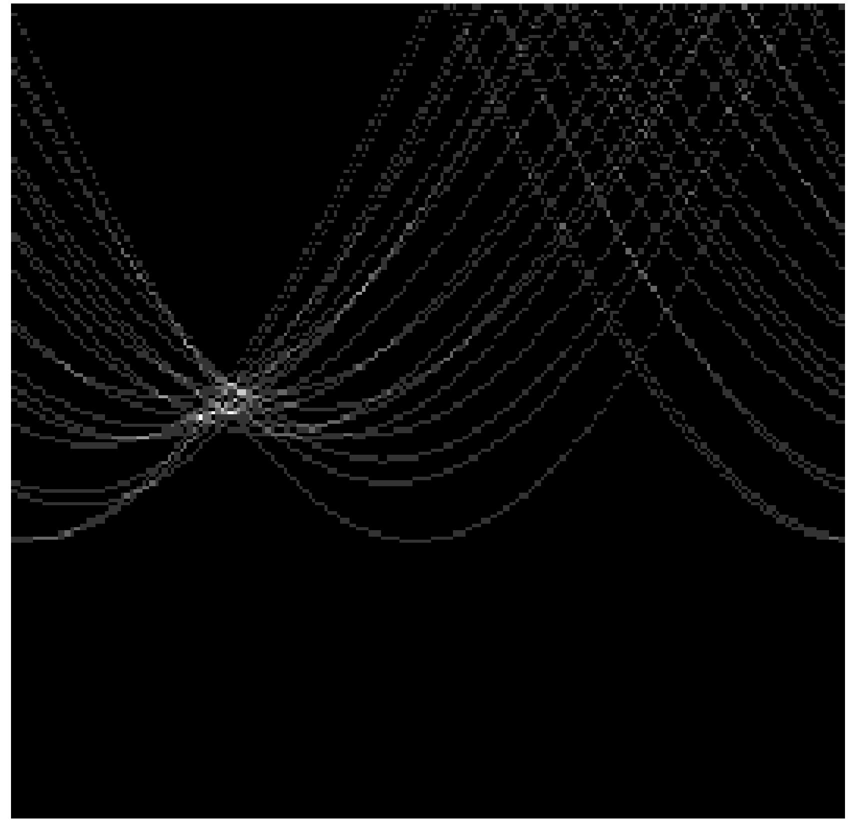
Effect of noise



features

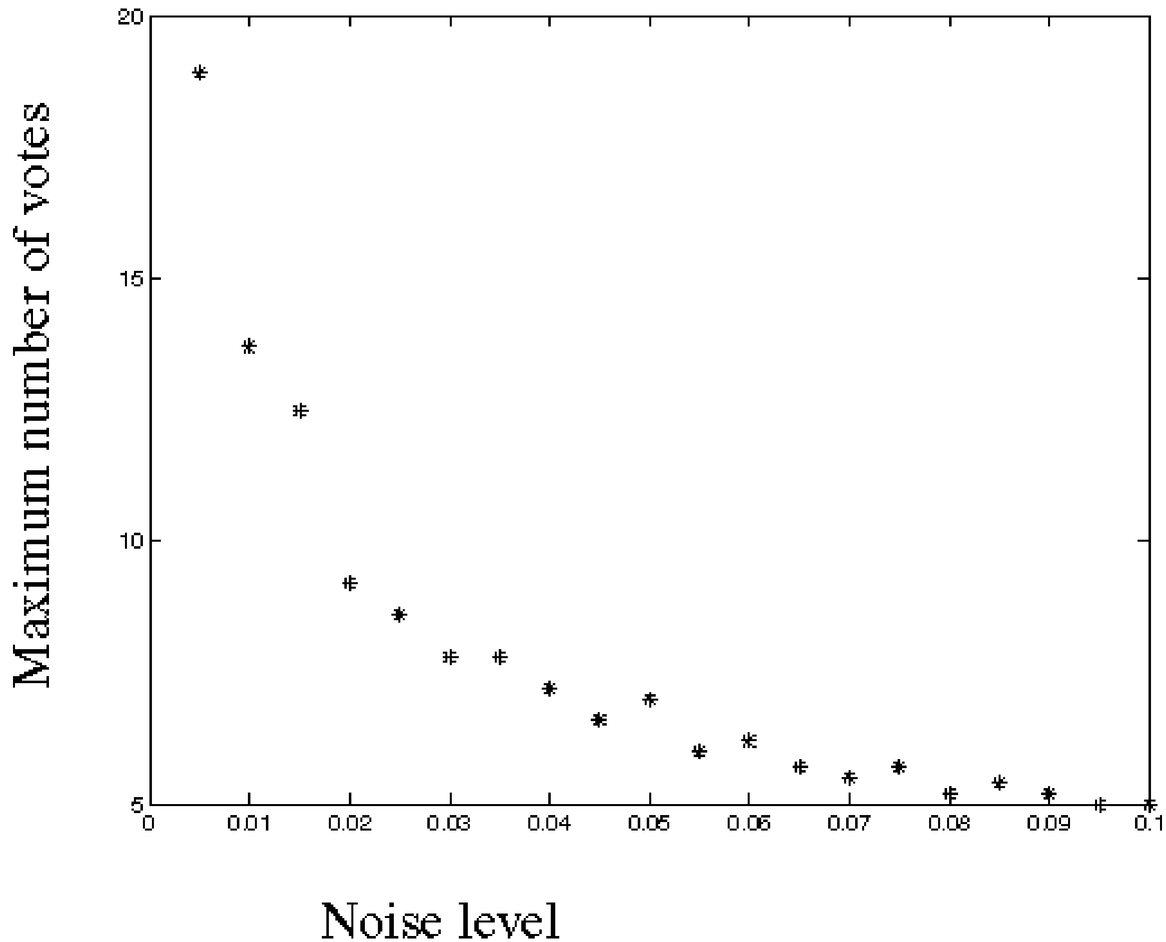


votes

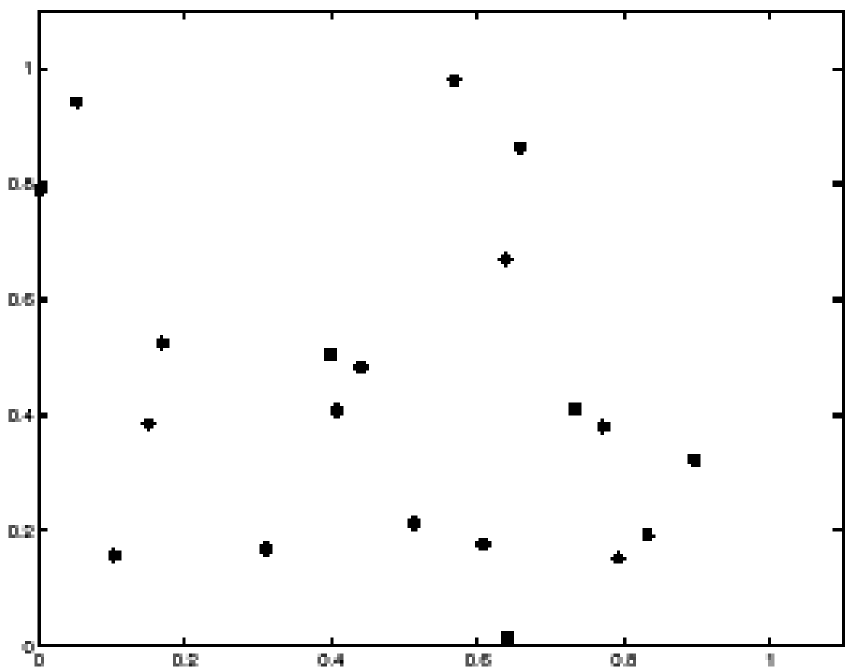
- Peak gets fuzzy and hard to locate

Effect of noise

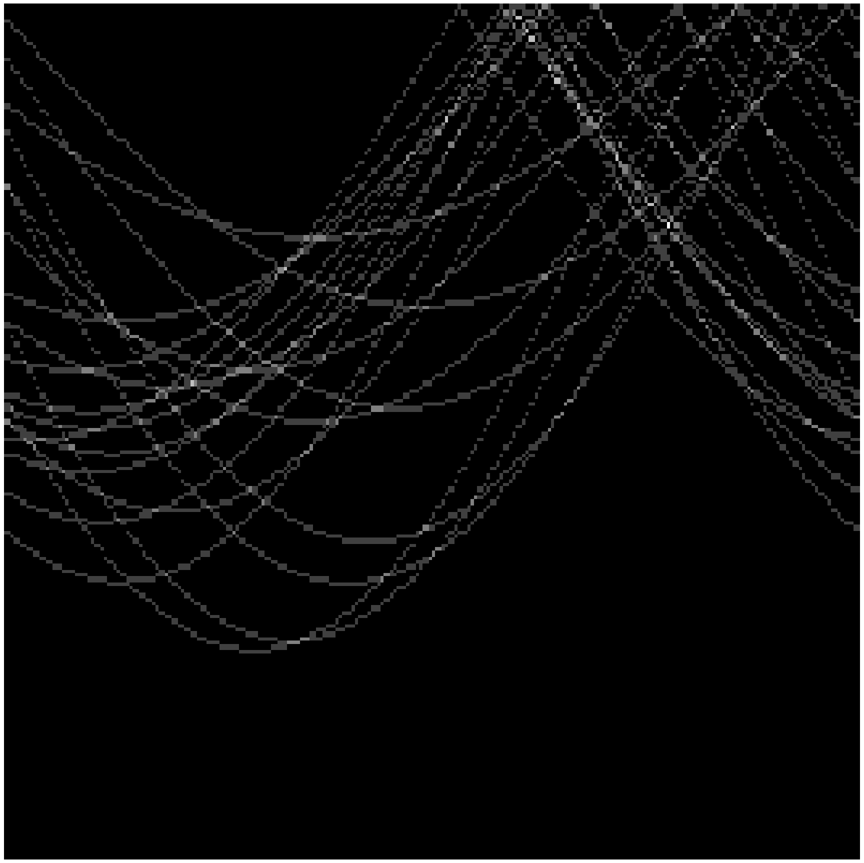
- Number of votes for a line of 20 points with increasing noise:



Random points



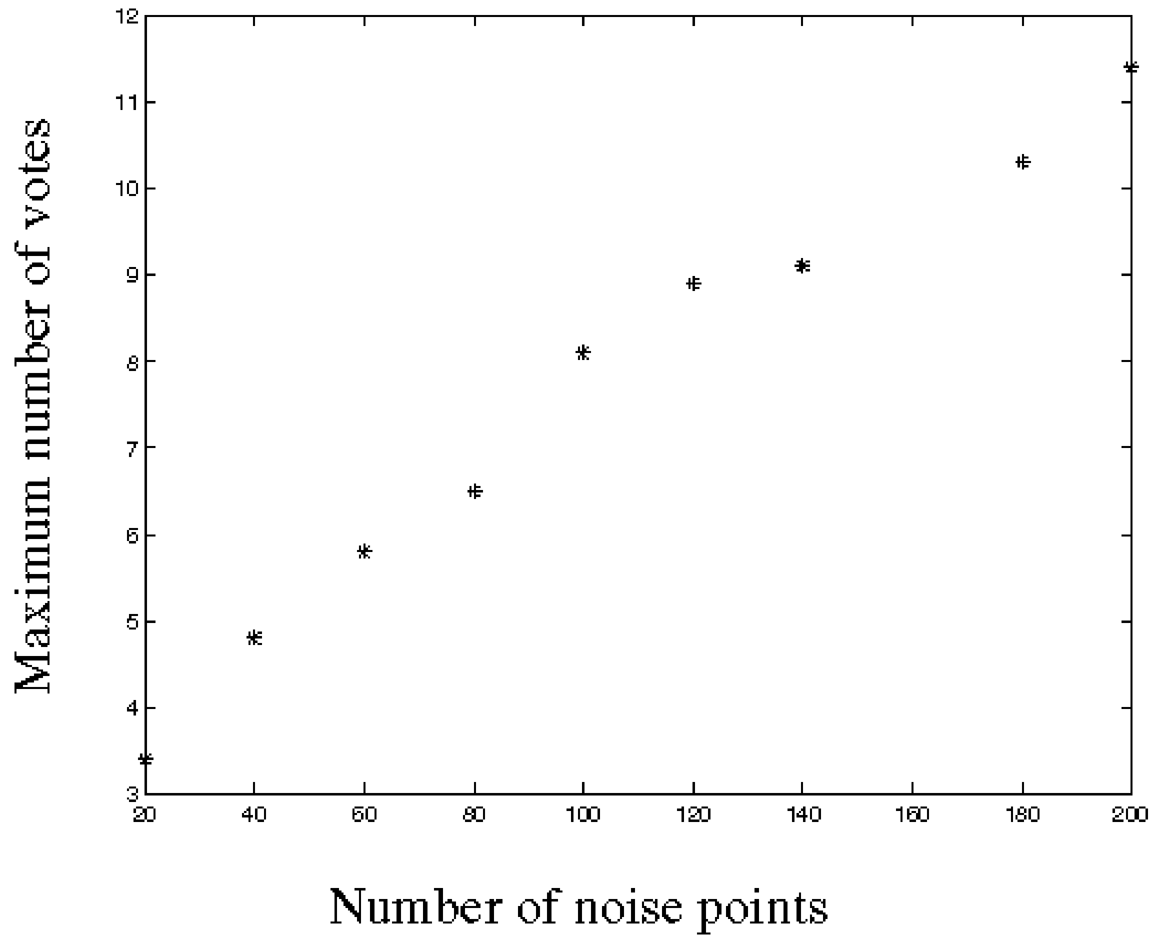
features


votes

- Uniform noise can lead to spurious peaks in the array

Random points

- As the level of uniform noise increases, the maximum number of votes increases too:

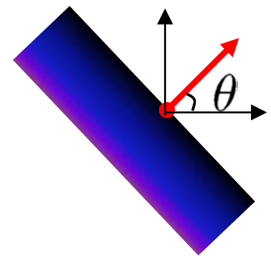


Dealing with noise

- Choose a good grid / discretization
  - **Too coarse:** large vote counts obtained when too many different lines correspond to a single bucket
  - **Too fine:** miss lines because some points that are not exactly collinear cast votes for different buckets
- Increment neighboring bins (smoothing in accumulator array)
- Try to get rid of irrelevant features
  - E.g., take only edge points with significant gradient magnitude

Incorporating image gradients

- Recall: when we detect an edge point, we also know its gradient direction
- But this means that the line is uniquely determined!



$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$$

- Modified Hough transform:

- For each edge point (x,y)
  θ = gradient orientation at (x,y)
  $\rho = x \cos \theta + y \sin \theta$
  $H(\theta, \rho) = H(\theta, \rho) + 1$
  end

Hough transform for circles

- How many dimensions will the parameter space have?
- Given an unoriented edge point, what are all possible bins that it can vote for?
- What about an *oriented* edge point?

Hough transform for circles

- Circle: center (a,b) and radius r

$$(x_i - a)^2 + (y_i - b)^2 = r^2$$

- For a fixed radius r, unknown gradient direction

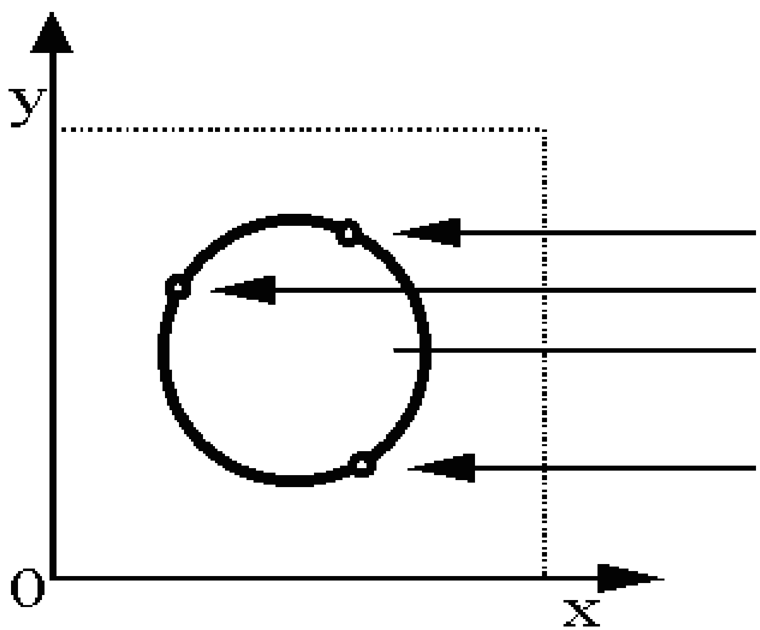
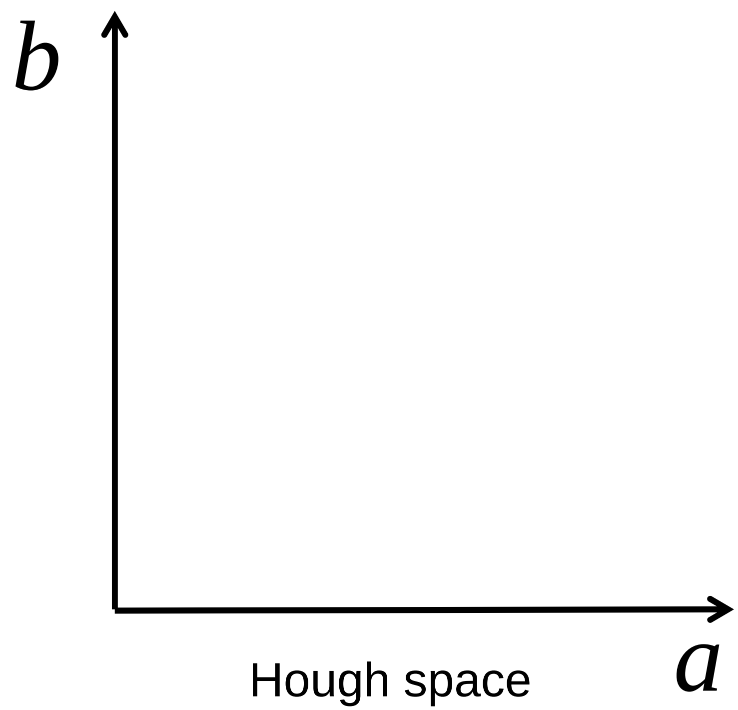


Image space



Hough space

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Hough transform for circles

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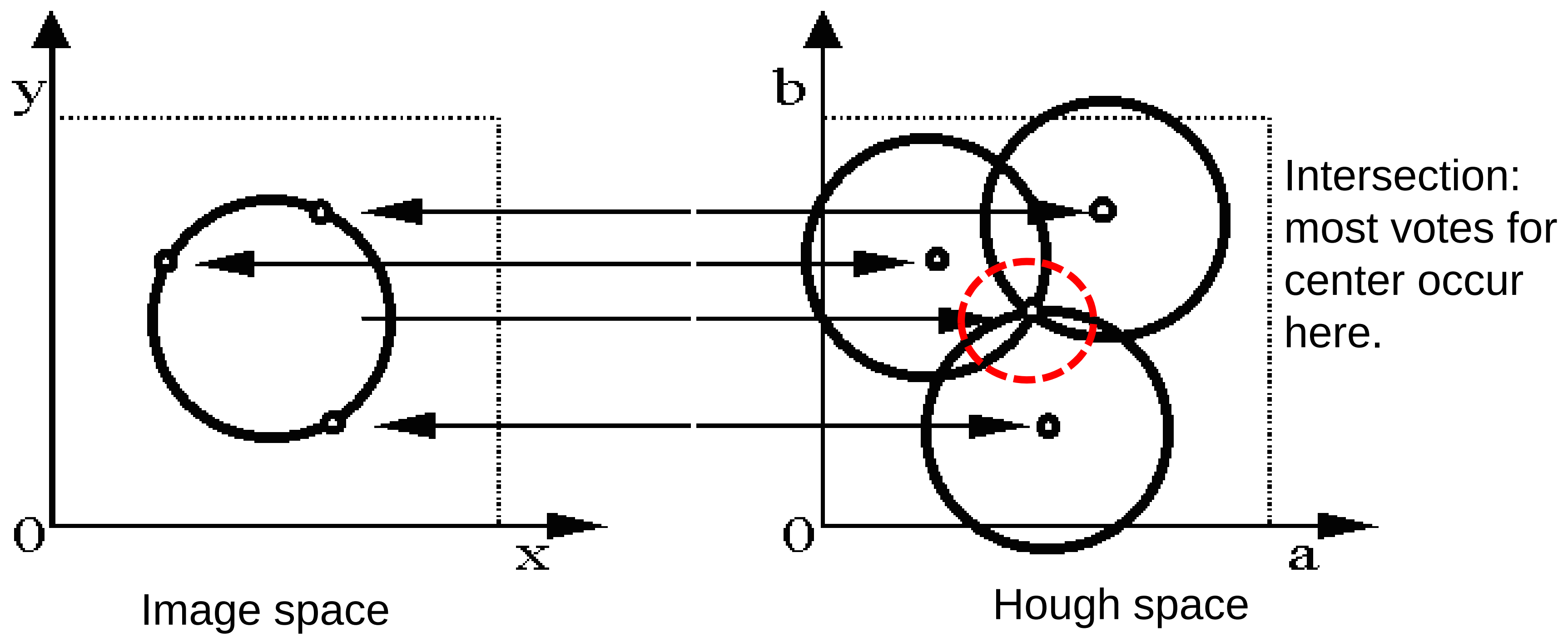

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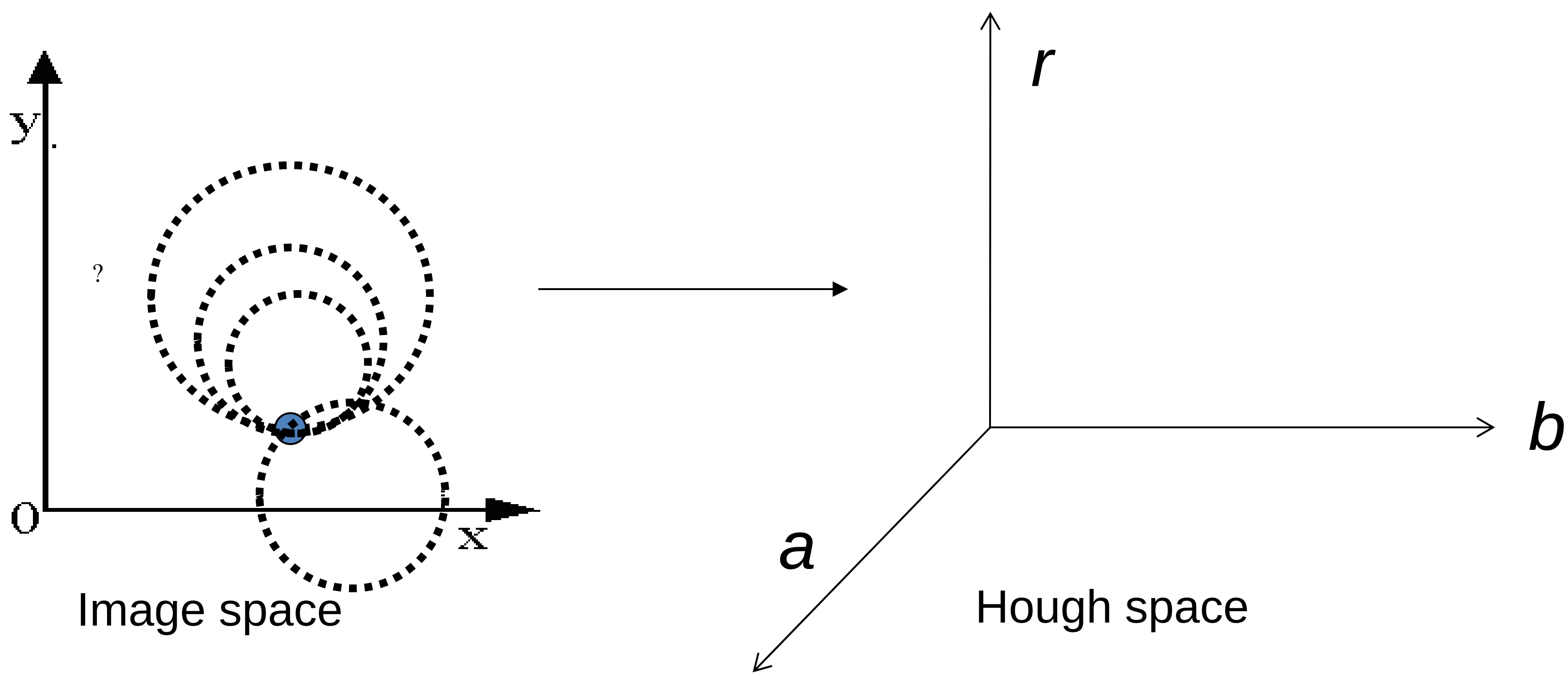

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Hough transform for circles

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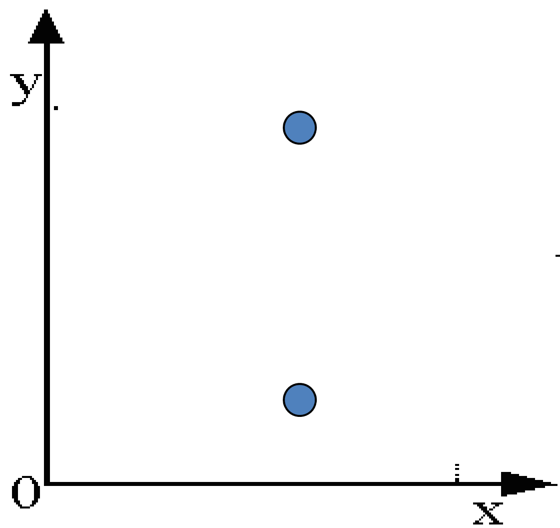
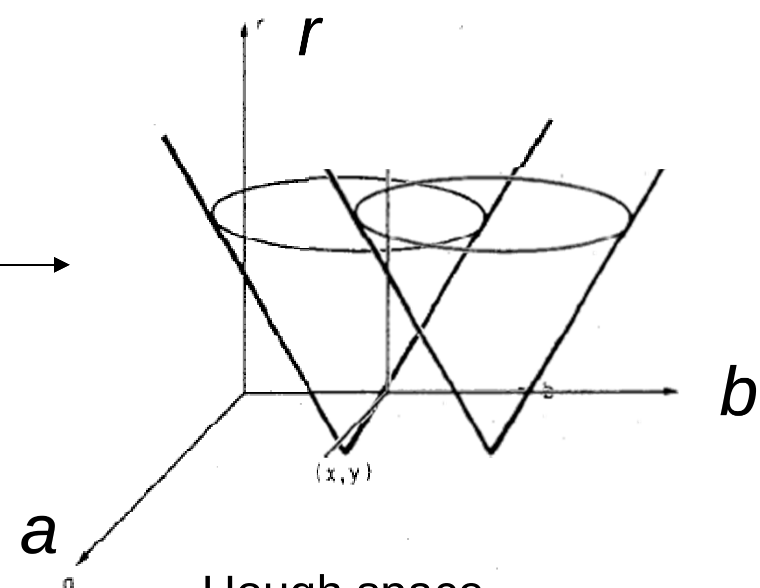


Image space



Hough space

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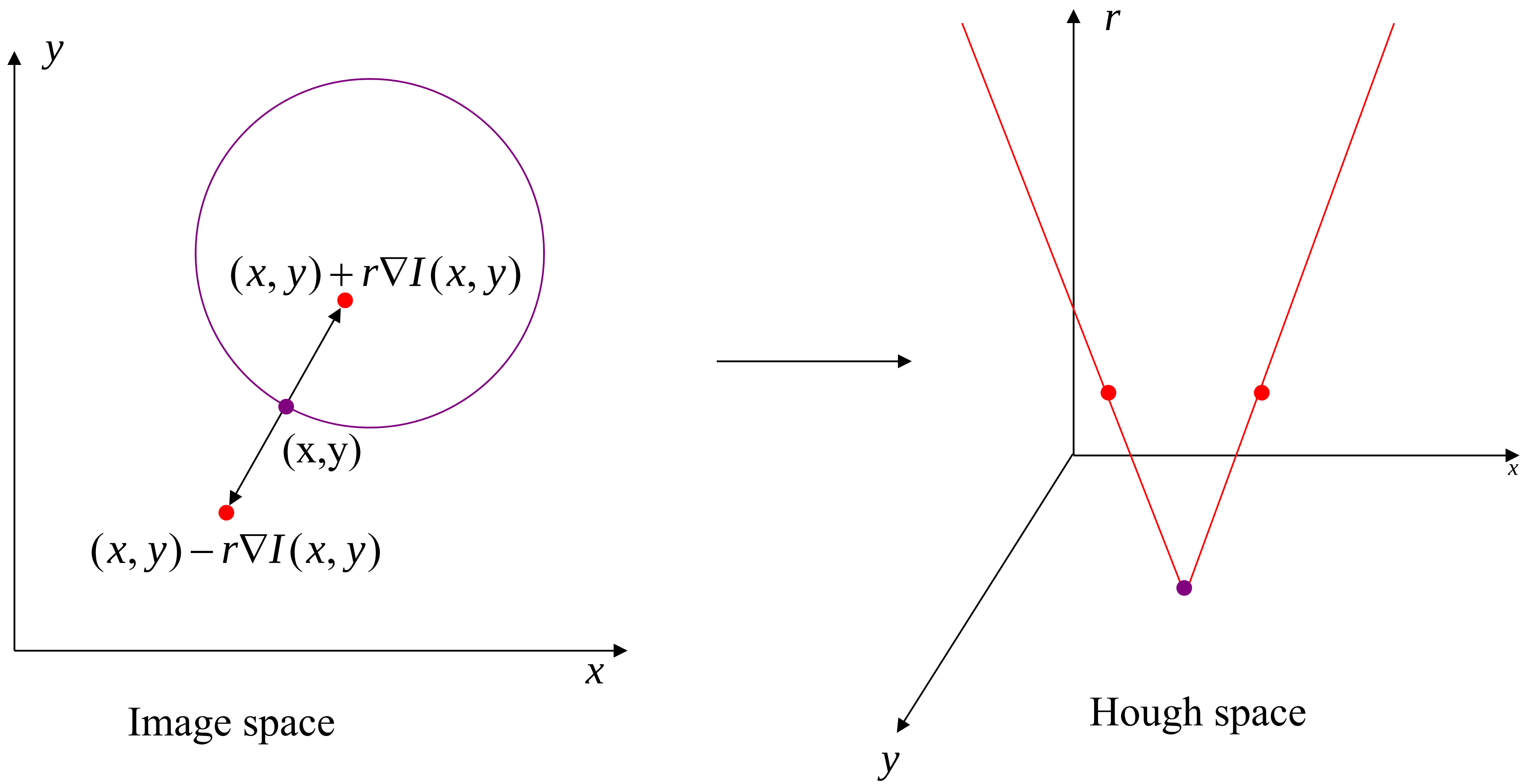
Hough transform for circles

- For an unknown radius r, **known** gradient direction



Hough transform for circles

For every edge pixel (x,y) :

  For each possible radius value r:

    For each possible gradient direction θ:

      // or use estimated gradient at (x,y)

$$a = x - r\cos(\theta) \quad \text{// column}$$

$$b = y + r\sin(\theta) \quad \text{// row}$$

$$H[a,b,r] \mathrel{+}= 1$$

  end

end

Time complexity per edgel?

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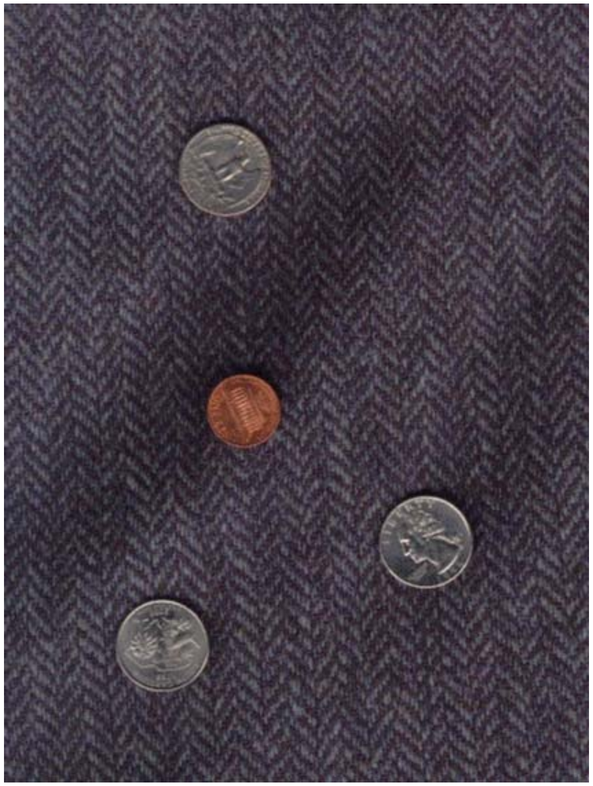
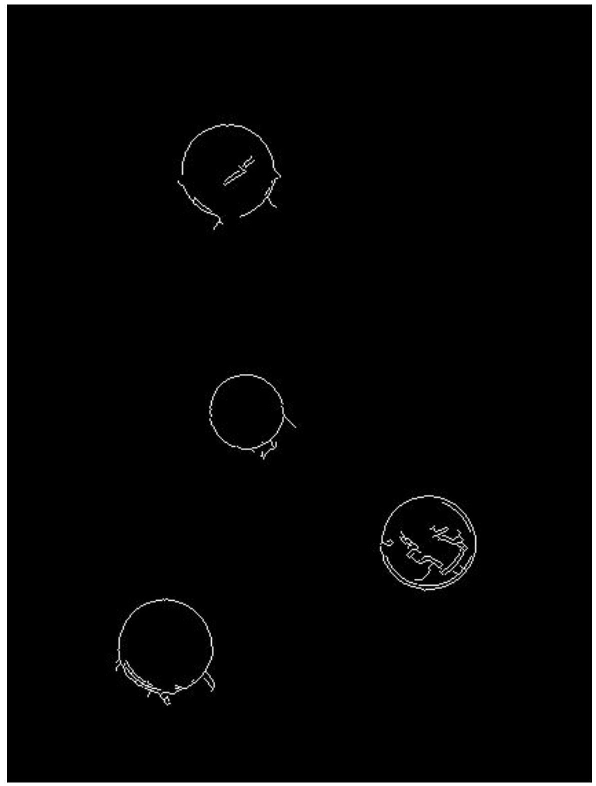
Example: detecting circles with Hough

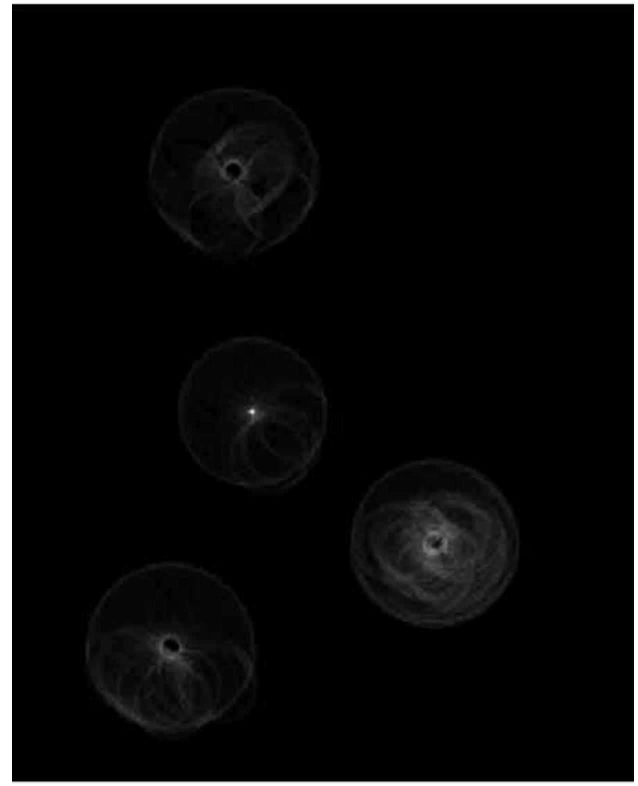
Original



Edges



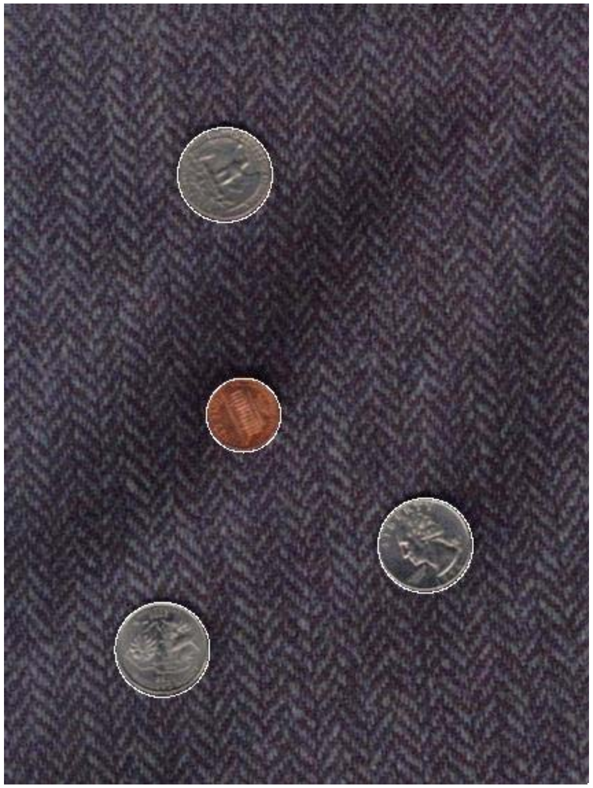
Votes: Penny



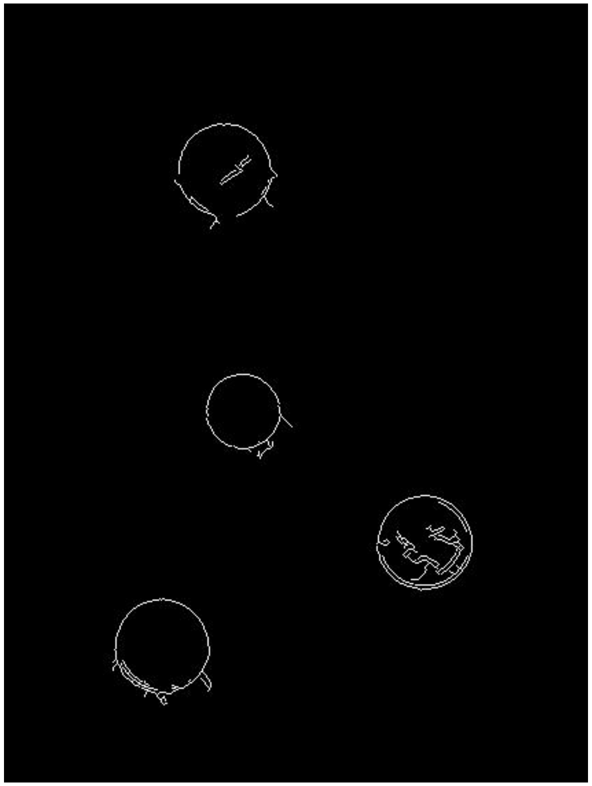
Note: a different Hough transform (with separate accumulators) was used for each circle radius (quarters vs. penny).

Example: detecting circles with Hough

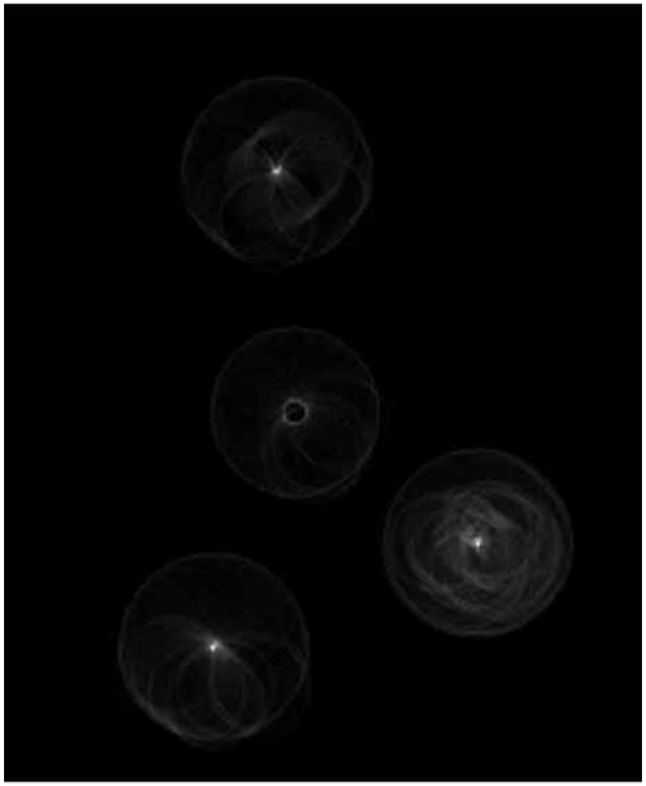
Combined detections



Edges



Votes: Quarter



Coin finding sample images from: Vivek Kwatra

Example: iris detection

Gradient+threshold Hough space (fixed radius) Max detections

- Hemerson Pistori and Eduardo Rocha Costa
  http://rsbweb.nih.gov/ij/plugins/hough-circles.html

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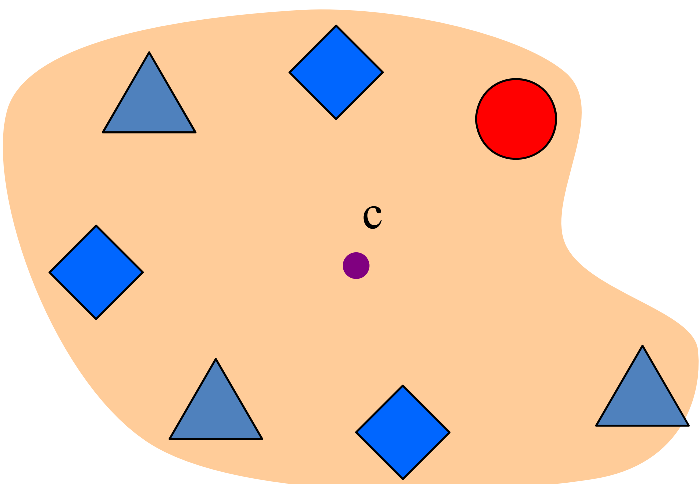
Generalized Hough transform

- We want to find a template defined by its reference point (center) and several distinct types of landmark points in stable spatial configuration

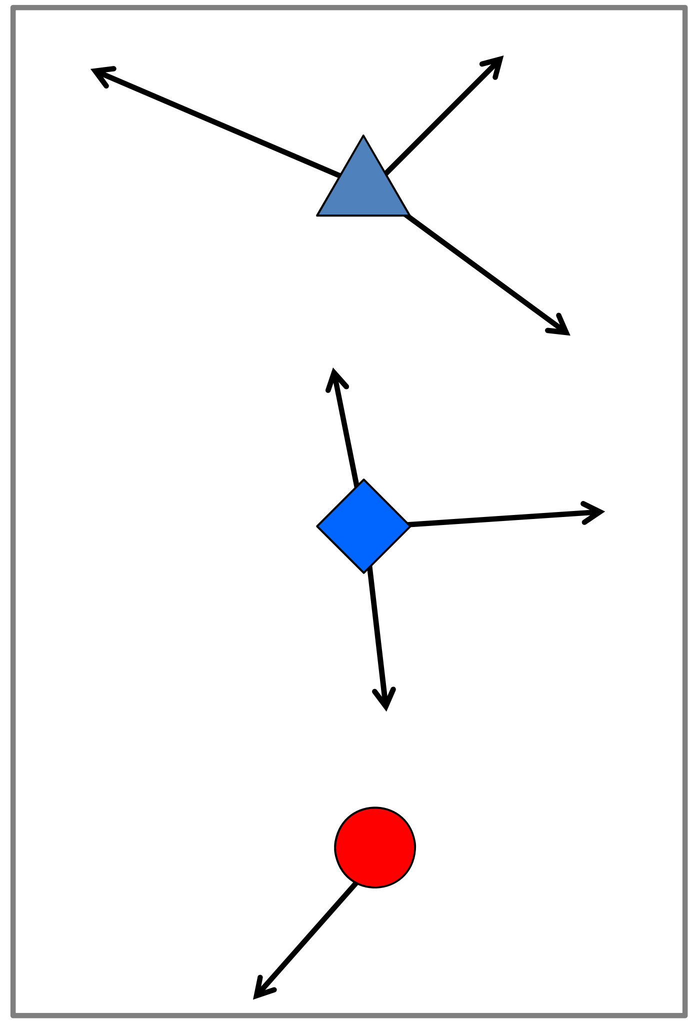
Template



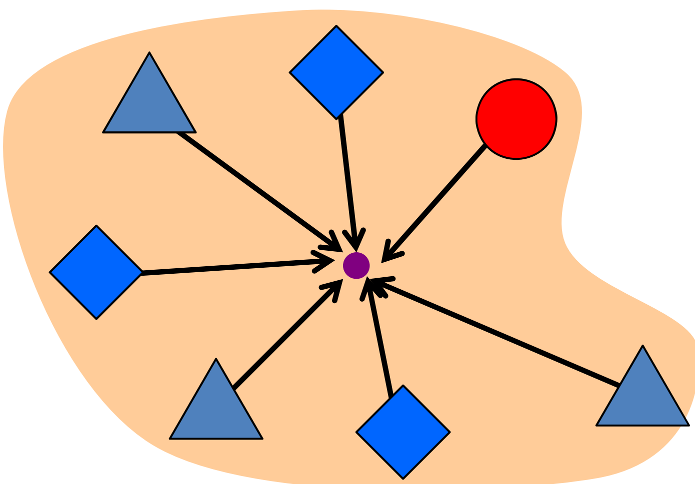
Generalized Hough transform

- Template representation: for each type of landmark point, store all possible displacement vectors towards the center

Model

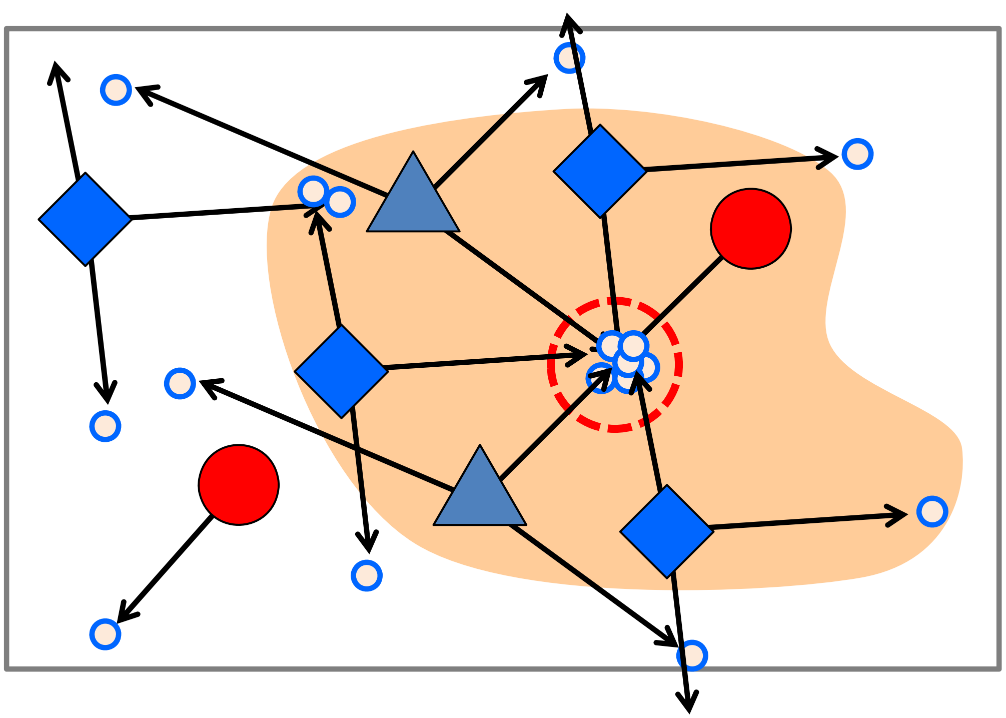


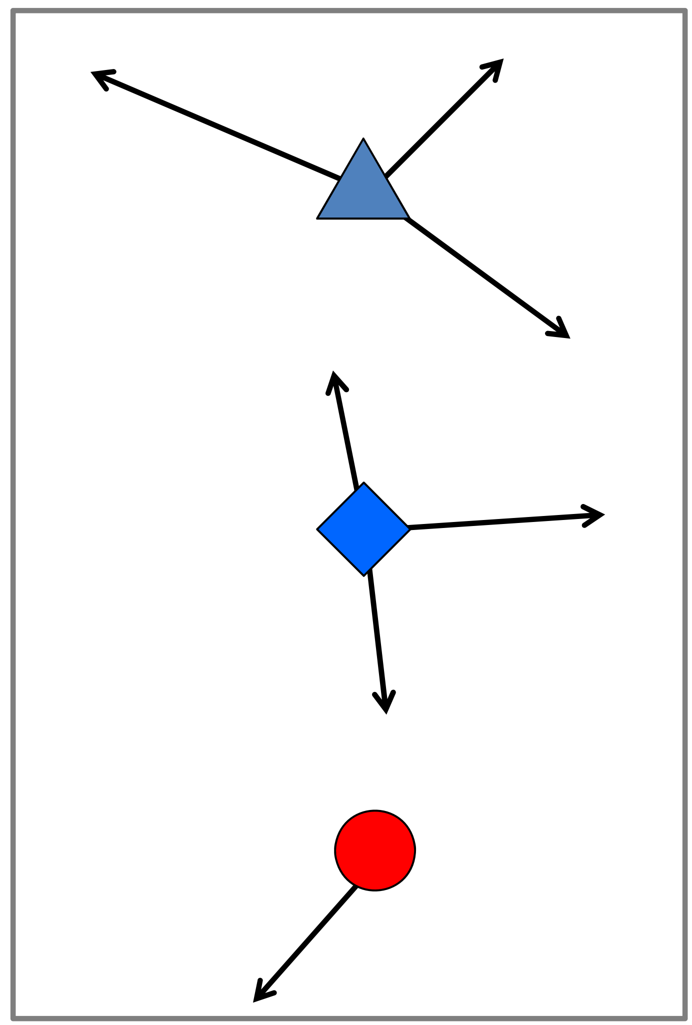
Template



Generalized Hough transform

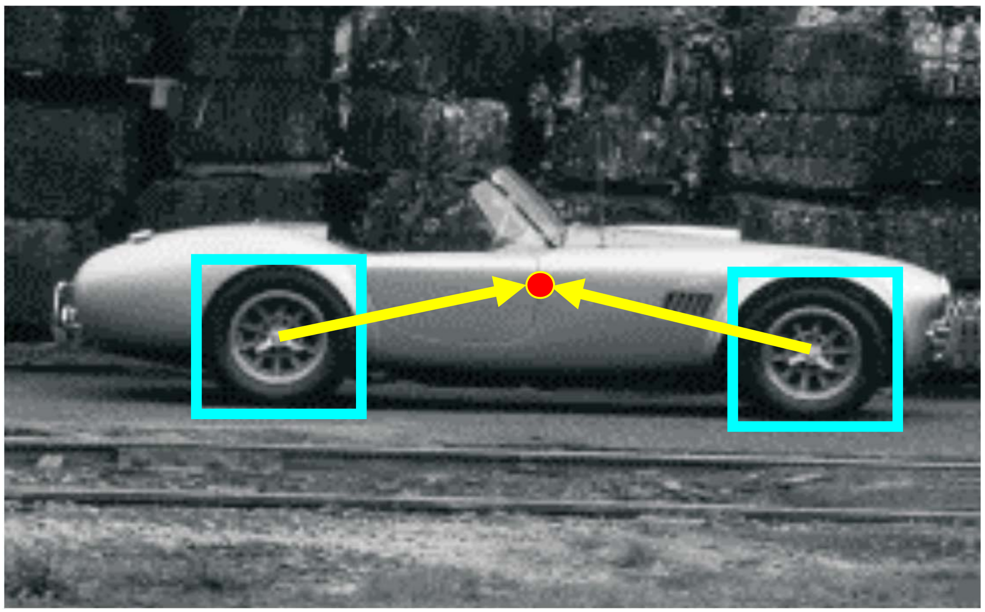
- Detecting the template:
  - For each feature in a new image, look up that feature type in the model and vote for the possible center locations associated with that type in the model



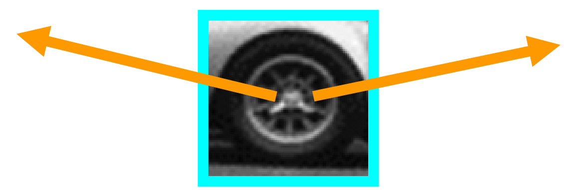


Application in recognition

- Index displacements by "visual codeword"



training image



visual codeword with
displacement vectors

B. Leibe, A. Leonardis, and B. Schiele, Combined Object Categorization and Segmentation with an Implicit Shape Model, ECCV Workshop on Statistical Learning in Computer Vision 2004

Application in recognition

- Index displacements by "visual codeword"

test image

Voting: practical tips

- Minimize irrelevant tokens first
- Choose a good grid / discretization

Too fine ? Too coarse

- Vote for neighbors, also (smoothing in accumulator array)
- Use direction of edge to reduce parameters by 1
- To read back which points voted for “winning” peaks, keep tags on the votes.

Kristen Grauman

Hough transform: Discussion

- Pros
  - All points processed independently
  - Can deal with occlusion and gaps
  - Can detect multiple instances of a model
  - Some robustness to noise: noise points unlikely to contribute consistently to any single bin
- Cons
  - Complexity of search time increases exponentially with the number of model parameters
  - Non-target shapes can produce spurious peaks in parameter space
  - It’s hard to pick a good grid size

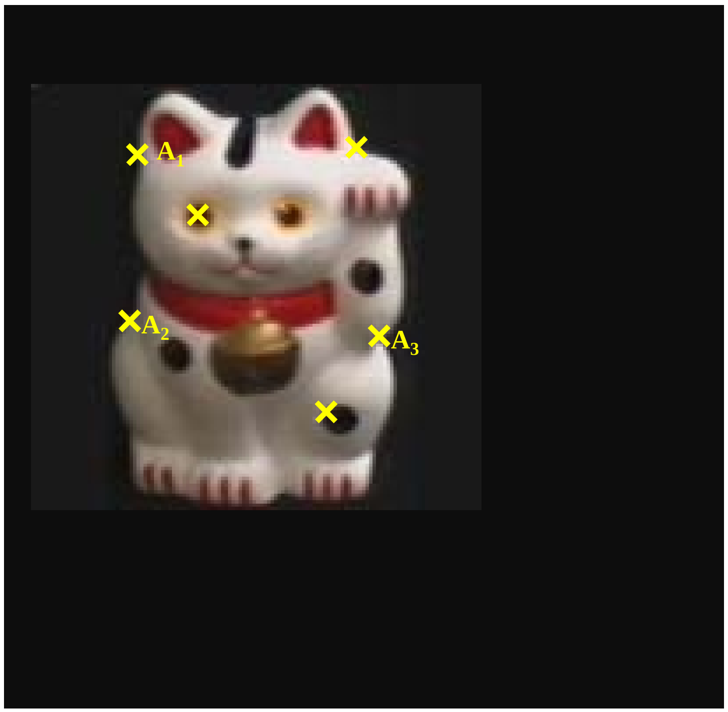
Kristen Grauman

Fitting Algorithm Summary

- Least Squares Fit
  - closed form solution
  - robust to noise
  - not robust to outliers
- Robust Least Squares
  - improves robustness to noise
  - requires iterative optimization
- Hough transform
  - robust to noise and outliers
  - can fit multiple models
  - only works for a few parameters (1-4 typically)
- RANSAC
  - robust to noise and outliers
  - works with a moderate number of parameters (e.g, 1-8)

Derek Hoiem

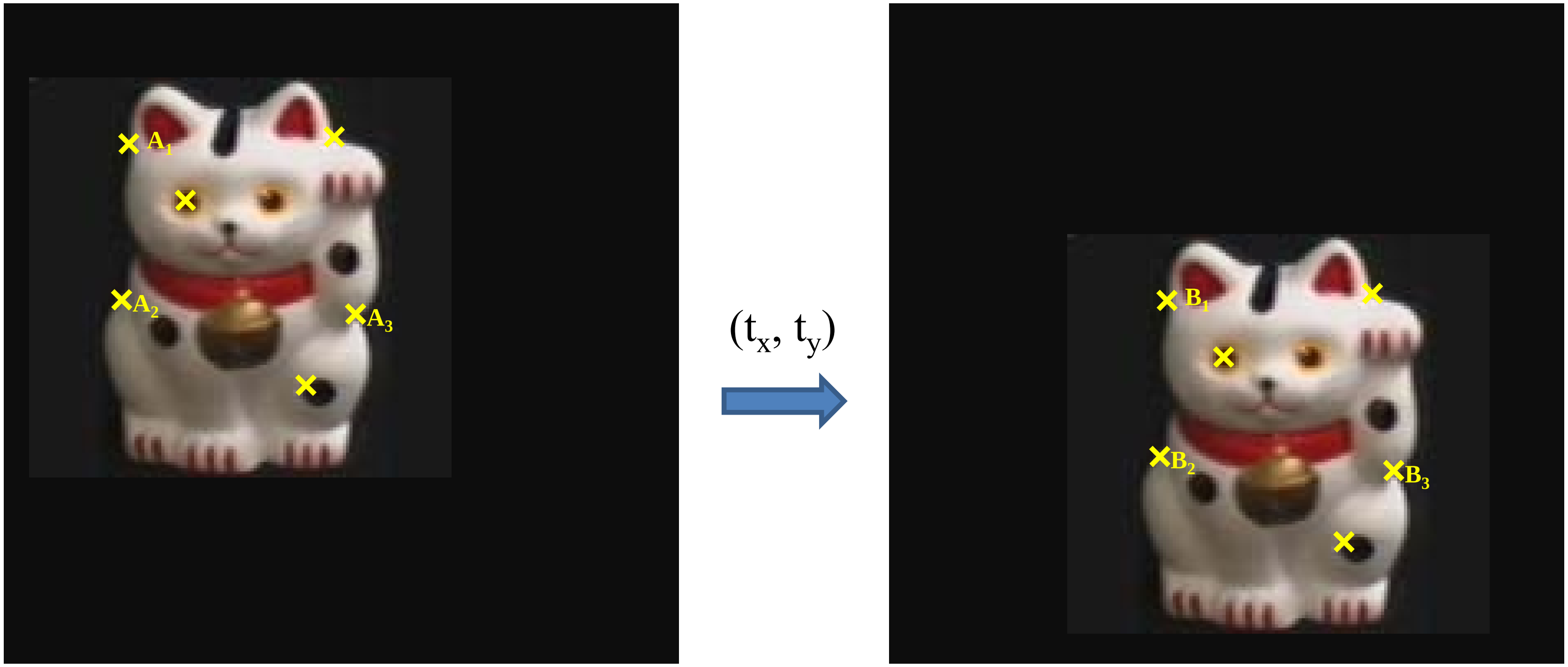
Example: solving for translation



Given matched points in {A} and {B}, estimate the translation of the object

$$\begin{bmatrix} x_i^B \\ y_i^B \end{bmatrix} = \begin{bmatrix} x_i^A \\ y_i^A \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

Example: solving for translation



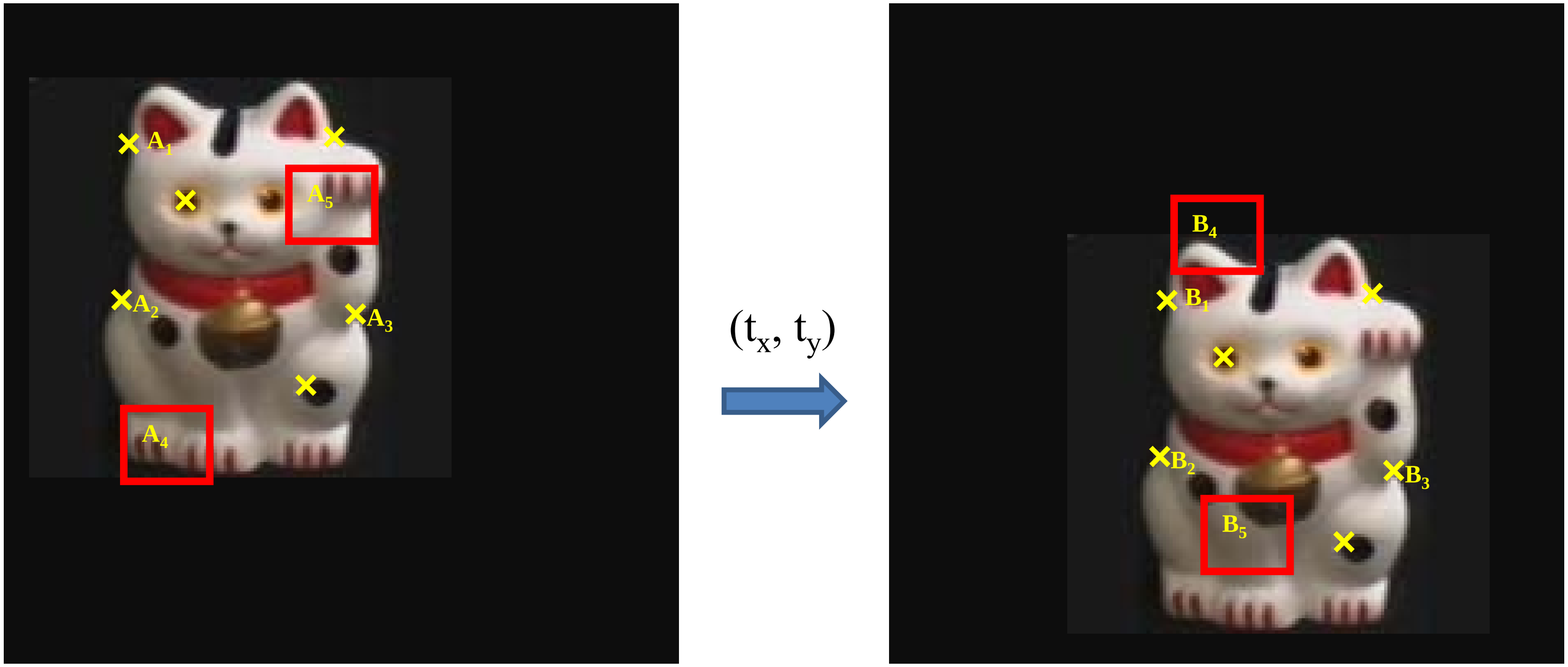
Least squares solution

1. Write down objective function
2. Derived solution
   a) Compute derivative
   b) Compute solution
3. Computational solution
   a) Write in form Ax=b
   b) Solve using pseudo-inverse or eigenvalue decomposition

$$\begin{bmatrix} x_i^B \\ y_i^B \end{bmatrix} = \begin{bmatrix} x_i^A \\ y_i^A \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \vdots & \vdots \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} t_x \\ t_y \end{bmatrix} = \begin{bmatrix} x_1^B - x_1^A \\ y_1^B - y_1^A \\ \vdots \\ x_n^B - x_n^A \\ y_n^B - y_n^A \end{bmatrix}$$

Example: solving for translation



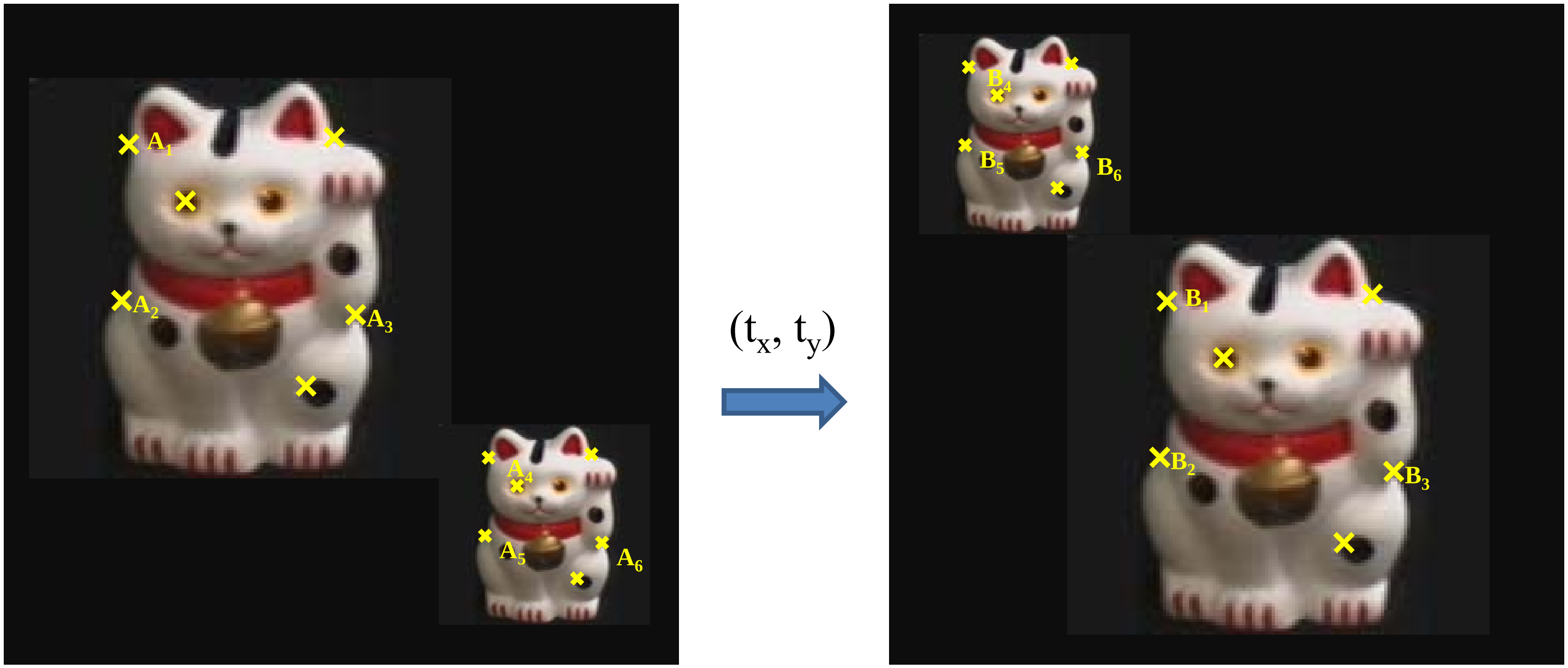
Problem: outliers

RANSAC solution

1. Sample a set of matching points (1 pair)
2. Solve for transformation parameters
3. Score parameters with number of inliers
4. Repeat steps 1-3 N times

$$\begin{bmatrix} x_i^B \\ y_i^B \end{bmatrix} = \begin{bmatrix} x_i^A \\ y_i^A \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

Example: solving for translation



Problem: outliers, multiple objects, and/or many-to-one matches

Hough transform solution

1. Initialize a grid of parameter values
2. Each matched pair casts a vote for consistent values
3. Find the parameters with the most votes
4. Solve using least squares with inliers

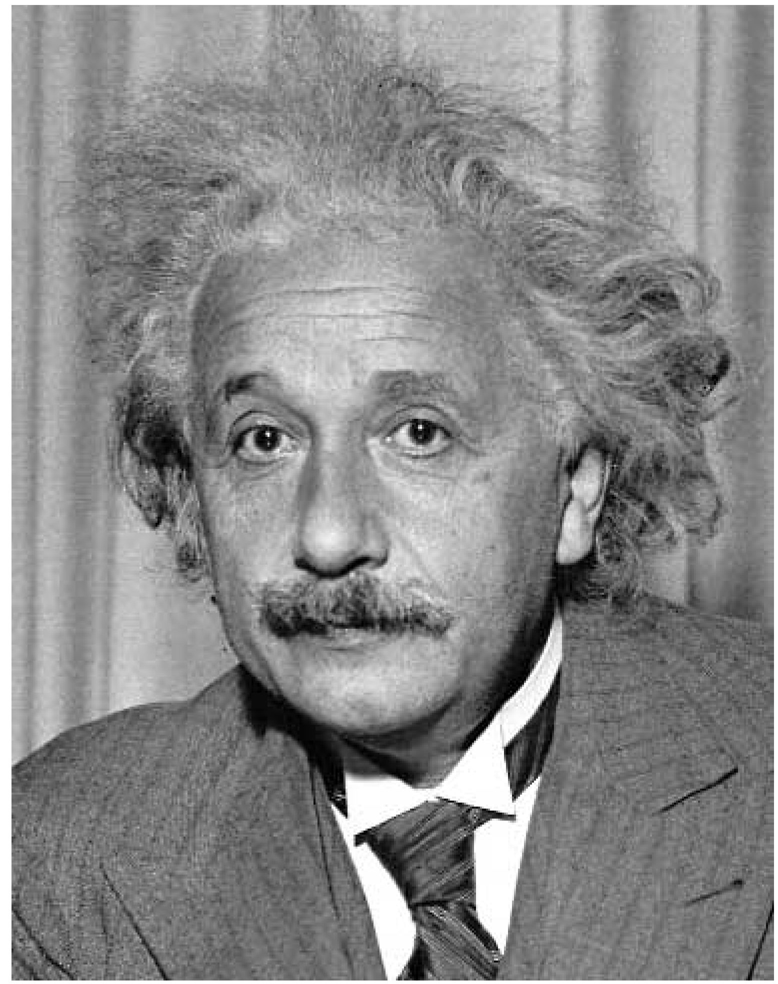
$$\begin{bmatrix} x_i^B \\ y_i^B \end{bmatrix} = \begin{bmatrix} x_i^A \\ y_i^A \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

Template Matching

Slides based on D. Hoiem's slides

Template matching

- Goal: find  in image



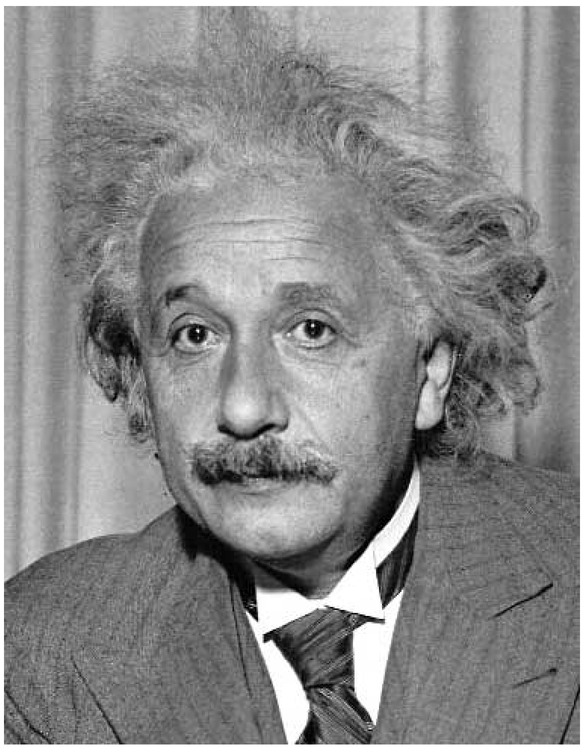
- Main challenge: What is a good similarity or distance measure between two patches?
  - Filtering
  - Zero-mean filtering
  - Sum of Squares Difference
  - Normalized Cross Correlation

Matching with filters

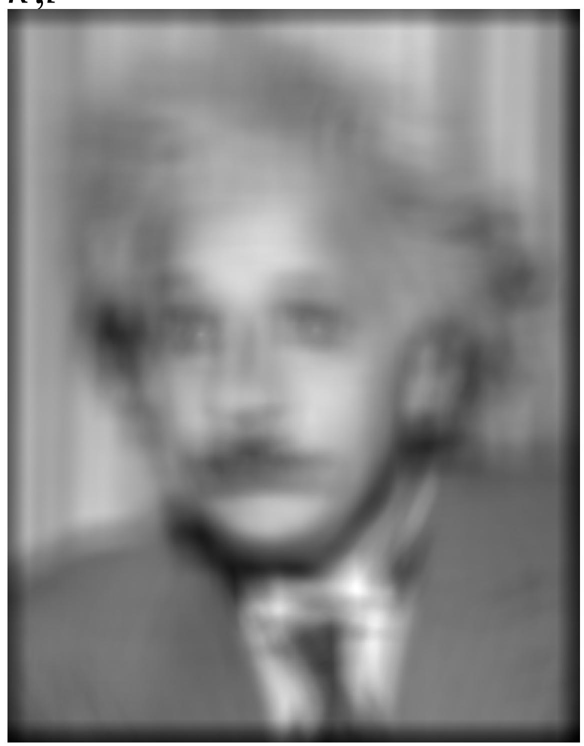
- Goal: find  in image
- Method 0: filter the image with eye patch

$$h[m,n]=\sum_{k,l} g[k,l]\ f[m+k,n+l]$$

f = image
g = filter



Input



Filtered Image

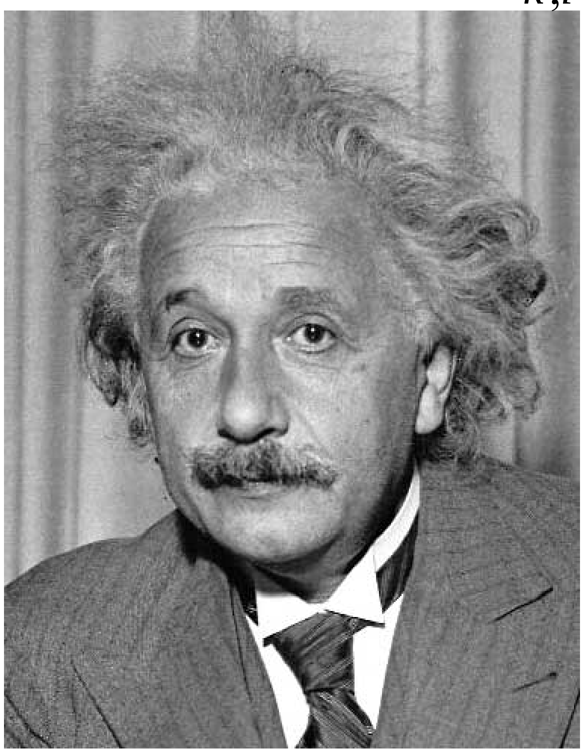
What went wrong?

Matching with filters

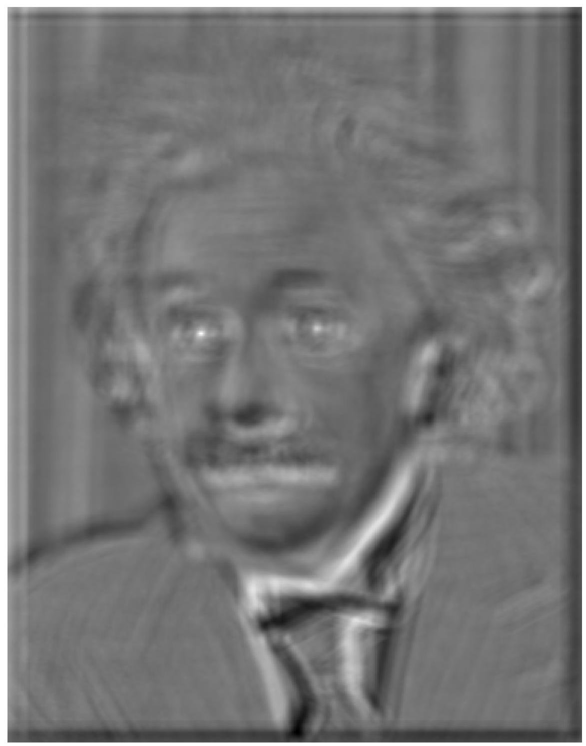
- Goal: find  in image
- Method 1: filter the image with zero-mean eye

$$h[m,n] = \sum_{k,l} (g[k,l] - \bar{g})\,(f[m+k,n+l])$$

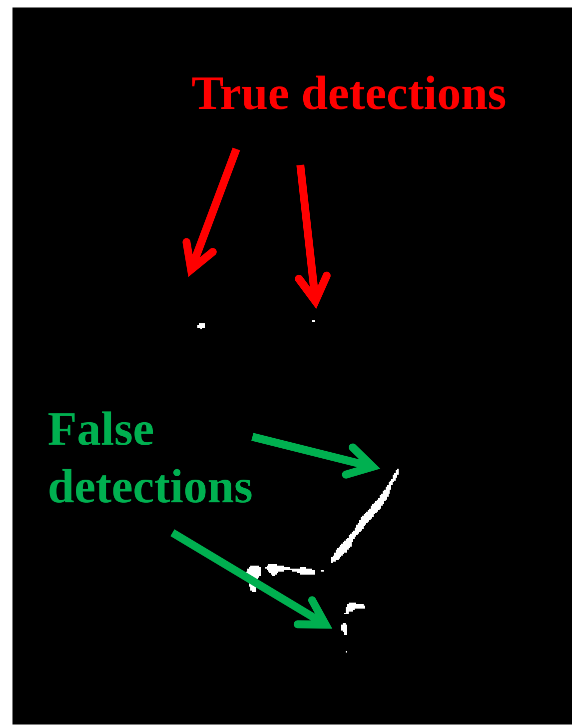
mean of template g



Input



Filtered Image (scaled)

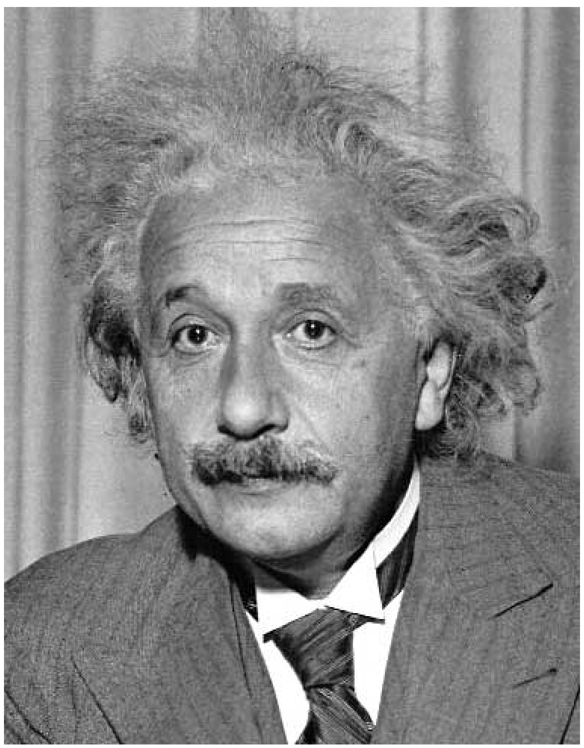


Thresholded Image

SSD

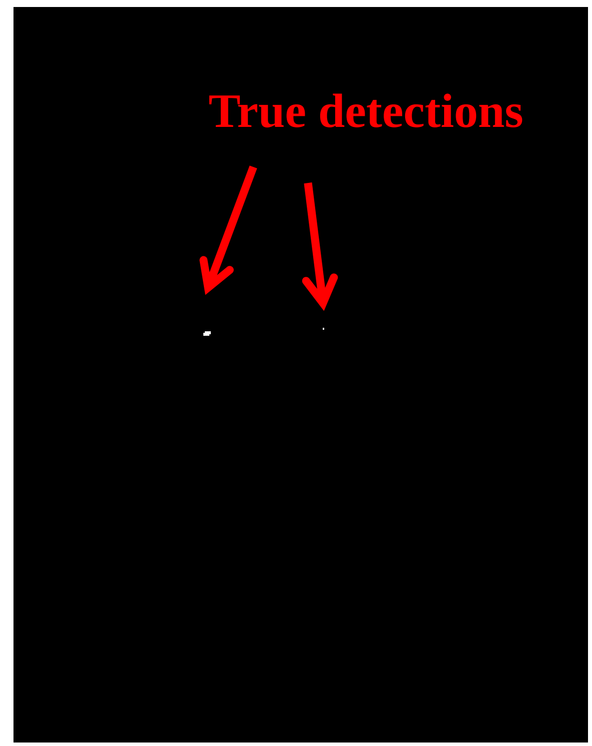
- Goal: find  in image
- Method 2: SSD

$$h[m,n] = \sum_{k,l} (g[k,l] - f[m+k, n+l])^2$$



Input

1- sqrt(SSD)



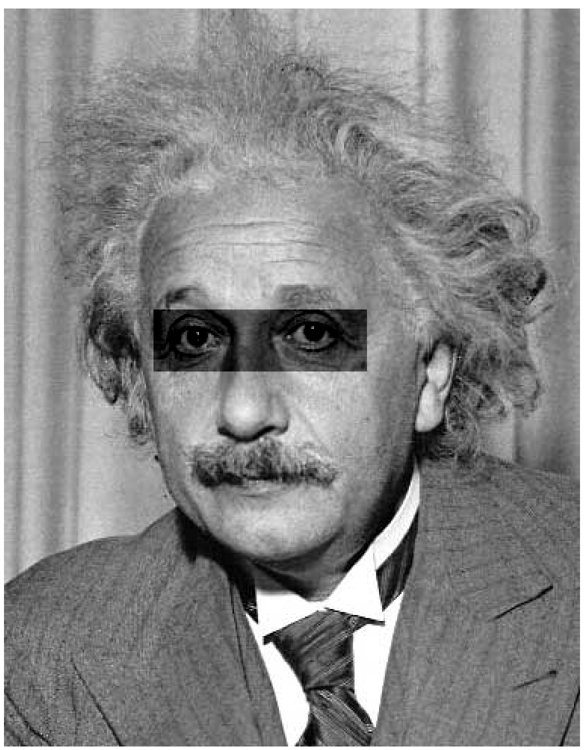
Thresholded Image

SSD

- Goal: find  in image
- Method 2: SSD

What's the potential downside of SSD?

$$h[m,n] = \sum_{k,l} (g[k,l] - f[m+k,n+l])^2$$



Input

1- sqrt(SSD)

NCC

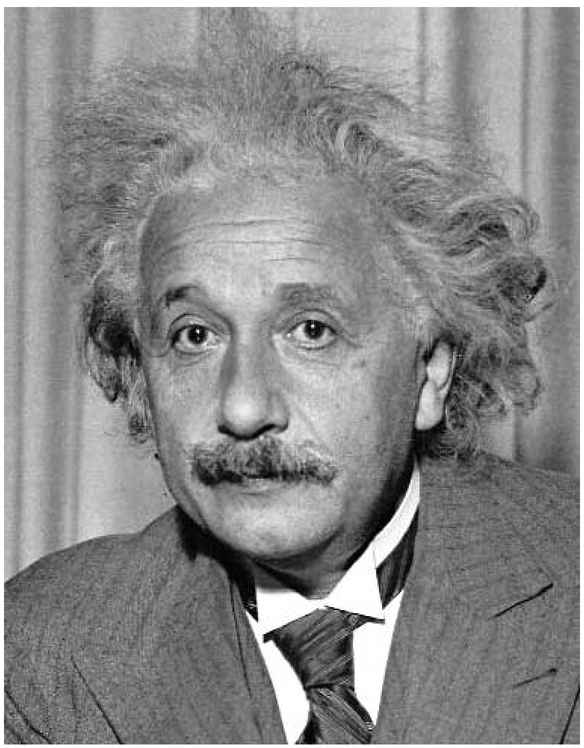
- Goal: find  in image
- Method 3: Normalized cross-correlation

$$h[m,n] = \frac{\sum_{k,l} (g[k,l] - \overset{\substack{\text{mean template} \\ \downarrow}}{\bar{g}})(f[m+k,n+l] - \overset{\substack{\text{mean image patch} \\ \downarrow}}{\bar{f}}_{m,n})}{\left(\sum_{k,l} (g[k,l] - \bar{g})^2 \sum_{k,l} (f[m+k,n+l] - \bar{f}_{m,n})^2 \right)^{0.5}}$$

Matlab: `normxcorr2(template, im)`

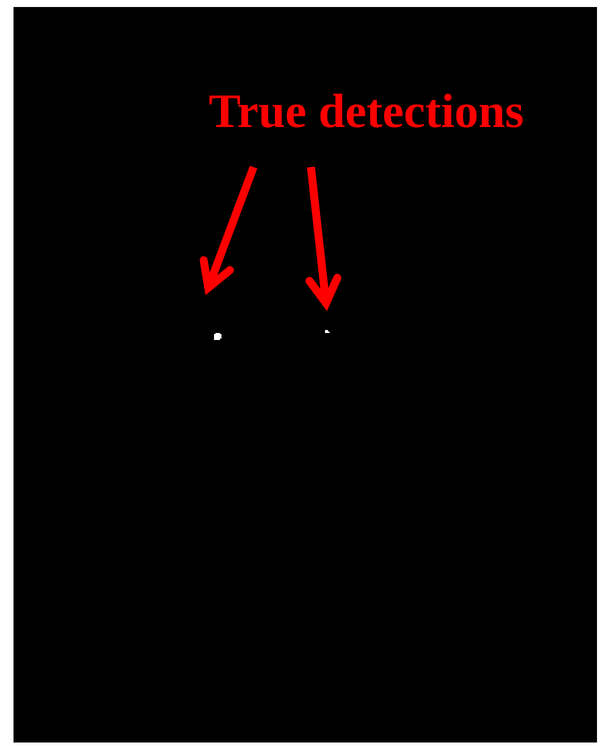
NCC

- Goal: find  in image
- Method 3: Normalized cross-correlation



Input

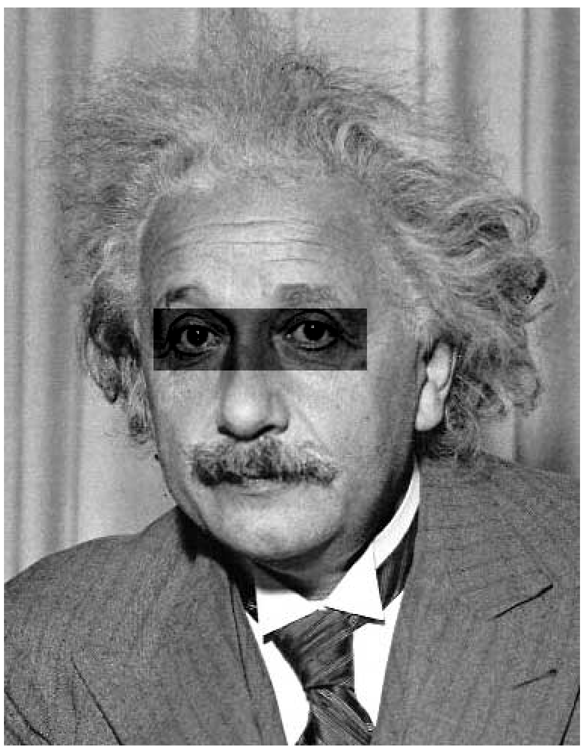
Normalized X-Correlation



Thresholded Image

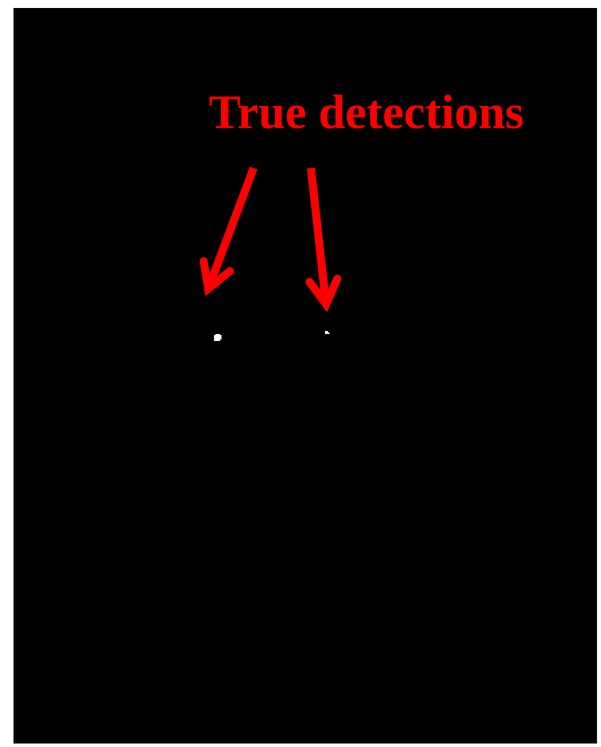
NCC

- Goal: find  in image
- Method 3: Normalized cross-correlation



Input

Normalized X-Correlation



Thresholded Image

Q: What is the best method to use?

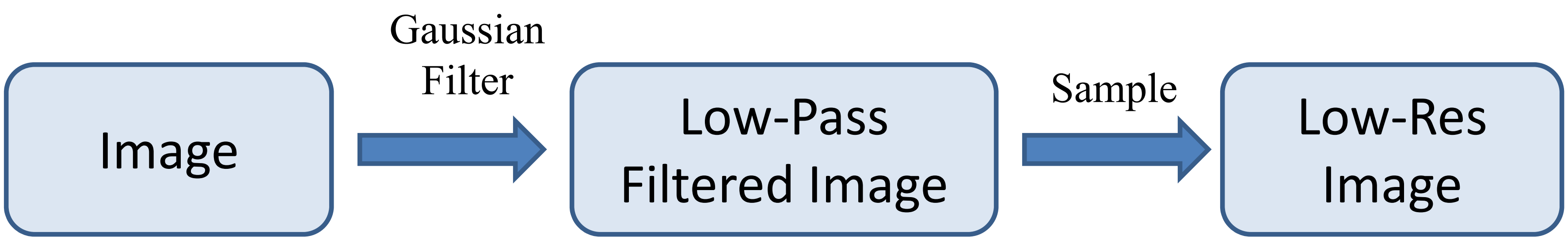
A: Depends

- Zero-mean filter: fastest but not a great matcher
- SSD: next fastest, sensitive to overall intensity
- Normalized cross-correlation: slowest, invariant to local average intensity and contrast

Q: What if we want to find larger or smaller eyes?

A: Image Pyramid

Review of Sampling



Gaussian pyramid

512 256 128 64 32 16 8

Source: D. Forsyth

Template Matching with Image Pyramids

Input: Image, Template

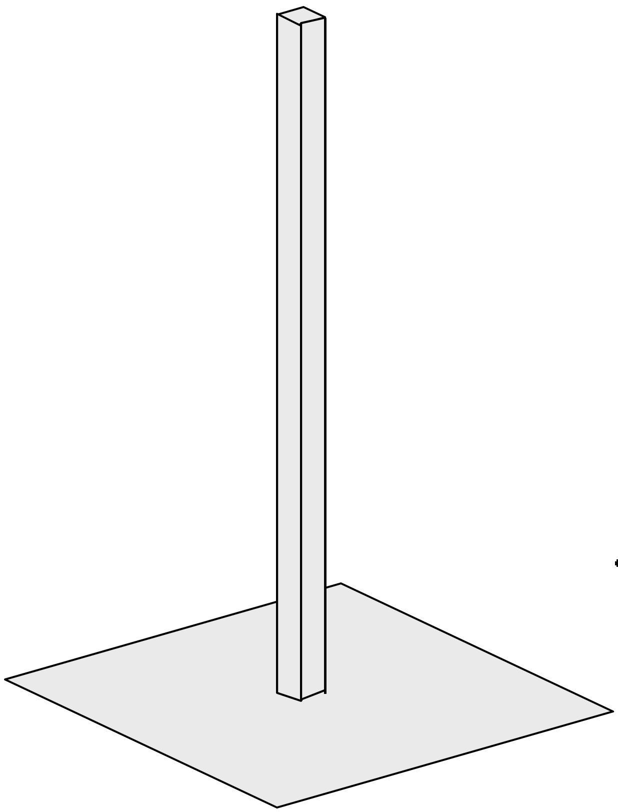
1. Match template at current scale

2. Downsample image
   – In practice, scale step of 1.1 to 1.2

3. Repeat 1-2 until image is very small

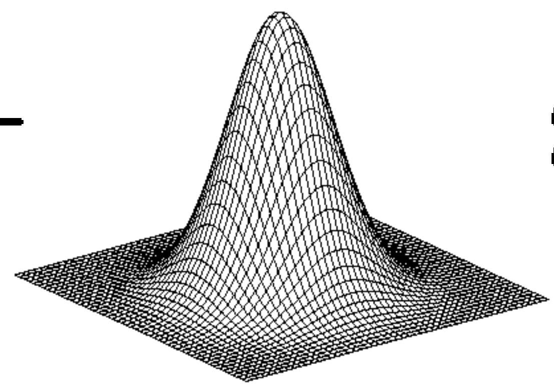
4. Take responses above some threshold, perhaps with non-maxima suppression

Laplacian filter



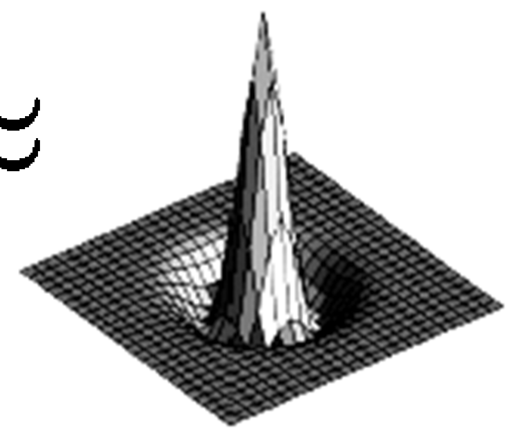
unit impulse

$-$



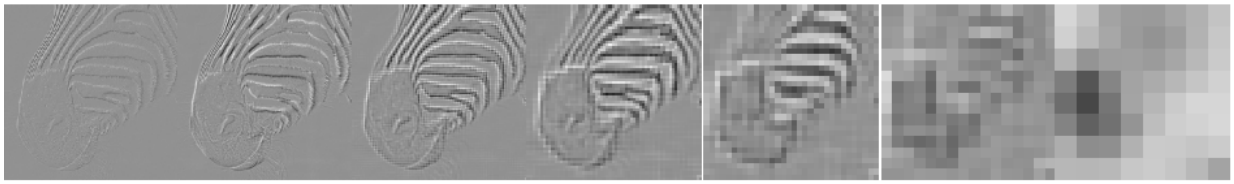
Gaussian

$\approx$

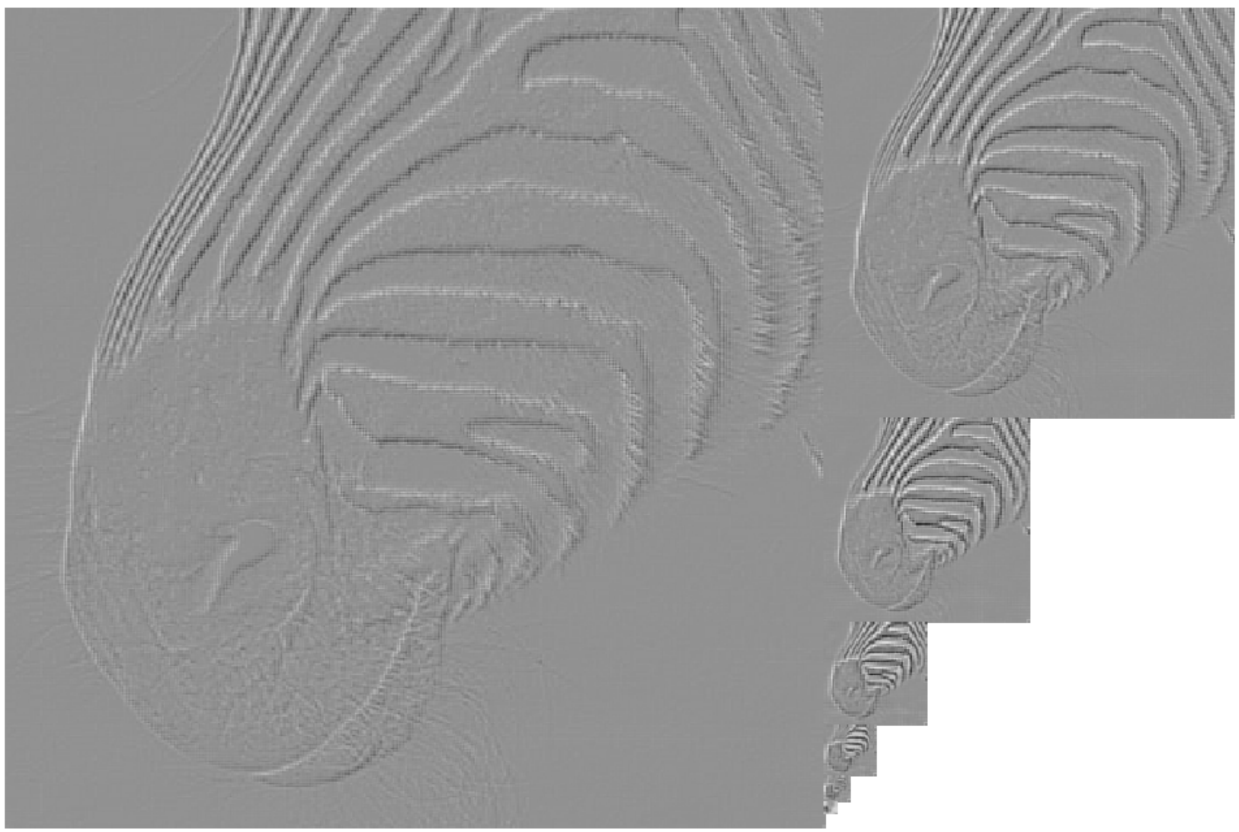


Laplacian of Gaussian

Laplacian pyramid

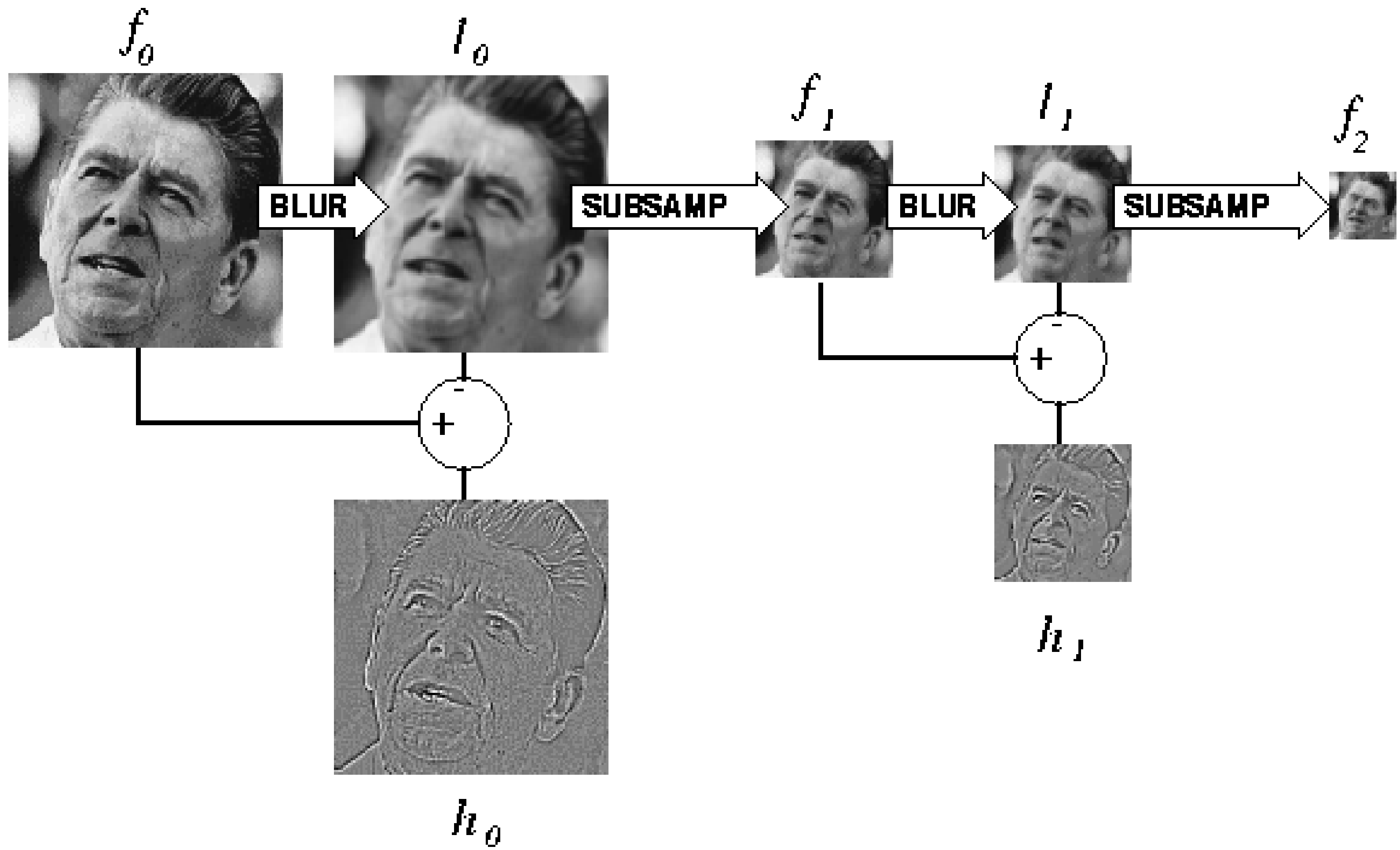


512 256 128 64 32 16 8



Source: D. Forsyth

Computing Gaussian/Laplacian Pyramid



Major uses of image pyramids

- Compression
- Object detection
  - Scale search
  - Features
- Detecting stable interest points
- Registration
  - Course-to-fine

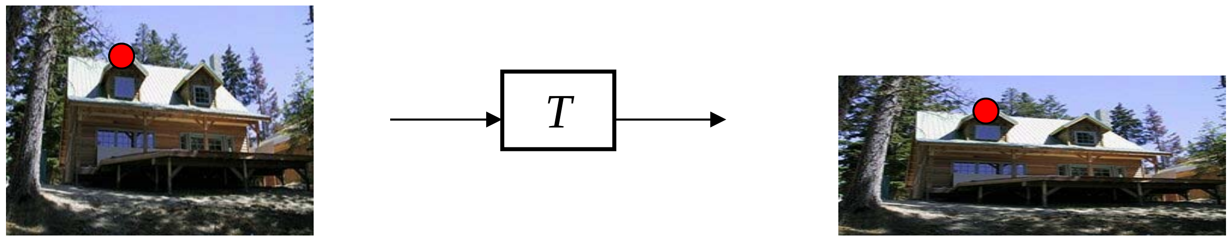
Alignment

Slides based on D. Hoiem's and S. Lazebnik's slides

Alignment

- Alignment: find parameters of model that maps one set of points to another
- Typically want to solve for a global transformation that accounts for most true correspondences
- Difficulties
  - Noise (perturbation around true features, matches, etc.)
  - Outliers
  - Many-to-one matches or multiple objects

Parametric (global) warping



p = (x,y) **p'** = (x',y')

Transformation T is a coordinate change

$$p' = T(p)$$

What does it mean that *T* is global?

- Is the same for any point p
- can be described by just a few numbers (parameters)

For linear transformations, we can represent T as a matrix

$$p' = Tp$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \mathbf{T} \begin{bmatrix} x \\ y \end{bmatrix}$$

Common transformations

original

Transformed

translation

rotation

aspect

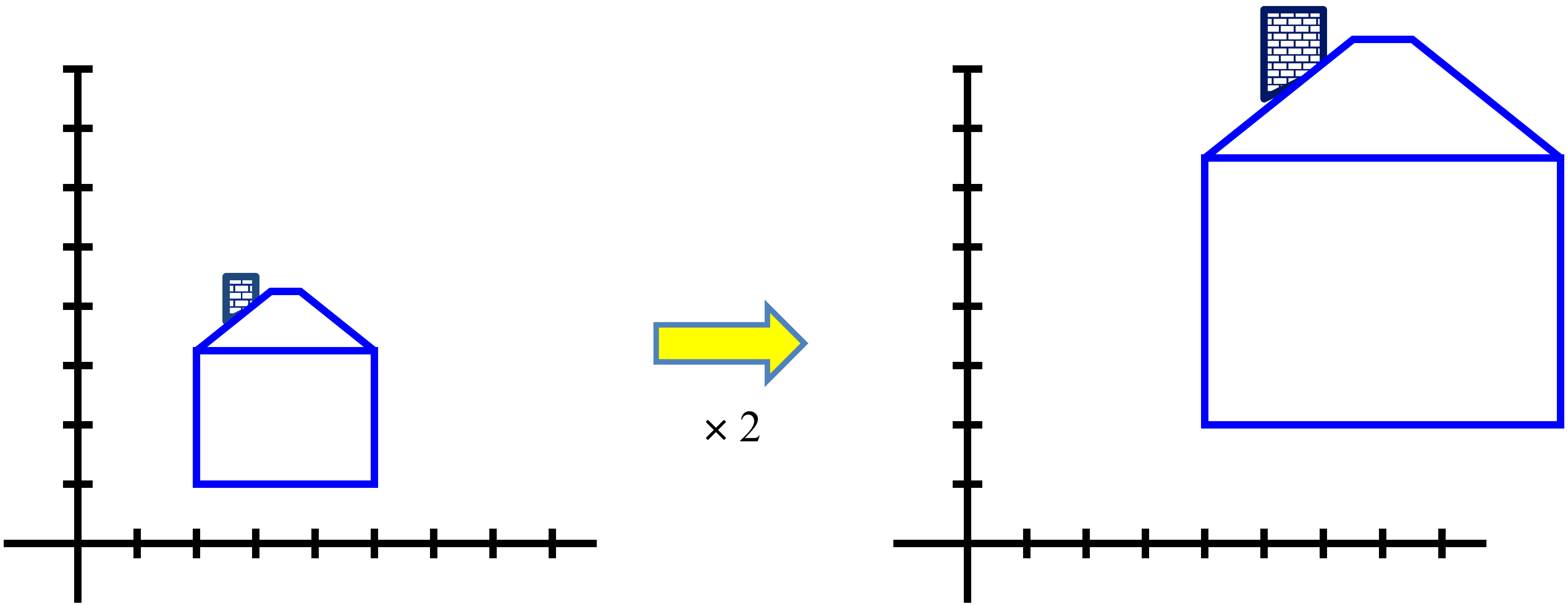
affine

perspective

Slide credit (next few slides): A. Efros and/or S. Seitz

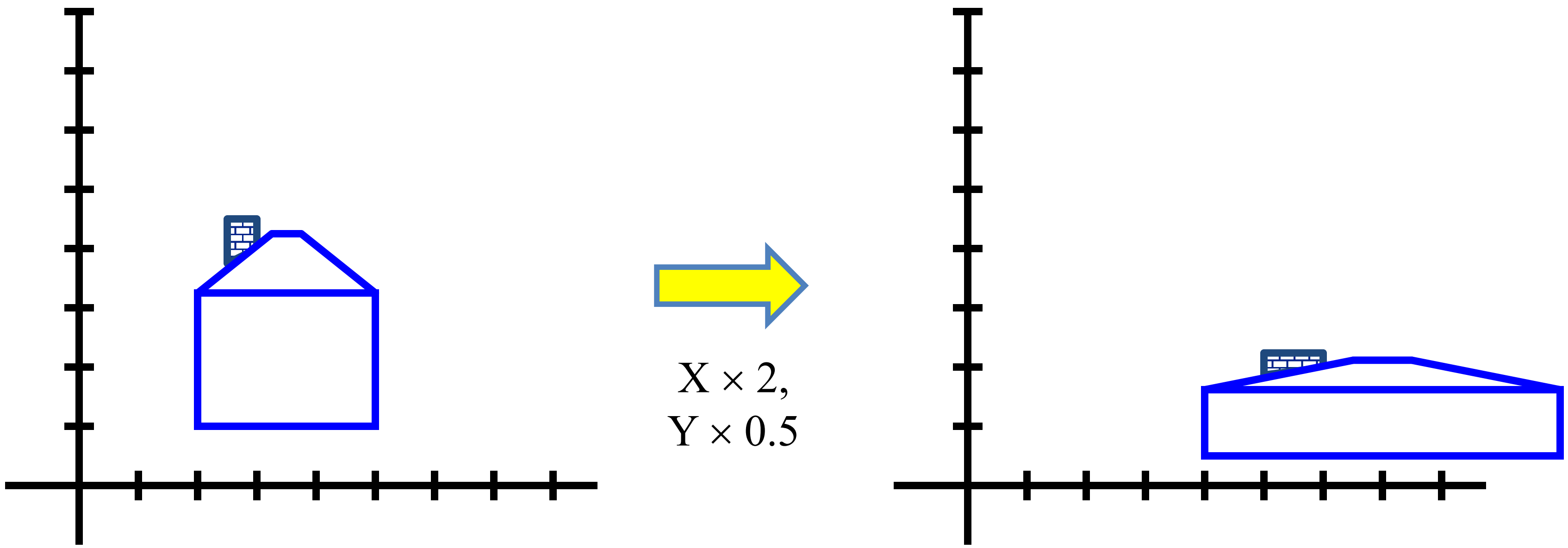
Scaling

- *Scaling* a coordinate means multiplying each of its components by a scalar
- *Uniform scaling* means this scalar is the same for all components:



Scaling

- *Non-uniform scaling*: different scalars per component:



Scaling

- Scaling operation:

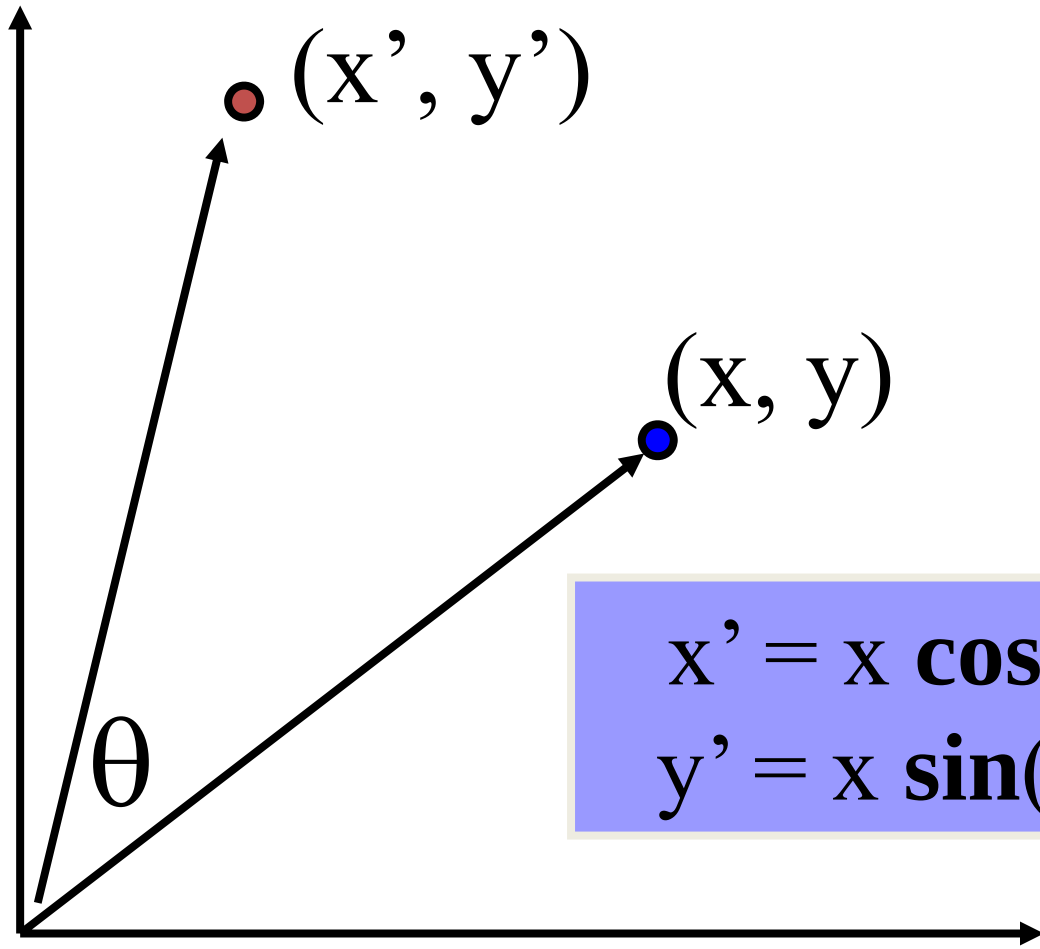
$$x' = ax$$

$$y' = by$$

- Or, in matrix form:

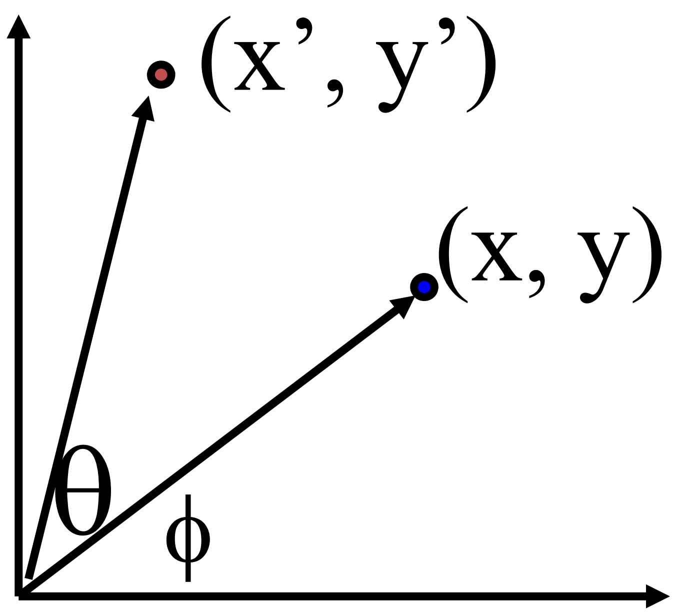
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}}_{\textit{scaling matrix } S} \begin{bmatrix} x \\ y \end{bmatrix}$$

2-D Rotation



$$x' = x\,\mathbf{cos}(\theta) - y\,\mathbf{sin}(\theta)$$
$$y' = x\,\mathbf{sin}(\theta) + y\,\mathbf{cos}(\theta)$$

2-D Rotation



Polar coordinates…

$x = r \cos(\phi)$

$y = r \sin(\phi)$

$x' = r \cos(\phi + \theta)$

$y' = r \sin(\phi + \theta)$

Trig Identity…

$x' = r \cos(\phi) \cos(\theta) - r \sin(\phi) \sin(\theta)$

$y' = r \sin(\phi) \cos(\theta) + r \cos(\phi) \sin(\theta)$

Substitute…

$x' = x \mathbf{cos}(\theta) - y \mathbf{sin}(\theta)$

$y' = x \mathbf{sin}(\theta) + y \mathbf{cos}(\theta)$

2-D Rotation

This is easy to capture in matrix form:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}}_{\mathbf{R}} \begin{bmatrix} x \\ y \end{bmatrix}$$

Even though sin(θ) and cos(θ) are nonlinear functions of θ,

- ***x' is a linear combination of x and y***
- ***y' is a linear combination of x and y***

What is the inverse transformation?

- Rotation by -θ
- For rotation matrices $\mathbf{R}^{-1} = \mathbf{R}^T$

Basic 2D transformations

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & \alpha_x \\ \alpha_y & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Shear

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\Theta & -\sin\Theta \\ \sin\Theta & \cos\Theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Rotate

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translate

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Affine

Affine is any combination of translation, scale, rotation, shearing

Affine Transformations

Affine transformations are combinations of

- Linear transformations, and
- Translations

Properties of affine transformations:

- Lines map to lines
- Parallel lines remain parallel
- Ratios are preserved
- Closed under composition

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

or

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Projective Transformations

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

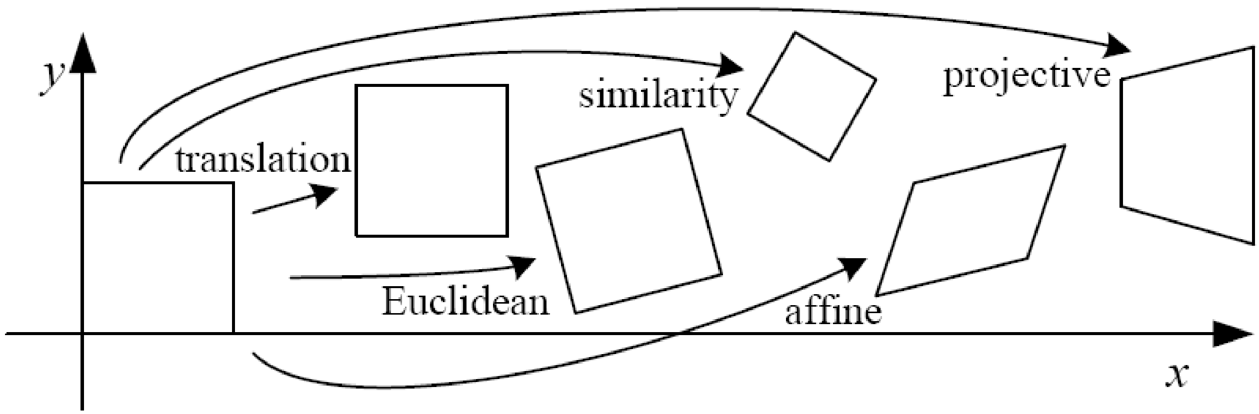
Projective transformations are combos of

- Affine transformations, and
- Projective warps

Properties of projective transformations:

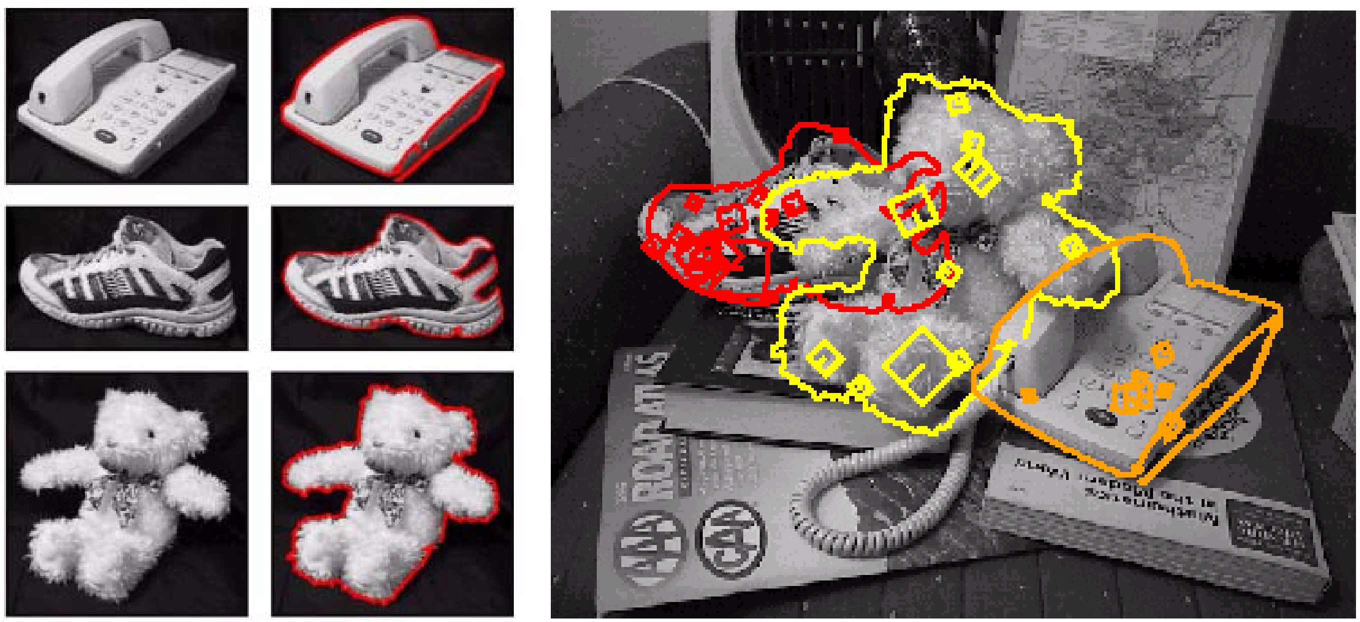
- Lines map to lines
- Parallel lines do not necessarily remain parallel
- Ratios are not preserved
- Closed under composition
- Models change of basis
- Projective matrix is defined up to a scale (8 DOF)

2D image transformations (reference table)



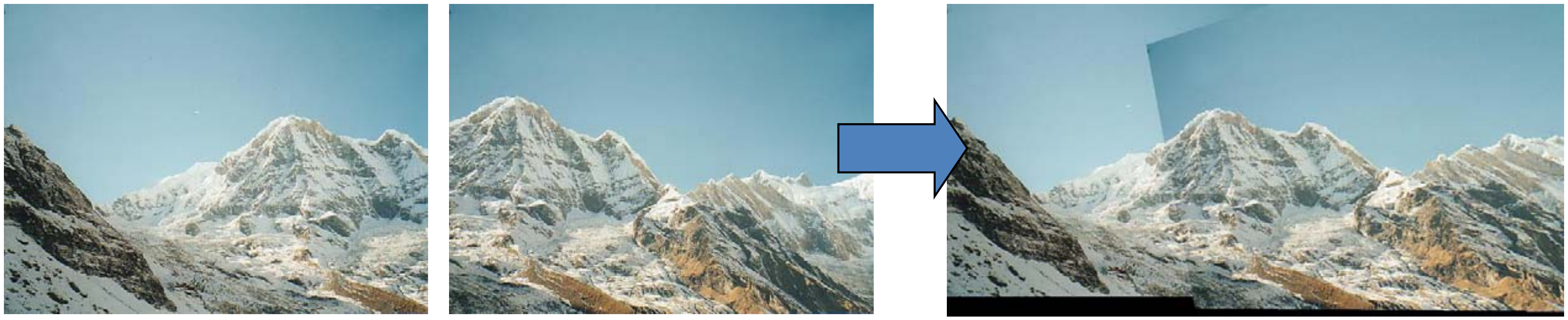
Name	Matrix	# D.O.F.	Preserves:	Icon
translation	$\left[\begin{array}{c\|c} \boldsymbol{I} & \boldsymbol{t} \end{array}\right]_{2\times 3}$	2	orientation $+\cdots$	
rigid (Euclidean)	$\left[\begin{array}{c\|c} \boldsymbol{R} & \boldsymbol{t} \end{array}\right]_{2\times 3}$	3	lengths $+\cdots$	
similarity	$\left[\begin{array}{c\|c} s\boldsymbol{R} & \boldsymbol{t} \end{array}\right]_{2\times 3}$	4	angles $+\cdots$	
affine	$\left[\begin{array}{c} \boldsymbol{A} \end{array}\right]_{2\times 3}$	6	parallelism $+\cdots$	
projective	$\left[\begin{array}{c} \tilde{\boldsymbol{H}} \end{array}\right]_{3\times 3}$	8	straight lines	

Image alignment: Applications



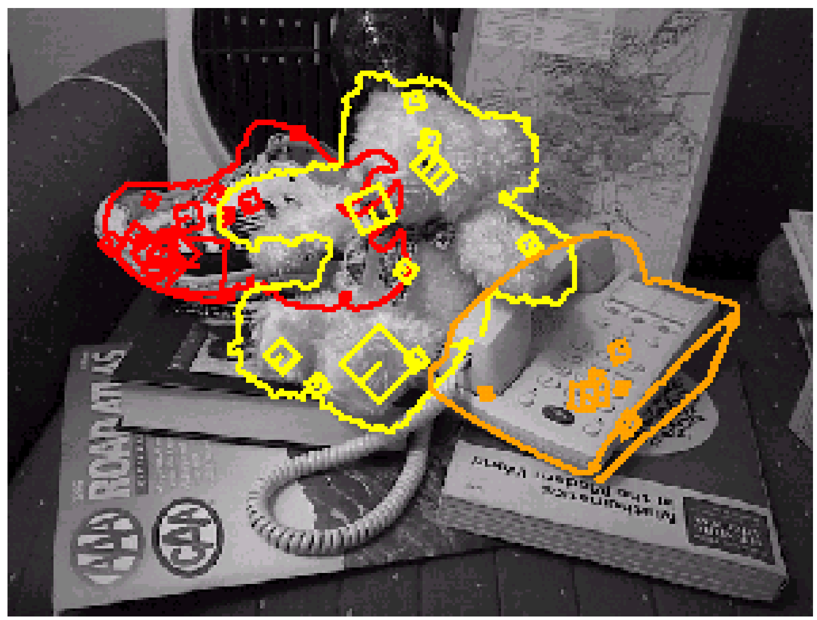
Recognition of object instances

Image alignment: Challenges



Small degree of overlap

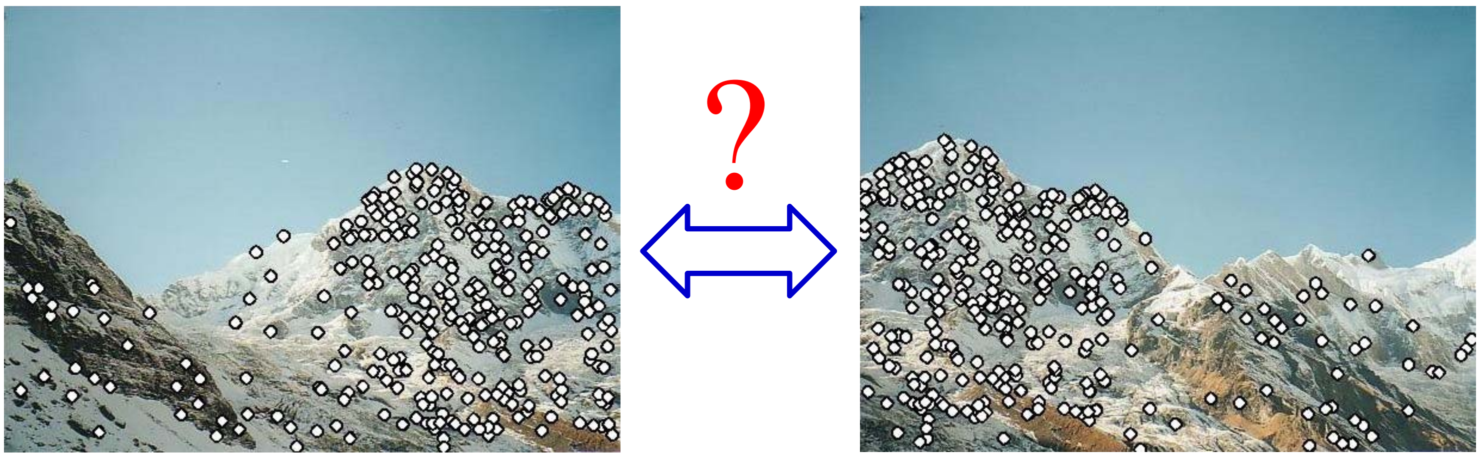
Intensity changes



Occlusion,
clutter

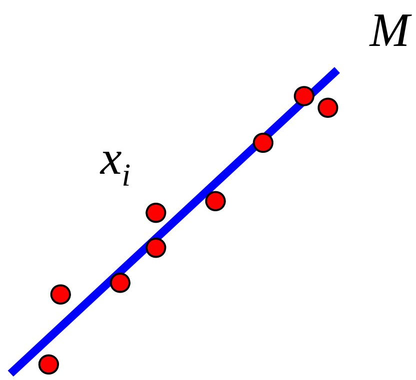
Feature-based alignment

- Search sets of feature matches that agree in terms of:
  a) Local appearance
  b) Geometric configuration



Alignment as fitting

- Previously: fitting a model to features in one image

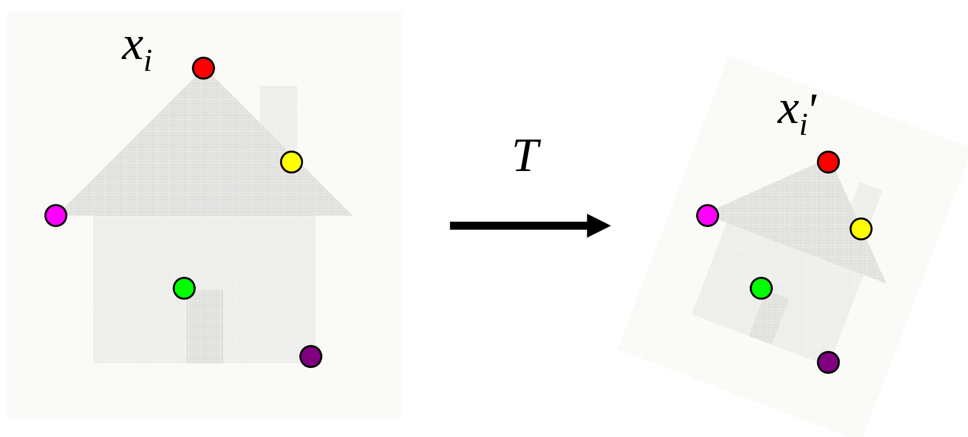


Find model *M* that minimizes

$$\sum_i \mathrm{residual}(x_i, M)$$

Alignment as fitting

- Alignment: fitting a model to a transformation between pairs of features (*matches*) in two images

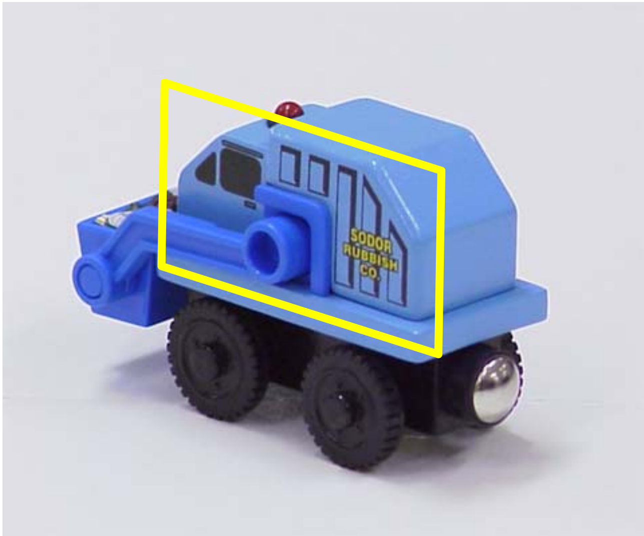


Find transformation T that minimizes

$$\sum_i \text{residual}(T(x_i), x_i')$$

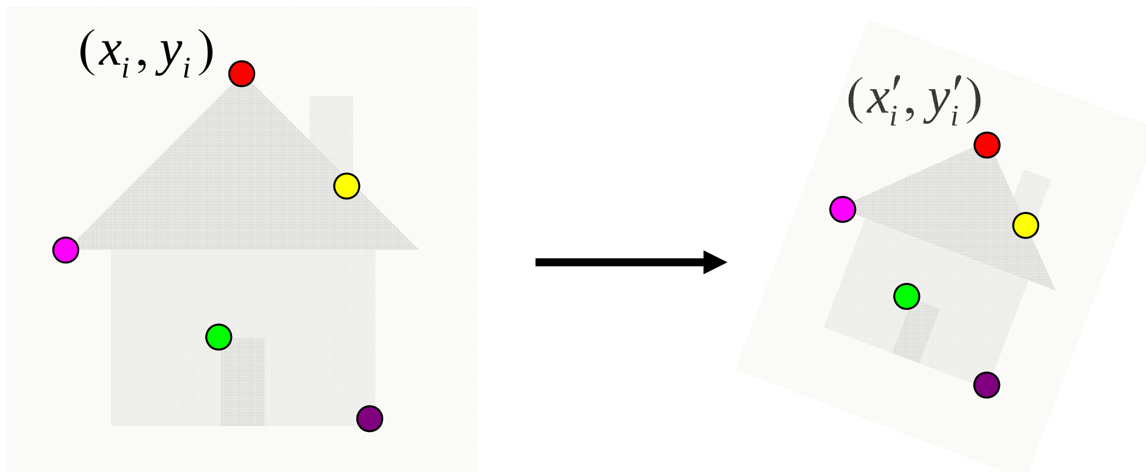
Let's start with affine transformations

- Simple fitting procedure (linear least squares)
- Approximates viewpoint changes for roughly planar objects
- Can be used to initialize fitting for more complex models



Fitting an affine transformation

- Assume we know the correspondences, how do we get the transformation?



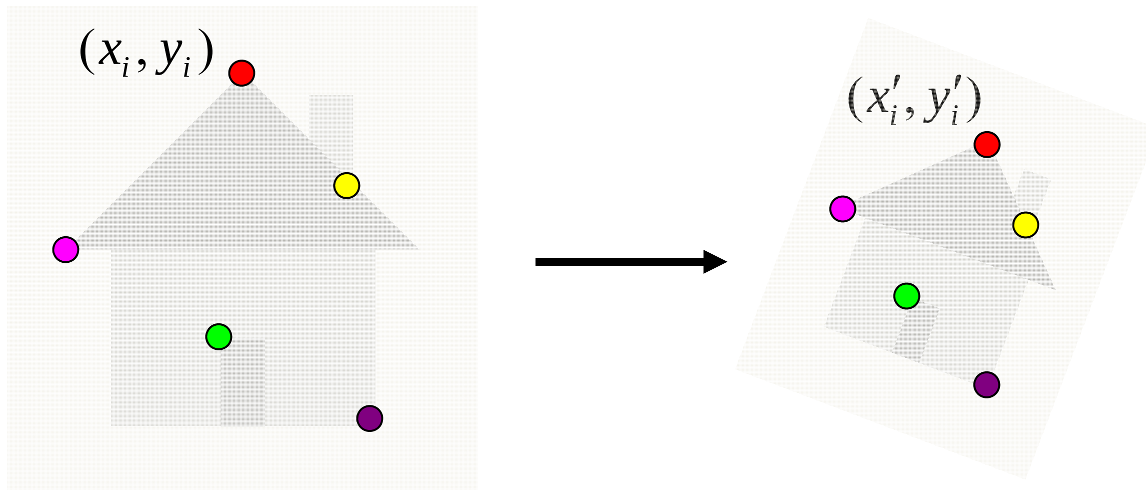
$$\mathbf{x}'_i = \mathbf{M}\mathbf{x}_i + \mathbf{t}$$

$$\begin{bmatrix} x'_i \\ y'_i \end{bmatrix} = \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$$

Want to find M, t to minimize

$$\sum_{i=1}^{n} \| \mathbf{x}'_i - \mathbf{M}\mathbf{x}_i - \mathbf{t} \|^2$$

Fitting an affine transformation



$$\begin{bmatrix} x_i' \\ y_i' \end{bmatrix} = \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$$

$$\begin{bmatrix} & & \cdots & & & \\ x_i & y_i & 0 & 0 & 1 & 0 \\ 0 & 0 & x_i & y_i & 0 & 1 \\ & & \cdots & & & \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} \cdots \\ x_i' \\ y_i' \\ \cdots \end{bmatrix}$$

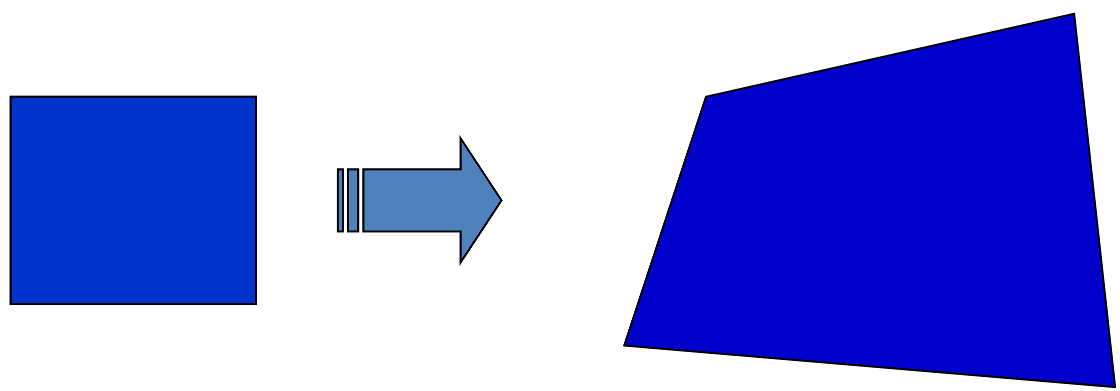
Fitting an affine transformation

$$\begin{bmatrix} & & \cdots & & & \\ x_i & y_i & 0 & 0 & 1 & 0 \\ 0 & 0 & x_i & y_i & 0 & 1 \\ & & \cdots & & & \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} \cdots \\ x_i' \\ y_i' \\ \cdots \end{bmatrix}$$

- Linear system with six unknowns
- Each match gives us two linearly independent equations: need at least three to solve for the transformation parameters

Fitting a plane projective transformation

- **Homography:** plane projective transformation (transformation taking a quad to another arbitrary quad)

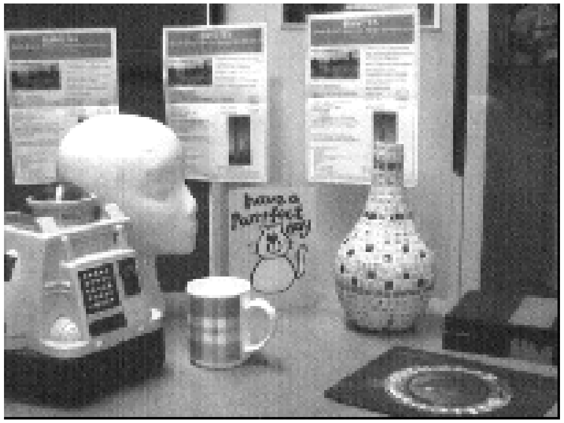


Homography

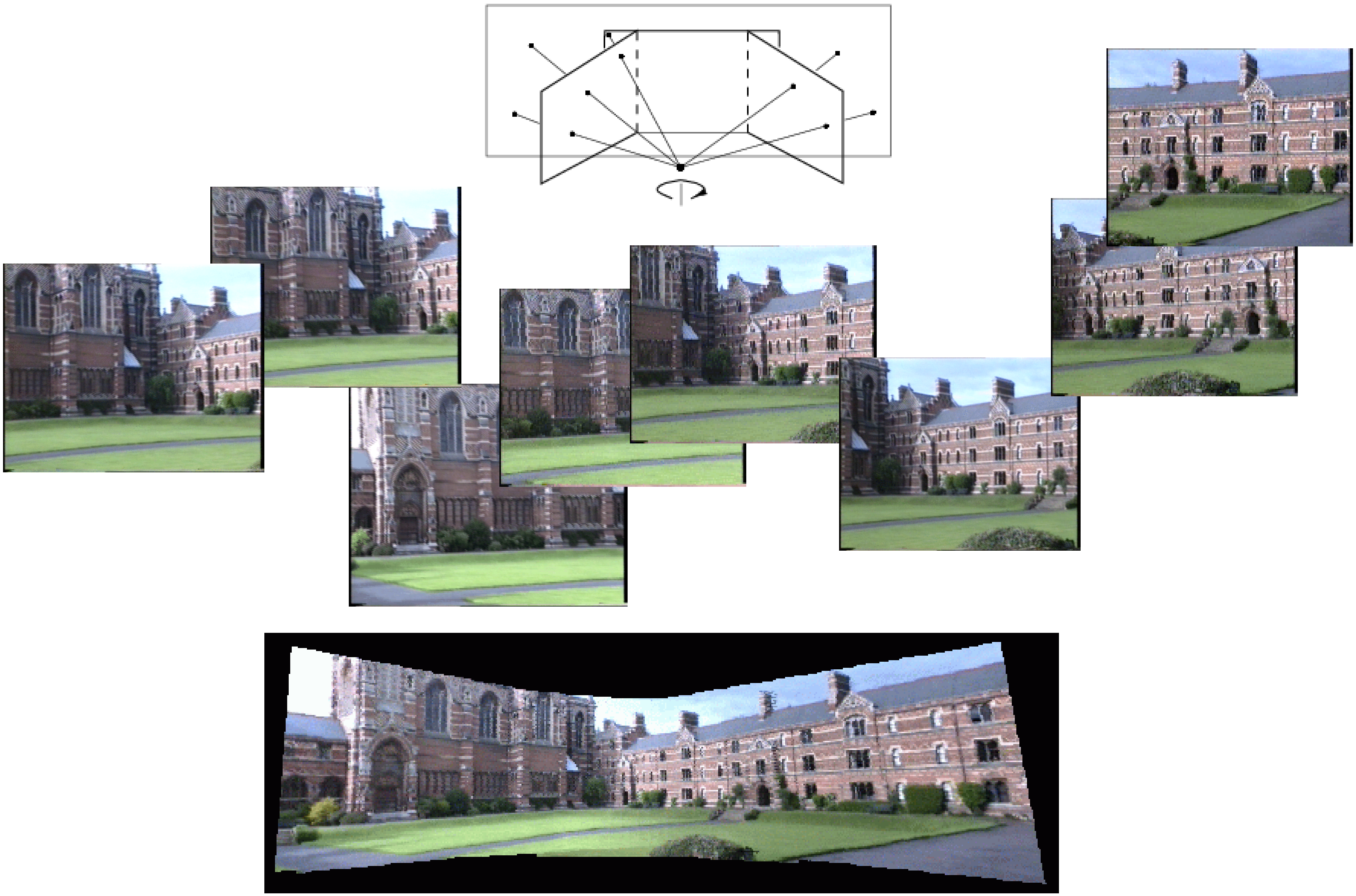
- The transformation between two views of a planar surface

- The transformation between images from two cameras that share the same center

Application: Panorama stitching



Source: Hartley & Zisserman

Fitting a homography

- Homogeneous coordinates (more later)

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Converting *to* homogeneous image coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

Converting *from* homogeneous image coordinates

$$\lambda \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Fitting a homography

- Equation for homography:

$$\lambda \begin{bmatrix} x_i' \\ y_i' \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

$$\lambda \mathbf{x}_i' = \mathbf{H}\,\mathbf{x}_i$$

$$\mathbf{x}_i' \times \mathbf{H}\,\mathbf{x}_i = 0$$

$$\begin{bmatrix} x_i' \\ y_i' \\ 1 \end{bmatrix} \times \begin{bmatrix} \mathbf{h}_1^T \mathbf{x}_i \\ \mathbf{h}_2^T \mathbf{x}_i \\ \mathbf{h}_3^T \mathbf{x}_i \end{bmatrix} = \begin{bmatrix} y_i' \mathbf{h}_3^T \mathbf{x}_i - \mathbf{h}_2^T \mathbf{x}_i \\ \mathbf{h}_1^T \mathbf{x}_i - x_i' \mathbf{h}_3^T \mathbf{x}_i \\ x_i' \mathbf{h}_2^T \mathbf{x}_i - y_i' \mathbf{h}_1^T \mathbf{x}_i \end{bmatrix}$$

$$\begin{bmatrix} 0^T & -\mathbf{x}_i^T & y_i' \mathbf{x}_i^T \\ \mathbf{x}_i^T & 0^T & -x_i' \mathbf{x}_i^T \\ -y_i' \mathbf{x}_i^T & x_i' \mathbf{x}_i^T & 0^T \end{bmatrix} \begin{pmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \\ \mathbf{h}_3 \end{pmatrix} = 0$$

3 equations,
only 2 linearly
independent

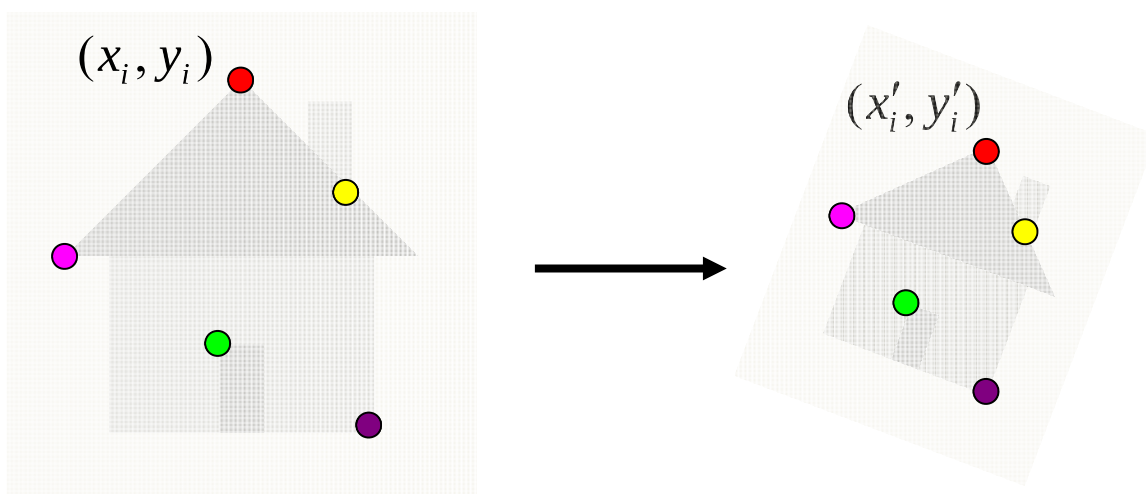
Direct linear transform

$$\begin{bmatrix} 0^T & \mathbf{x}_1^T & -y_1' \mathbf{x}_1^T \\ \mathbf{x}_1^T & 0^T & -x_1' \mathbf{x}_1^T \\ \cdots & \cdots & \cdots \\ 0^T & \mathbf{x}_n^T & -y_n' \mathbf{x}_n^T \\ \mathbf{x}_n^T & 0^T & -x_n' \mathbf{x}_n^T \end{bmatrix} \begin{pmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \\ \mathbf{h}_3 \end{pmatrix} = 0 \qquad \mathbf{A}\,\mathbf{h} = 0$$

- H has 8 degrees of freedom (9 parameters, but scale is arbitrary)
- One match gives us two linearly independent equations
- Homogeneous least squares: find $\mathbf{h}$ minimizing $||\mathbf{Ah}||^2$
  - Eigenvector of $\mathbf{A}^\mathrm{T}\mathbf{A}$ corresponding to smallest eigenvalue
  - Four matches needed for a minimal solution

Robust feature-based alignment

- So far, we've assumed that we are given a set of "ground-truth" correspondences between the two images we want to align
- What if we don't know the correspondences?



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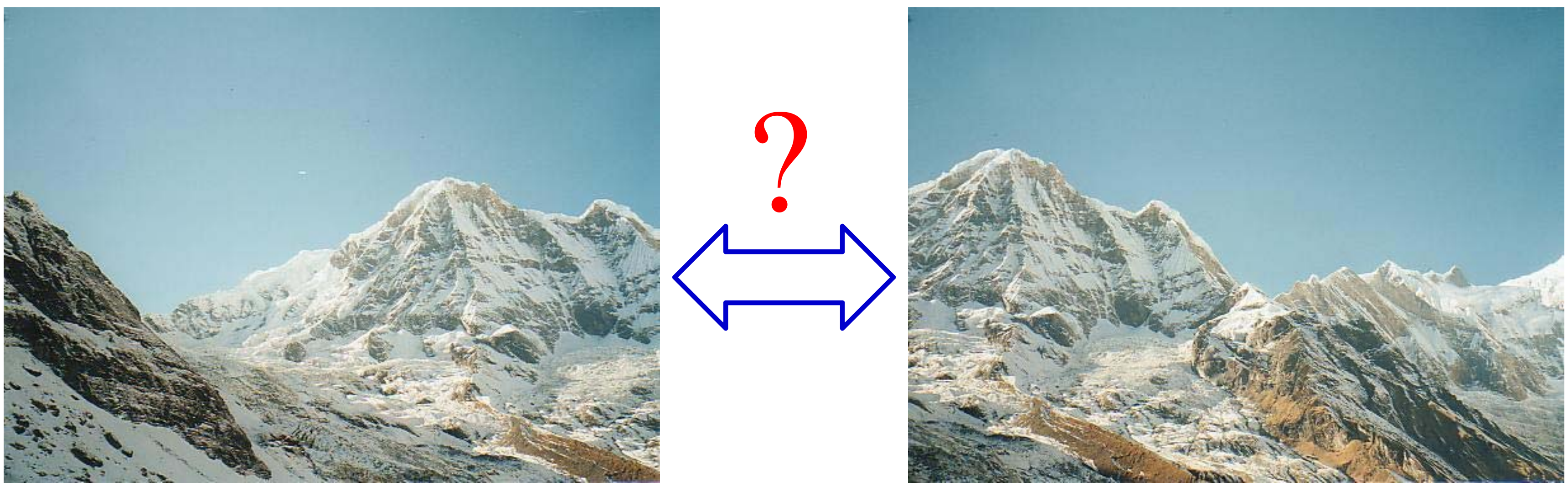
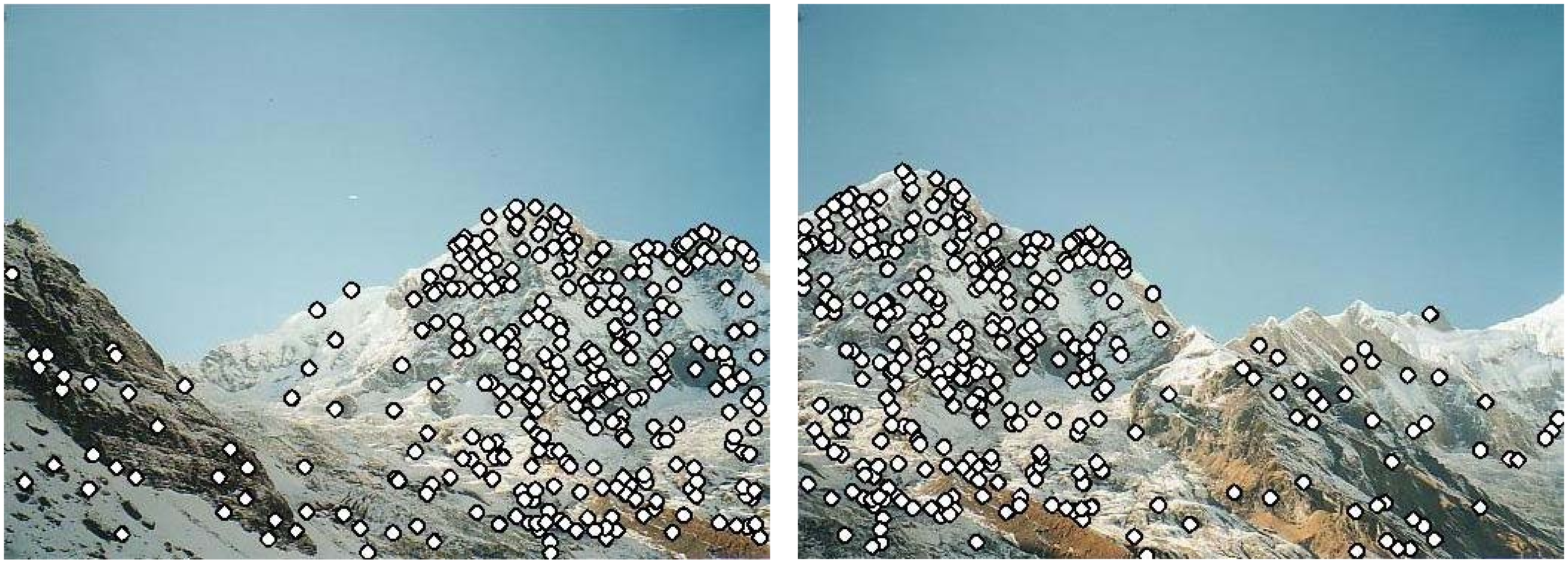


Robust feature-based alignment

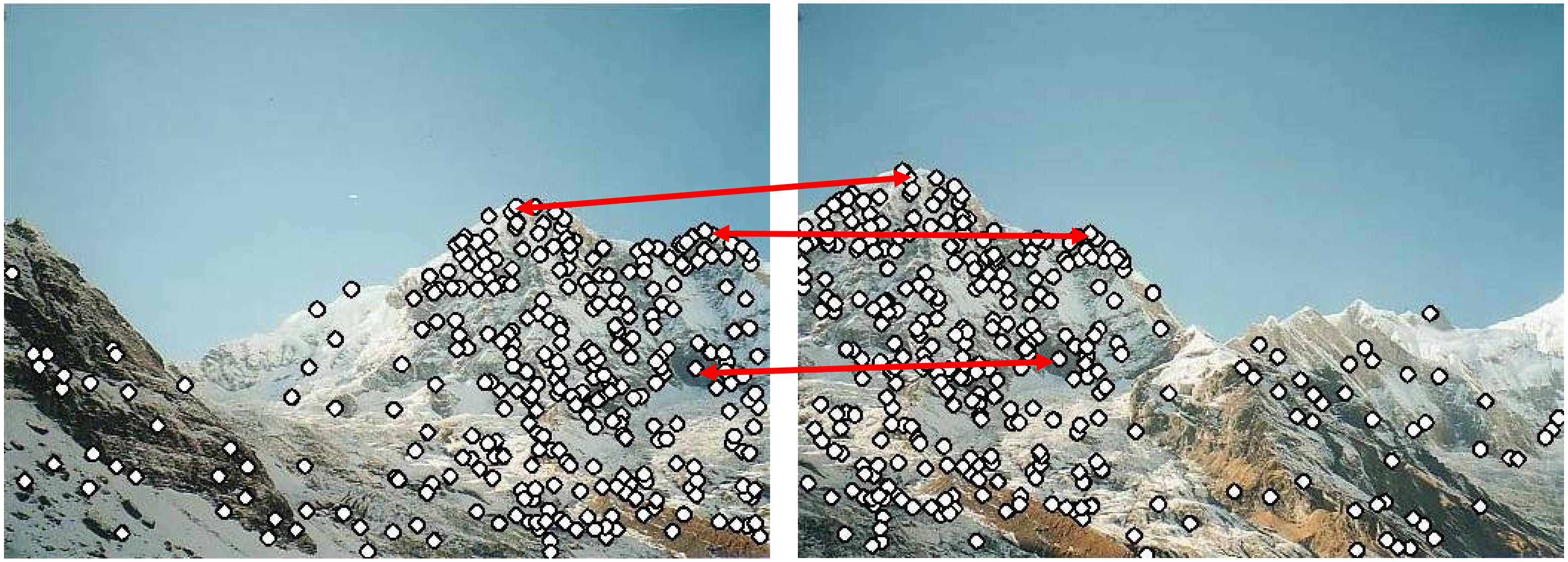


- Extract features

Robust feature-based alignment

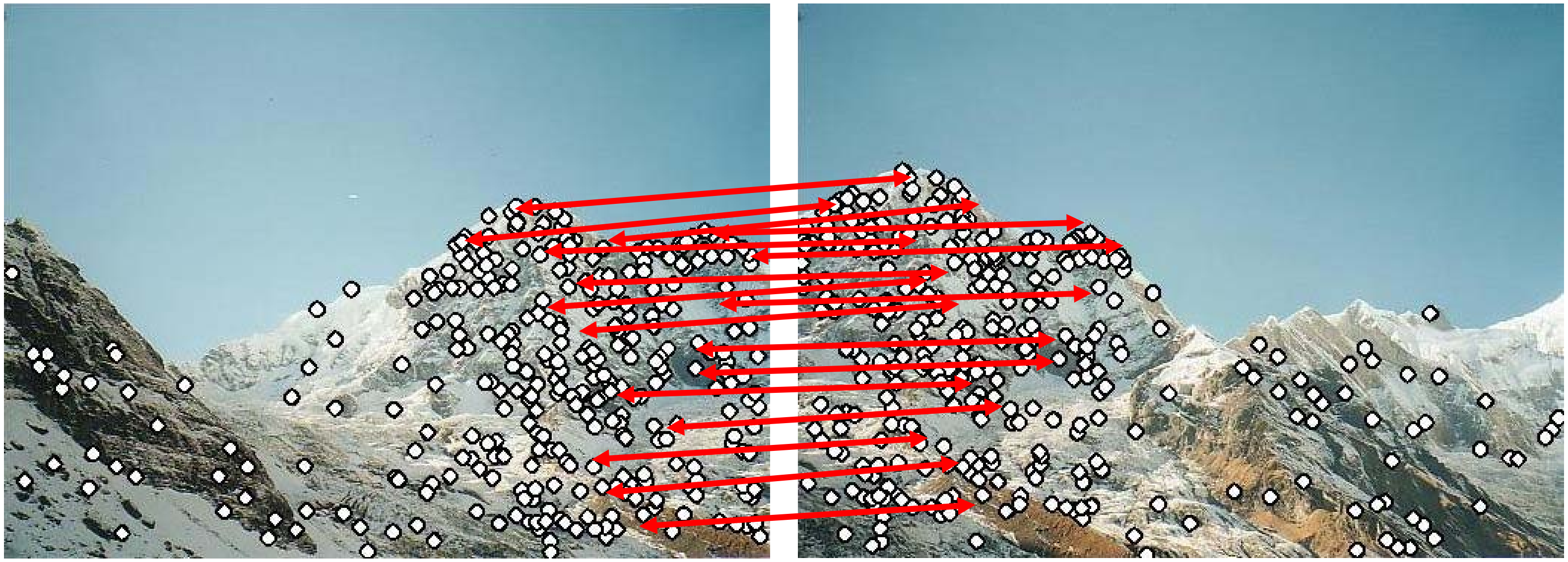
- Extract features
- Compute *putative matches*

Robust feature-based alignment



- Extract features
- Compute *putative matches*
- Loop:
  - *Hypothesize* transformation *T*

Robust feature-based alignment

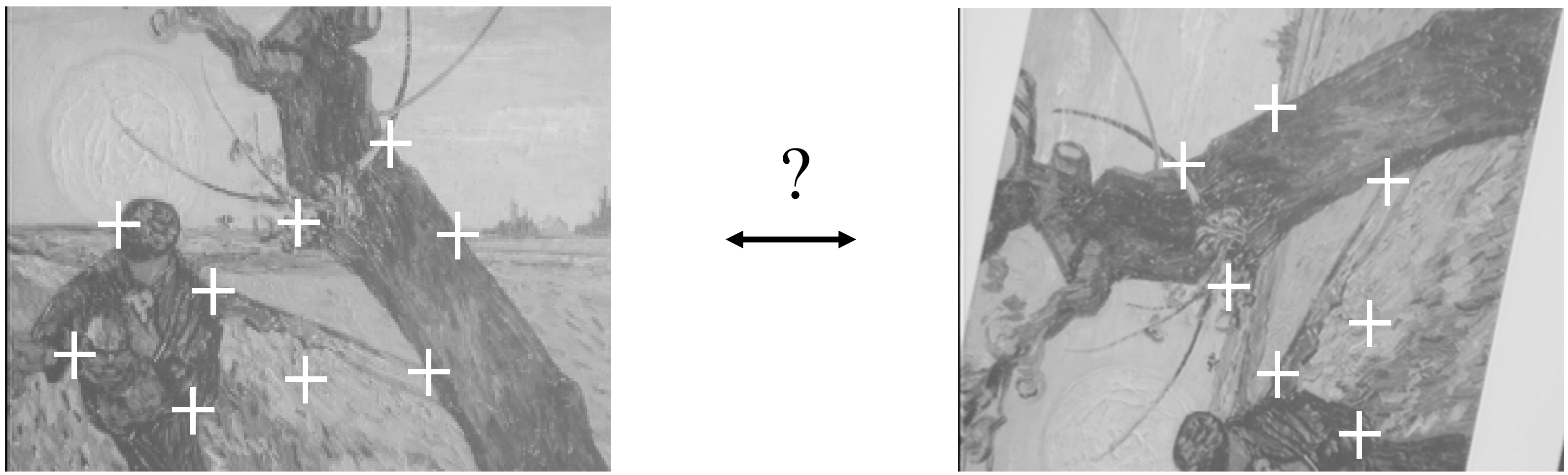


- Extract features
- Compute *putative matches*
- Loop:
  - *Hypothesize* transformation *T*
  - *Verify* transformation (search for other matches consistent with *T*)

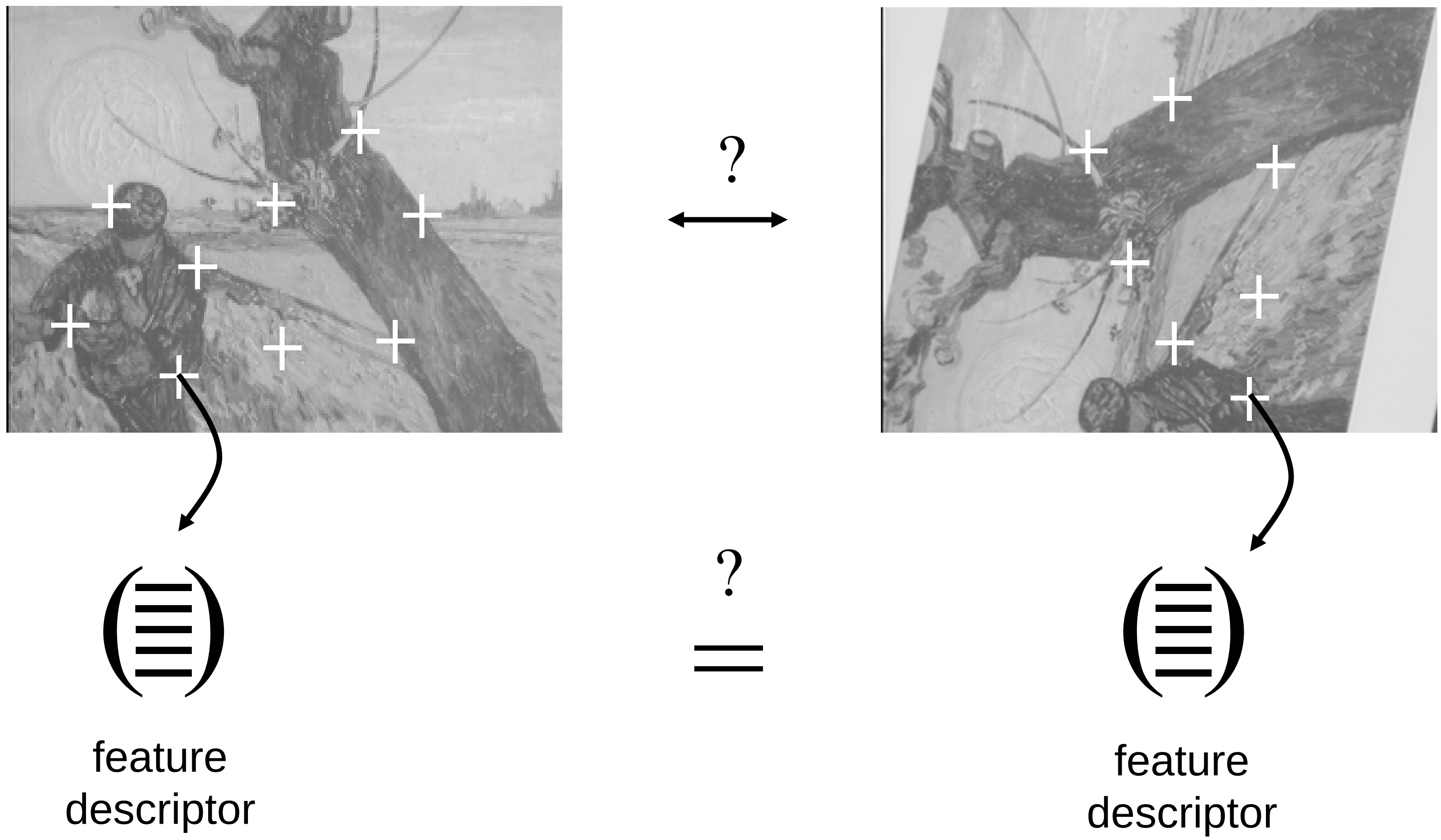
Robust feature-based alignment

- Extract features
- Compute *putative matches*
- Loop:
  - *Hypothesize* transformation T
  - *Verify* transformation (search for other matches consistent with T)

Generating putative correspondences



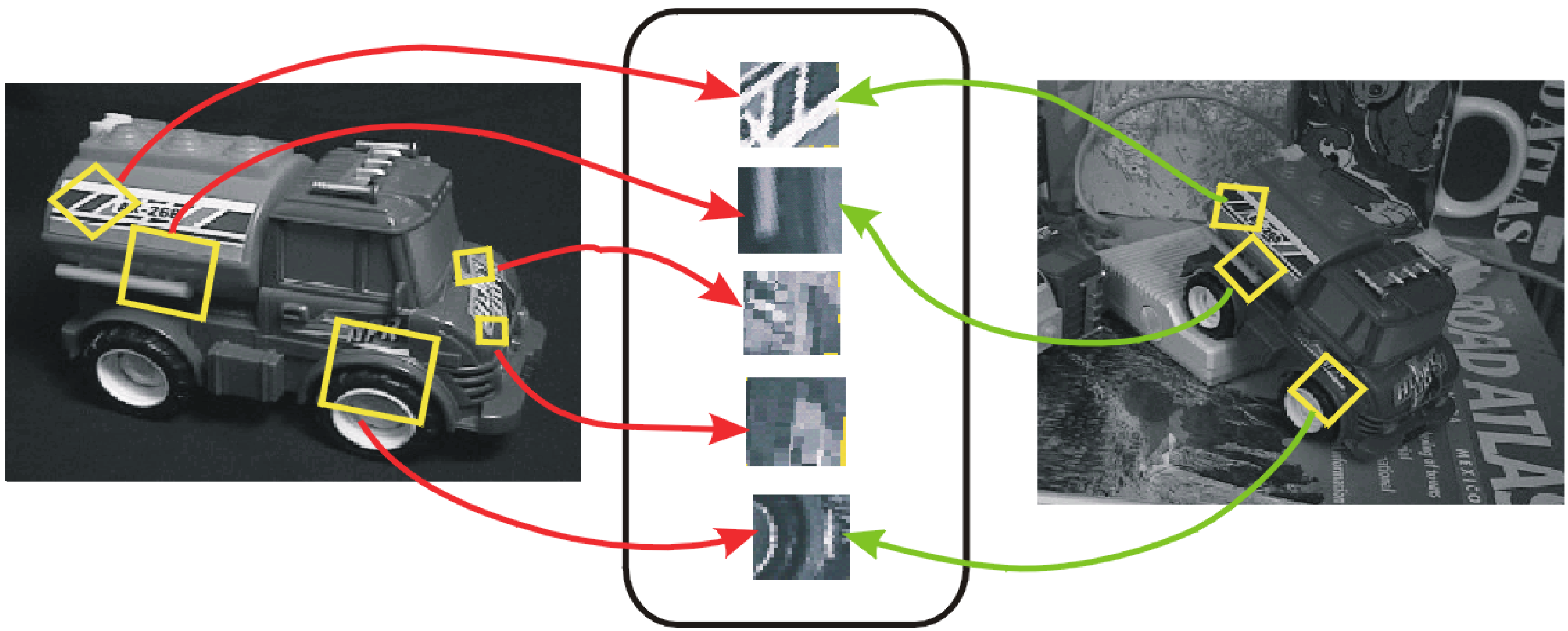
Generating putative correspondences



- Need to compare *feature descriptors* of local patches surrounding interest points

Feature descriptors

- Recall: feature detection and description



Feature descriptors

- Simplest descriptor: vector of raw intensity values
- How to compare two such vectors?
  - Sum of squared differences (SSD)

$$\mathrm{SSD}(\mathbf{u},\mathbf{v}) = \sum_i \left(u_i - v_i\right)^2$$

    - Not invariant to intensity change

  - Normalized correlation

$$\rho(\mathbf{u},\mathbf{v}) = \frac{(\mathbf{u}-\overline{\mathbf{u}})}{\|\mathbf{u}-\overline{\mathbf{u}}\|} \cdot \frac{(\mathbf{v}-\overline{\mathbf{v}})}{\|\mathbf{v}-\overline{\mathbf{v}}\|} = \frac{\sum_i (u_i - \overline{\mathbf{u}})(v_i - \overline{\mathbf{v}})}{\sqrt{\left(\sum_j (u_j - \overline{\mathbf{u}})^2\right)\left(\sum_j (v_j - \overline{\mathbf{v}})^2\right)}}$$

    - Invariant to affine intensity change

Disadvantage of intensity vectors as descriptors

- Small deformations can affect the matching score a lot

Slide Credits

- This set of sides contains contributions kindly made available by the following authors
  - Derek Hoiem
  - Svetlana Lazebnik
  - Kristen Grauman
  - Alexei Efros
  - David Forsyth
  - Steve Seitz