CS 558: Computer Vision
4th Set of Notes

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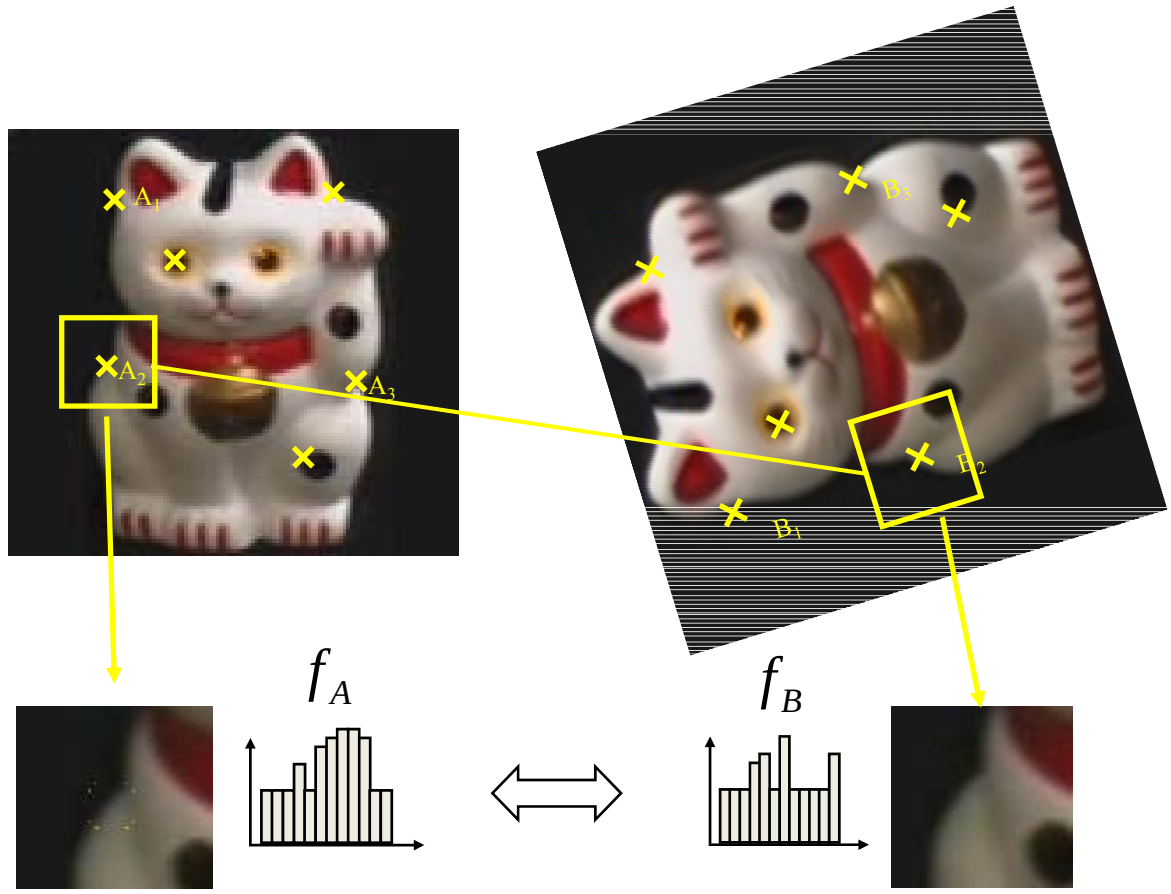
Overview

- Keypoint matching
- Hessian detector
- Blob detection
- Feature descriptors
- Fitting
- RANSAC
  - Based on slides by S. Lazebnik and D. Hoiem

Overview of Keypoint Matching

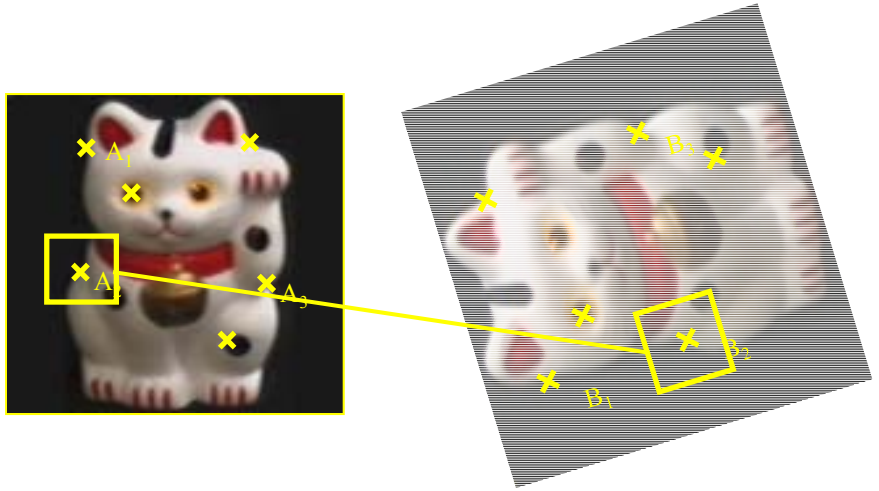


1. **Find a set of distinctive key-points**
2. **Define a region around each keypoint**
3. **Extract and normalize the region content**
4. **Compute a local descriptor from the normalized region**
5. **Match local descriptors**

Goals for Keypoints

Detect points that are *repeatable* and *distinctive*

Key trade-offs



Detection

More Repeatable

Robust detection
Precise localization

More Points

Robust to occlusion
Works with less texture

Description

More Distinctive

Minimize wrong matches

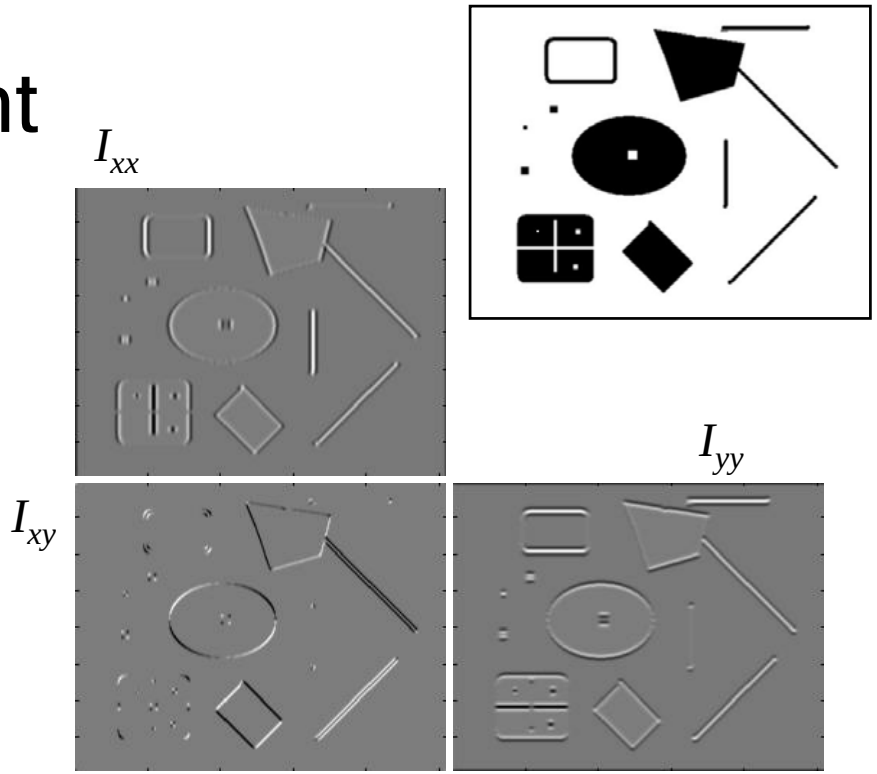
More Flexible

Robust to expected variations
Maximize correct matches

Hessian Detector [Beaudet78]

- Hessian determinant

$$Hessian(I) = \begin{bmatrix} I_{xx} & I_{xy} \\ I_{xy} & I_{yy} \end{bmatrix}$$



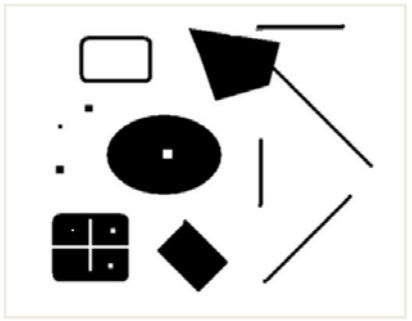
Intuition: **Search for strong curvature in two orthogonal directions**

Hessian Detector [Beaudet78]

- Hessian determinant

$$Hessian(x,\sigma) = \begin{bmatrix} I_{xx}(x,\sigma) & I_{xy}(x,\sigma) \\ I_{xy}(x,\sigma) & I_{yy}(x,\sigma) \end{bmatrix}$$

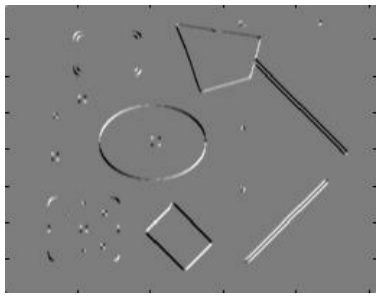
$$\det M = \lambda_1 \lambda_2$$
$$\text{trace}\, M = \lambda_1 + \lambda_2$$

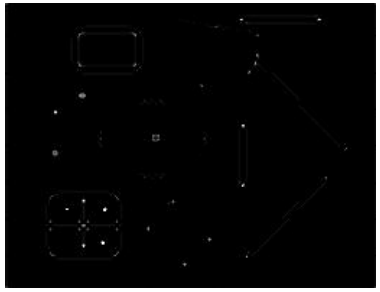


I_{xx}

I_{yy}

I_{xy}





Find maxima of determinant

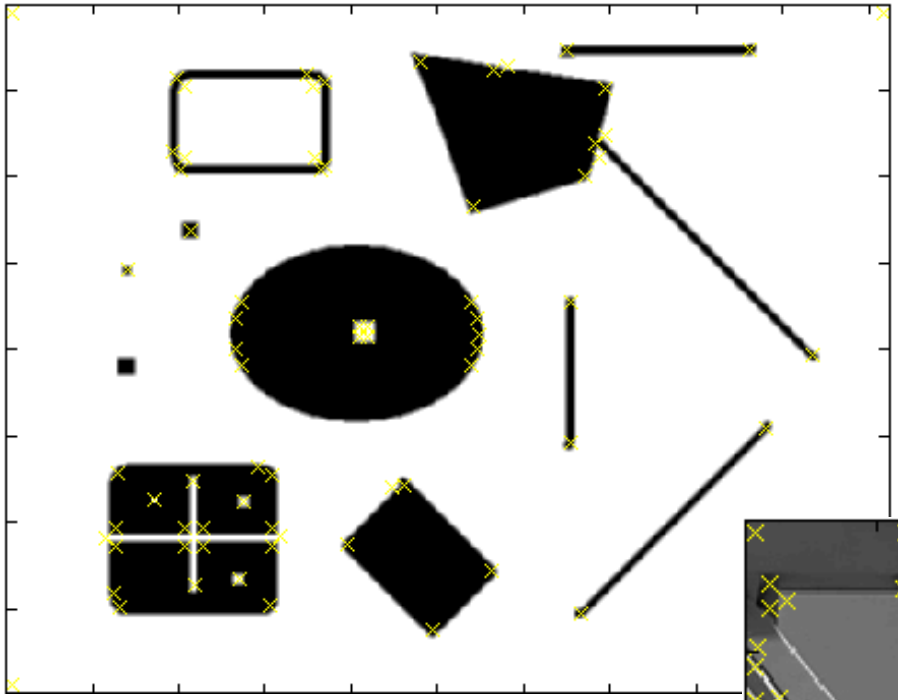
$$\det(Hessian(x)) = I_{xx}(x)I_{yy}(x) - I_{xy}^2(x)$$

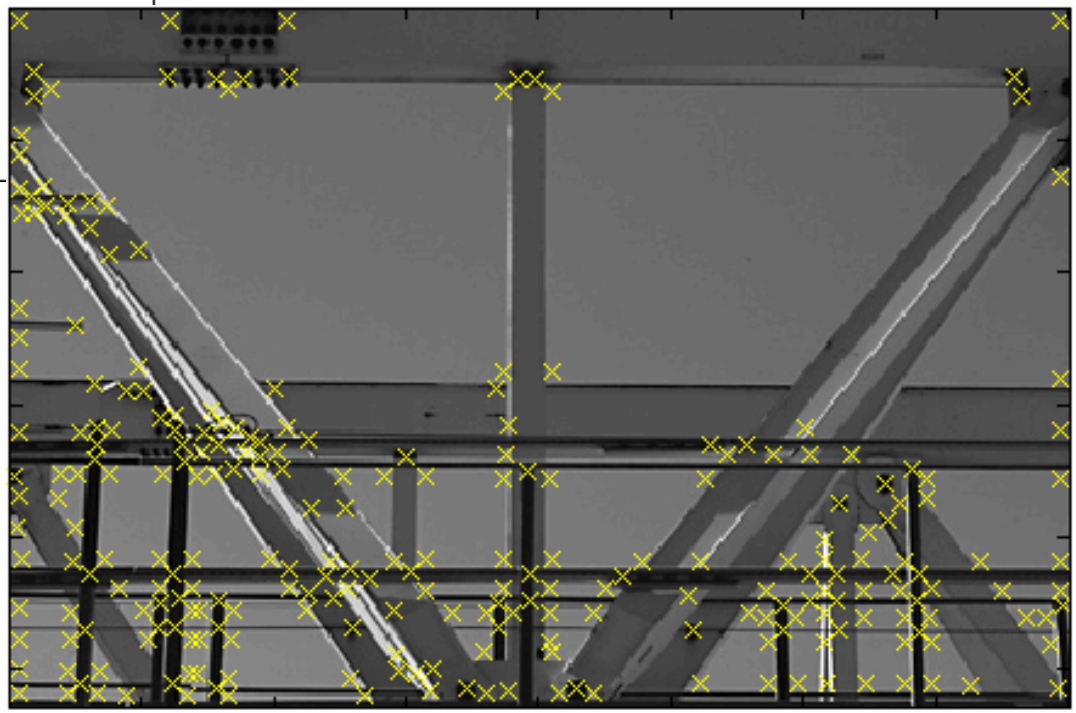
In Matlab:

$$I_{xx}.*I_{yy} - (I_{xy})\text{^}2$$

K. Grauman, B. Leibe

Hessian Detector – Responses [Beaudet78]





Effect: **Responses mainly on corners and strongly textured areas.**

Hessian Detector – Responses [Beaudet78]

So far: can localize in x-y, but not scale

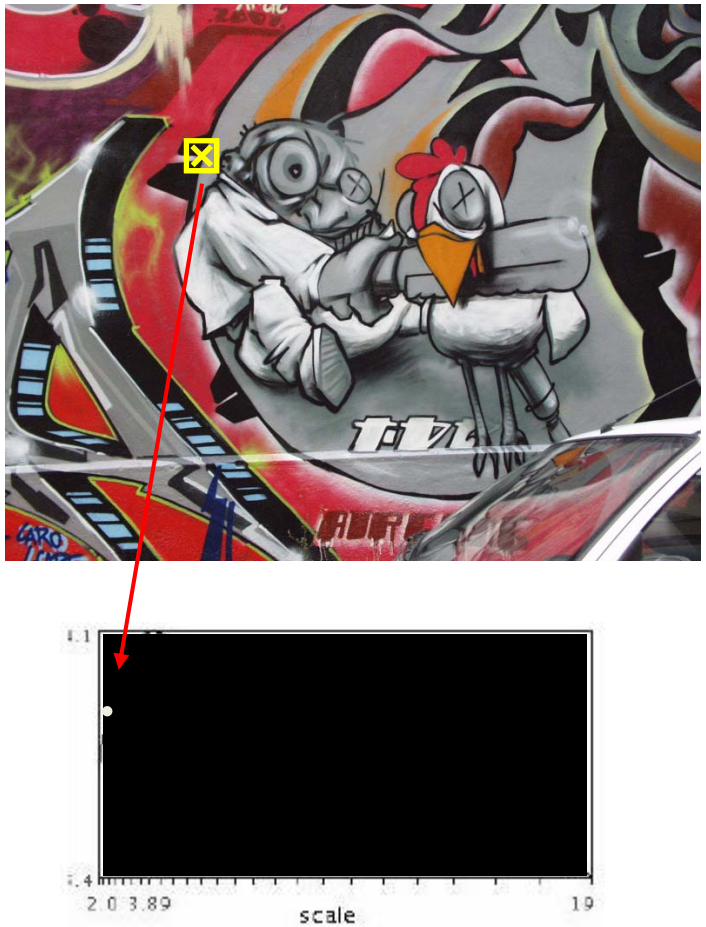
Automatic Scale Selection

$$f(I_{i_1 \ldots i_m}(x,\sigma)) \quad = \quad f(I_{i_1 \ldots i_m}(x',\sigma'))$$

How to find corresponding patch sizes?

Automatic Scale Selection

- Function responses for increasing scale (scale signature)



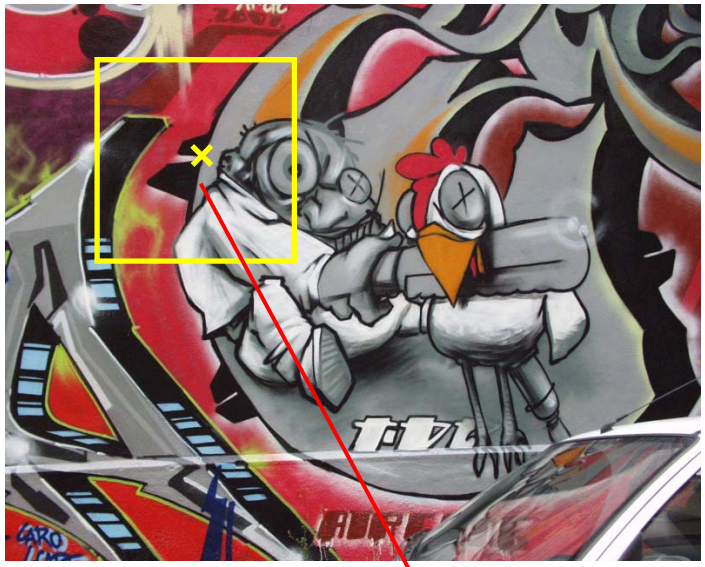
$f(I_{i_1 \ldots i_m}(x,\sigma))$

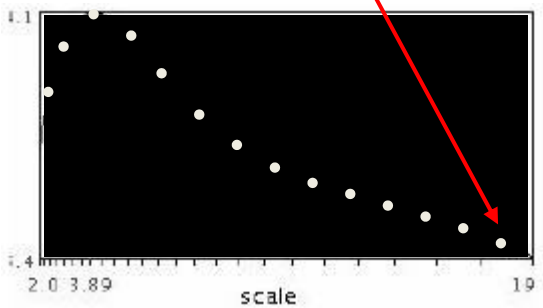
$f(I_{i_1 \ldots i_m}(x',\sigma))$

K. Grauman, B. Leibe

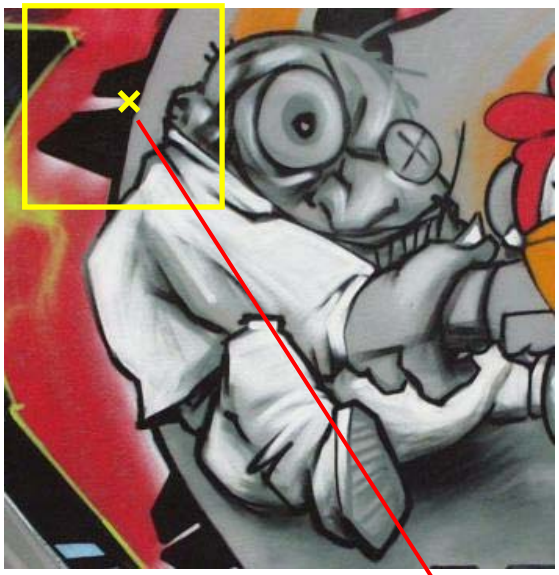
Automatic Scale Selection

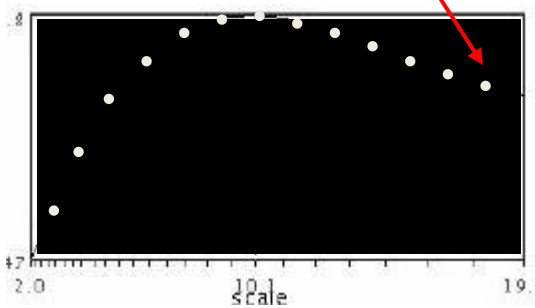
- Function responses for increasing scale (scale signature)





$f(I_{i_1 \ldots i_m}(x,\sigma))$

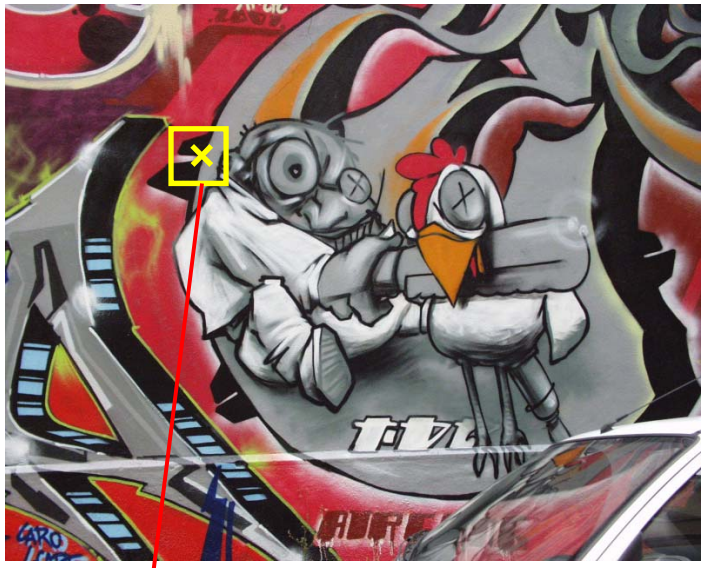


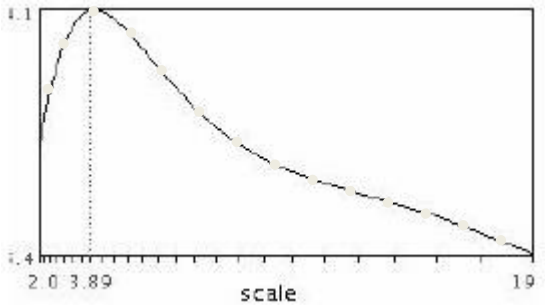


$f(I_{i_1 \ldots i_m}(x',\sigma))$

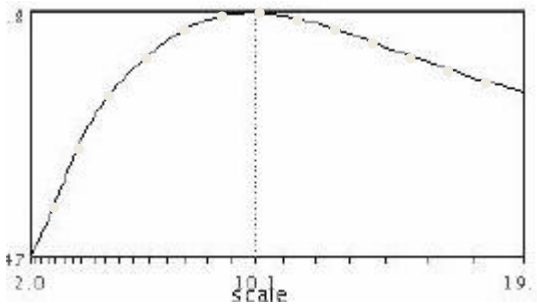
Automatic Scale Selection

- Function responses for increasing scale (scale signature)





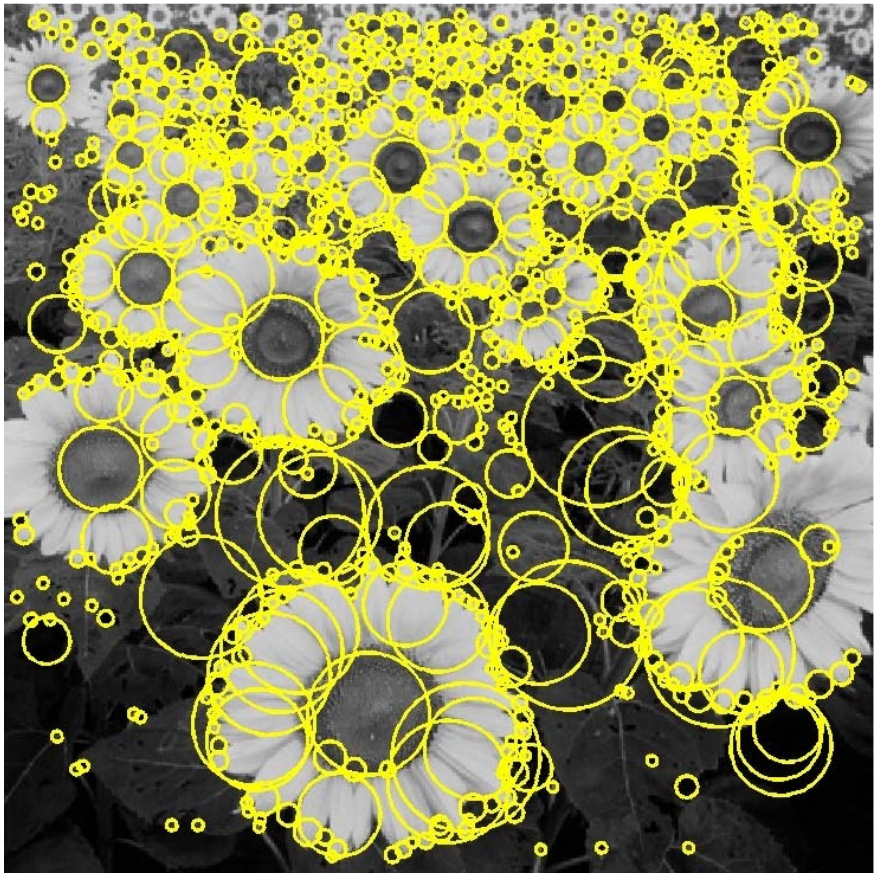
$f(I_{i_1 \ldots i_m}(x,\sigma))$



$f(I_{i_1 \ldots i_m}(x',\sigma'))$

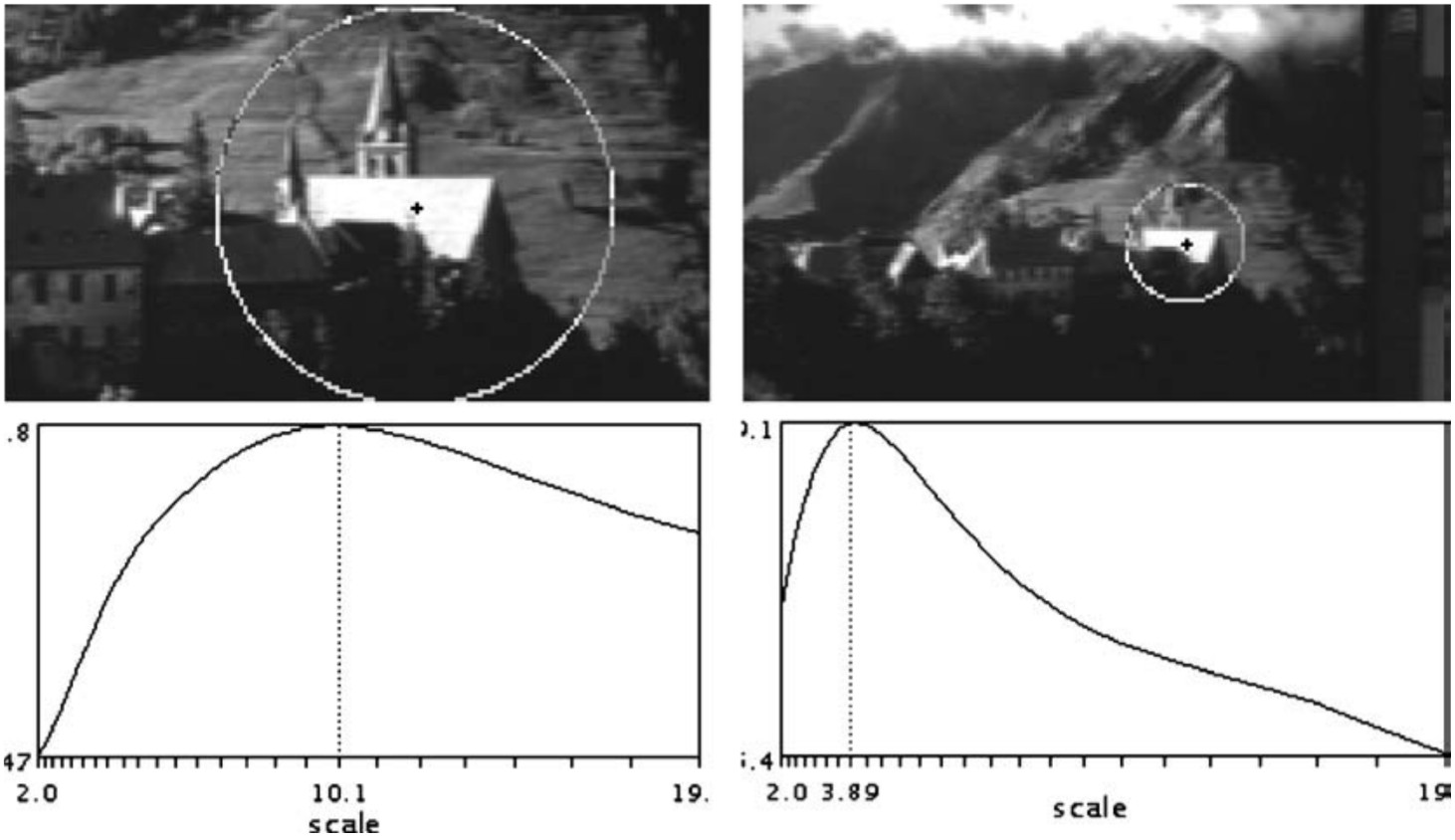
K. Grauman, B. Leibe

Blob detection



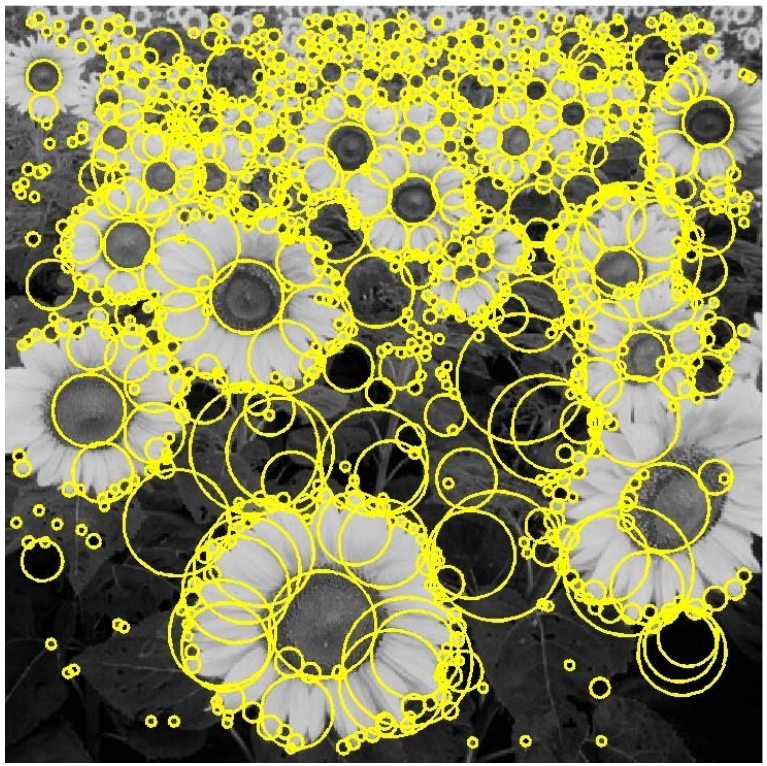
Feature detection with scale selection

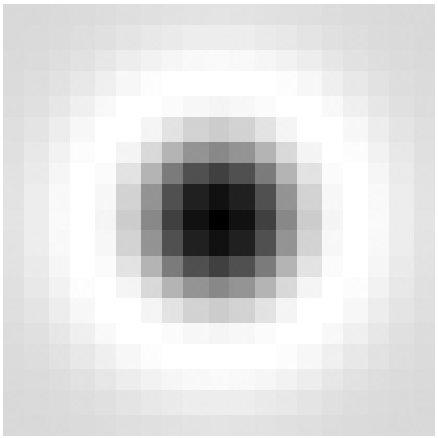
- We want to extract features with characteristic scale that is *covariant* with the image transformation



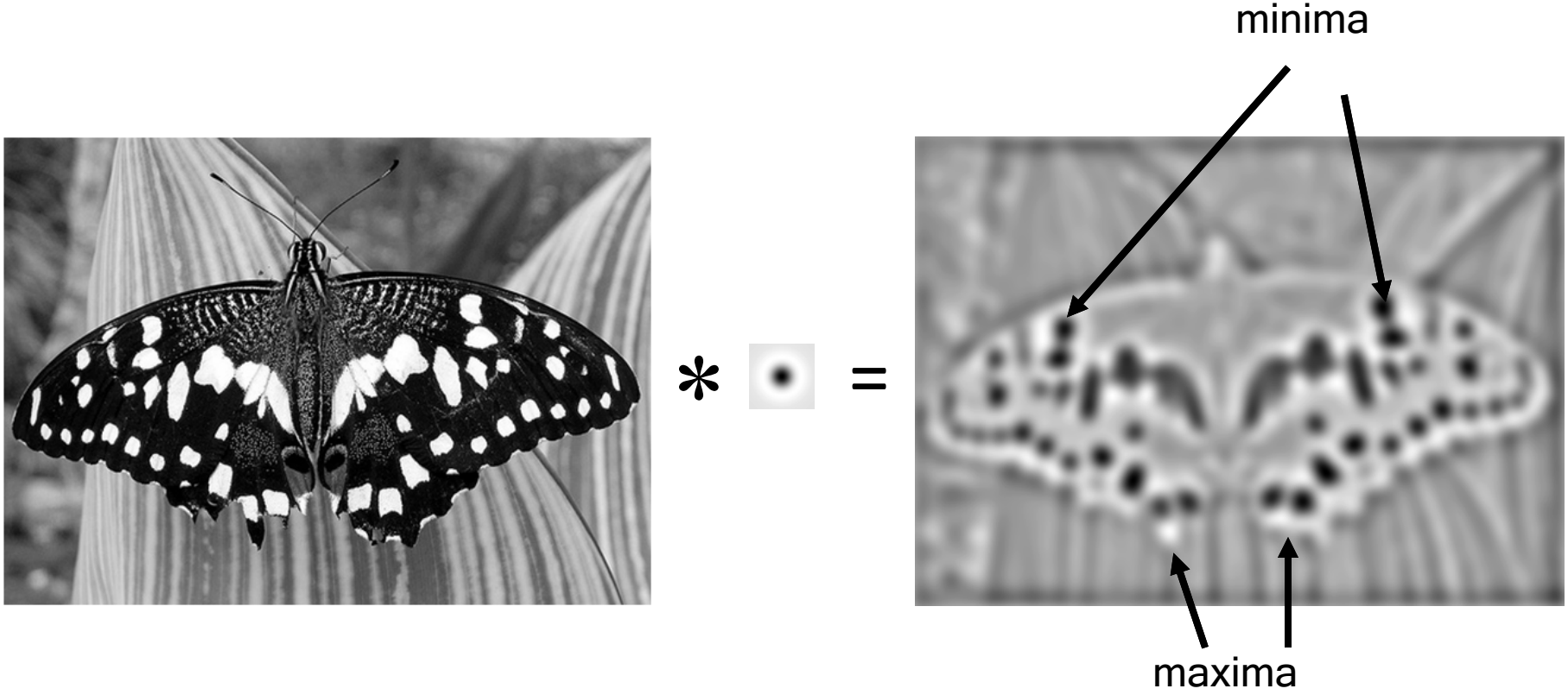
Blob detection: Basic idea

- To detect blobs, convolve the image with a “blob filter” at multiple scales and look for extrema of filter response in the resulting *scale space*





Blob detection: Basic idea

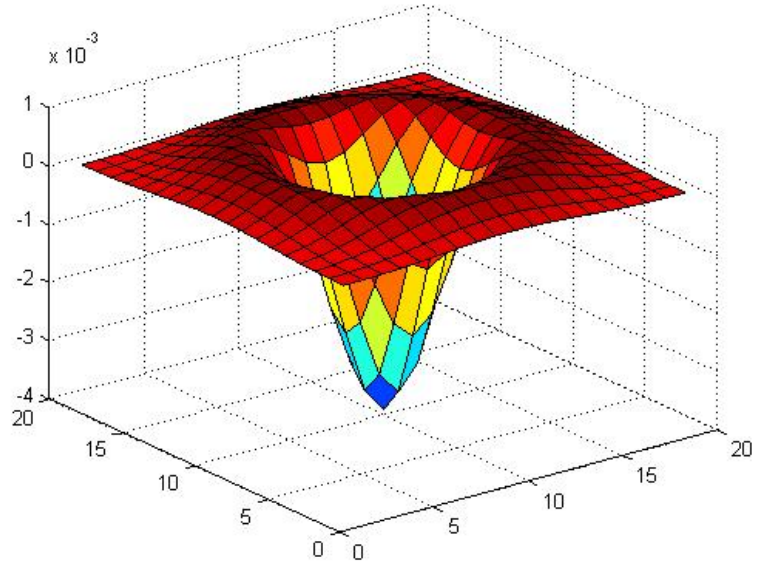


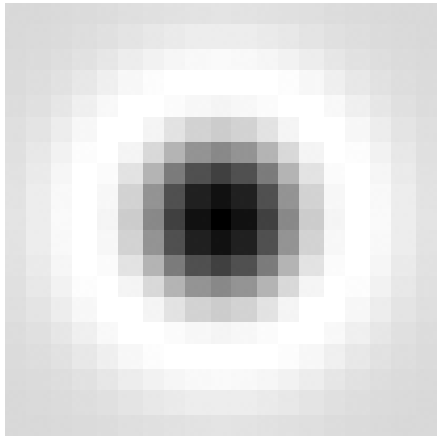
- Find maxima *and minima* of blob filter response in space *and scale*

Source: N. Snavely

Blob filter

- Laplacian of Gaussian: Circularly symmetric operator for blob detection in 2D

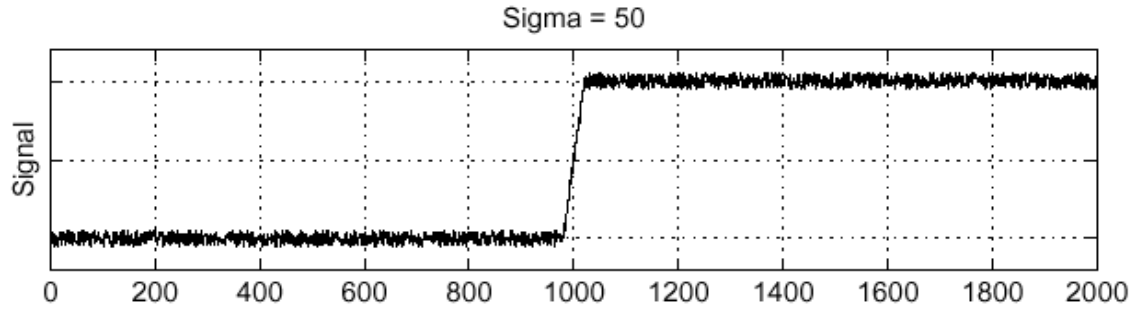




$$\nabla^2 g = \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2}$$

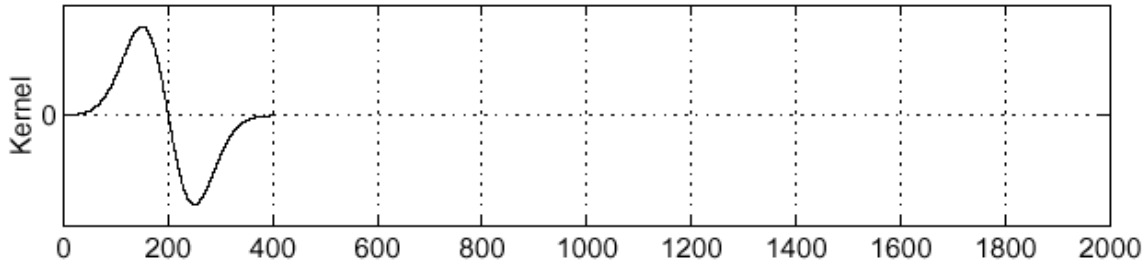
Recall: Edge detection

f



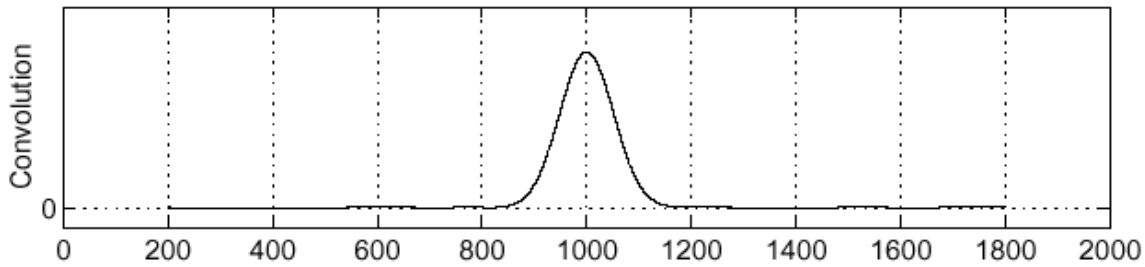
Edge

$\frac{d}{dx} g$



Derivative of Gaussian

$f * \frac{d}{dx} g$

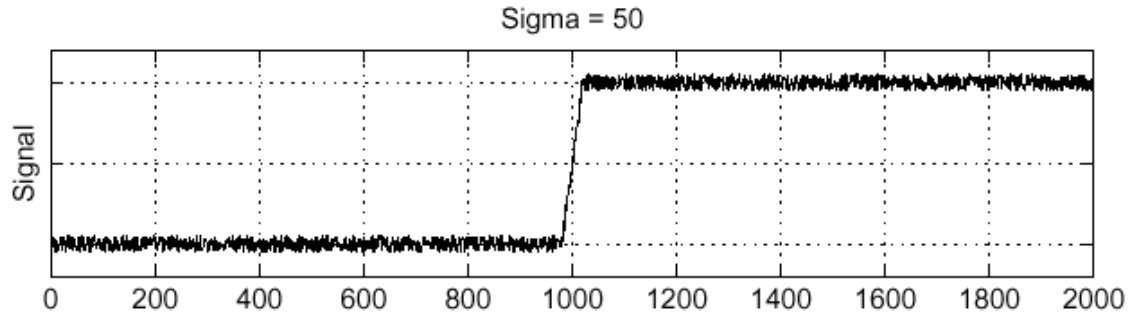


Edge = maximum of derivative

Source: S. Seitz

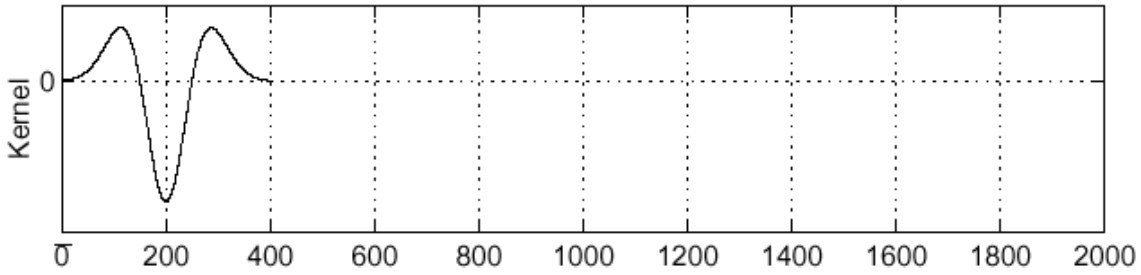
Edge detection, Take 2

f



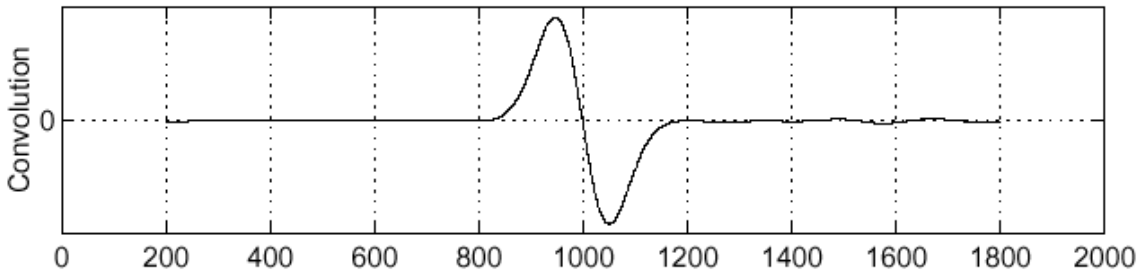
Edge

$\frac{d^2}{dx^2} g$



Second derivative of Gaussian (Laplacian)

$f * \frac{d^2}{dx^2} g$

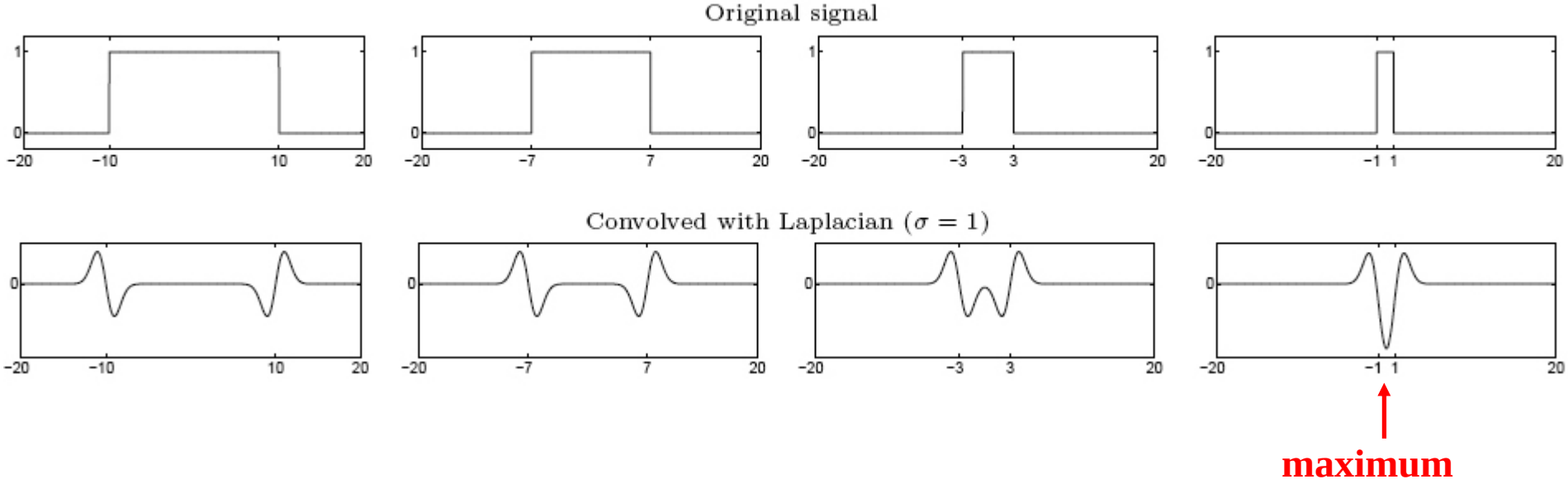


Edge = zero crossing of second derivative

Source: S. Seitz

From edges to blobs

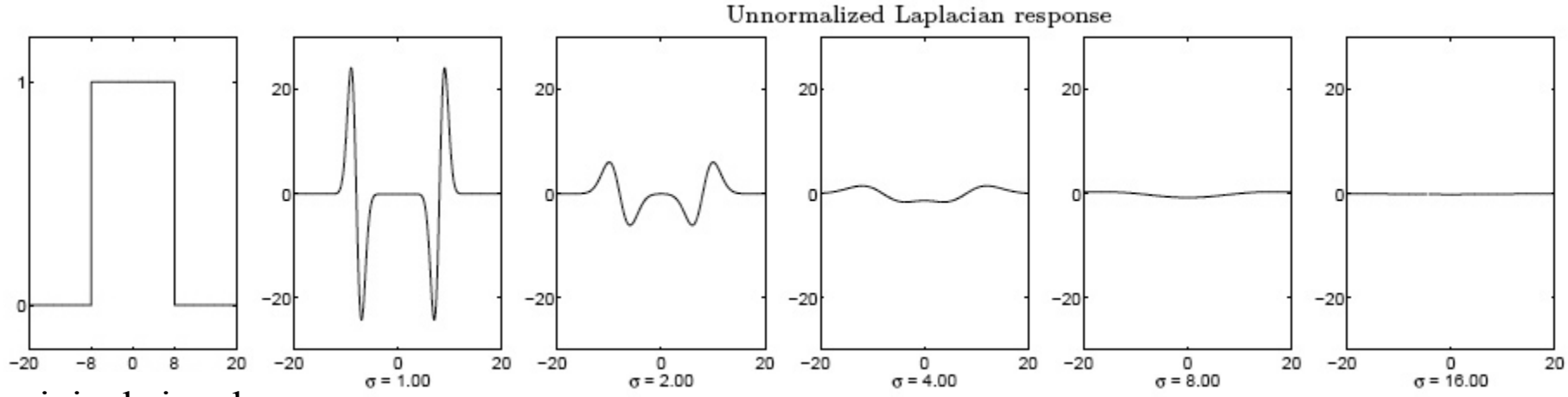
- Edge = ripple
- Blob = superposition of two ripples



Spatial selection: the magnitude of the Laplacian response will achieve a maximum at the center of the blob, provided the scale of the Laplacian is "matched" to the scale of the blob

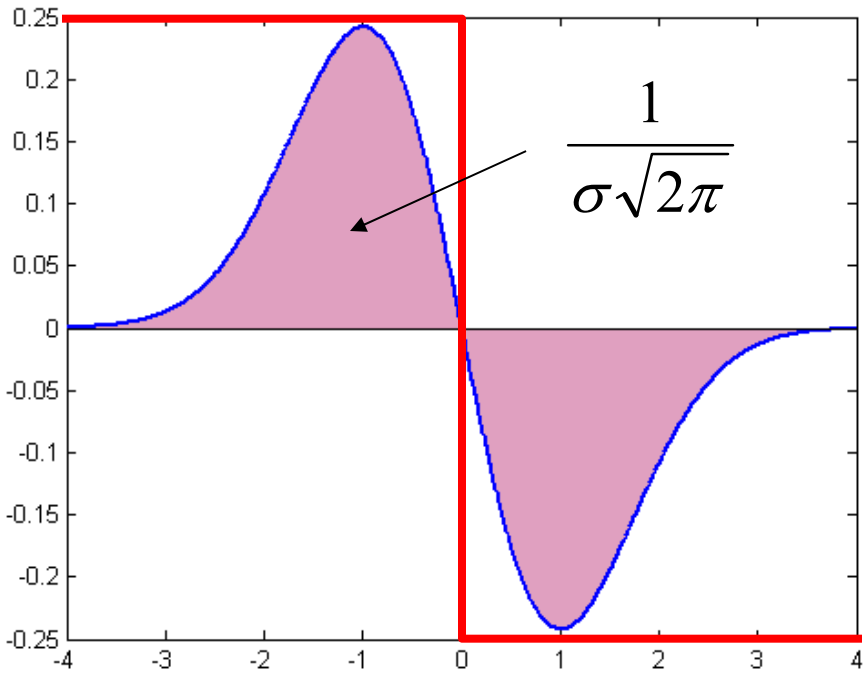
Scale selection

- We want to find the characteristic scale of the blob by convolving it with Laplacians at several scales and looking for the maximum response
- However, Laplacian response decays as scale increases:



Scale normalization

- The response of a derivative of Gaussian filter to a perfect step edge decreases as σ increases



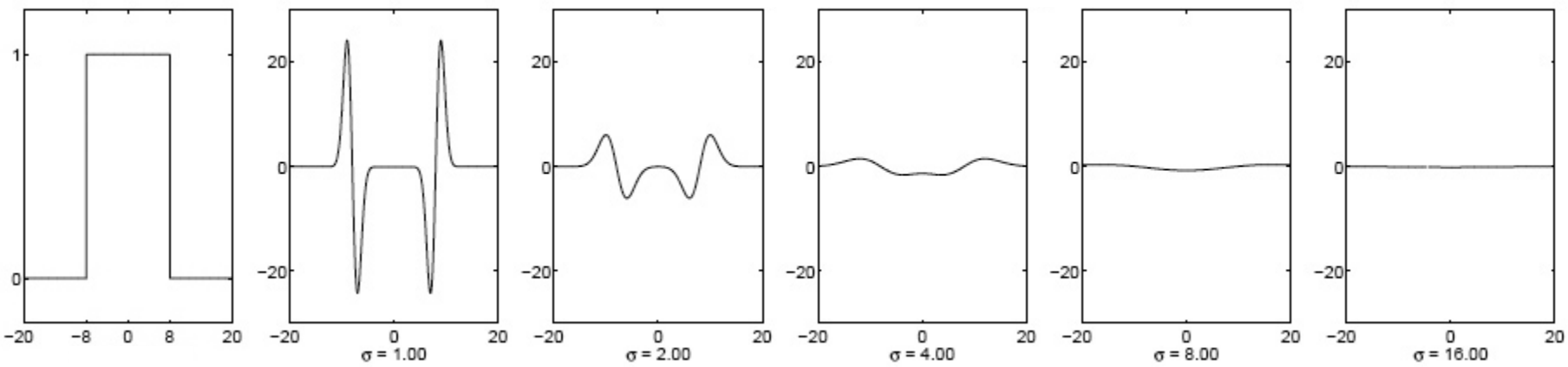
Scale normalization

- The response of a derivative of Gaussian filter to a perfect step edge decreases as σ increases
- To keep response the same (scale-invariant), must multiply Gaussian derivative by σ
- Laplacian is the second Gaussian derivative, so it must be multiplied by σ^2

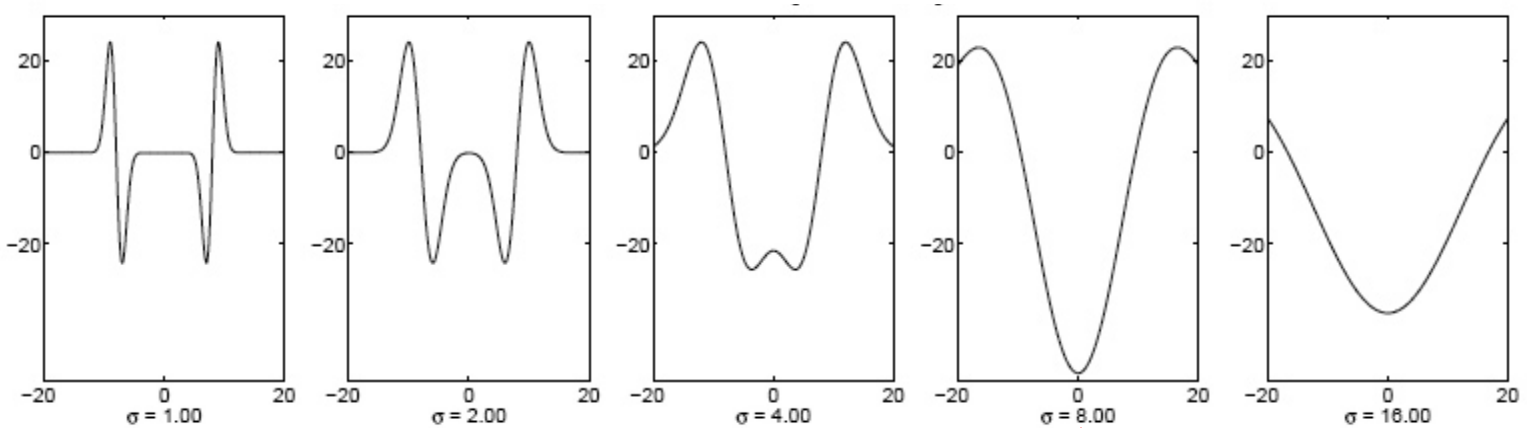
Effect of scale normalization

Original signal

Unnormalized Laplacian response



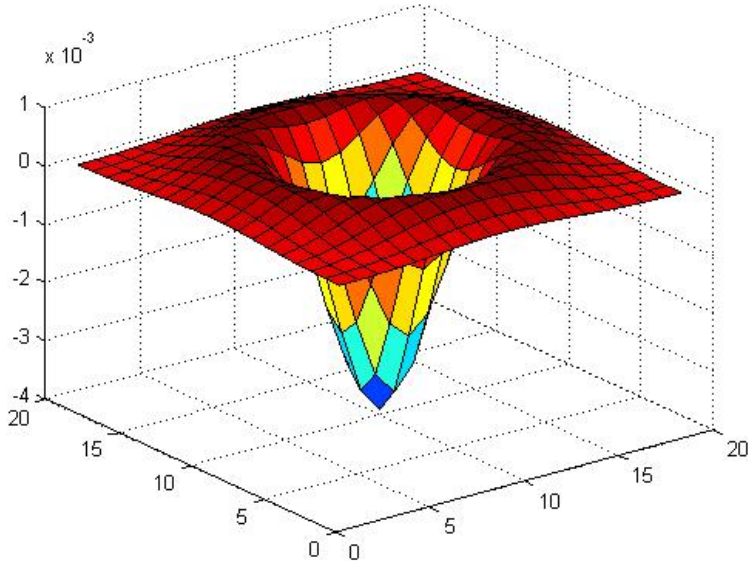
Scale-normalized Laplacian response

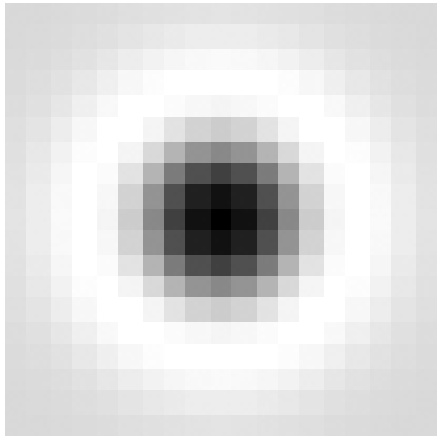


maximum

Blob detection in 2D

- Laplacian of Gaussian: Circularly symmetric operator for blob detection in 2D





Scale-normalized:

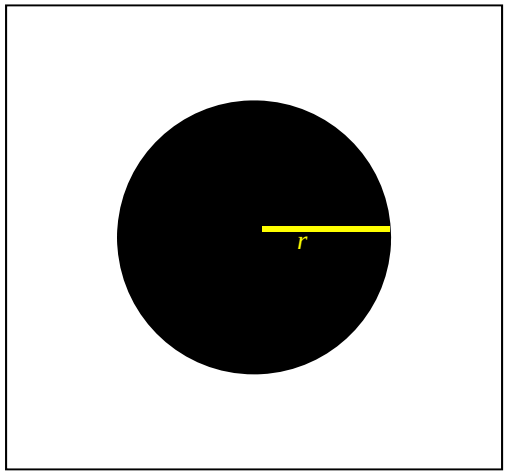
$$\nabla^2_{\text{norm}} g = \sigma^2 \left(\frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} \right)$$

Scale selection

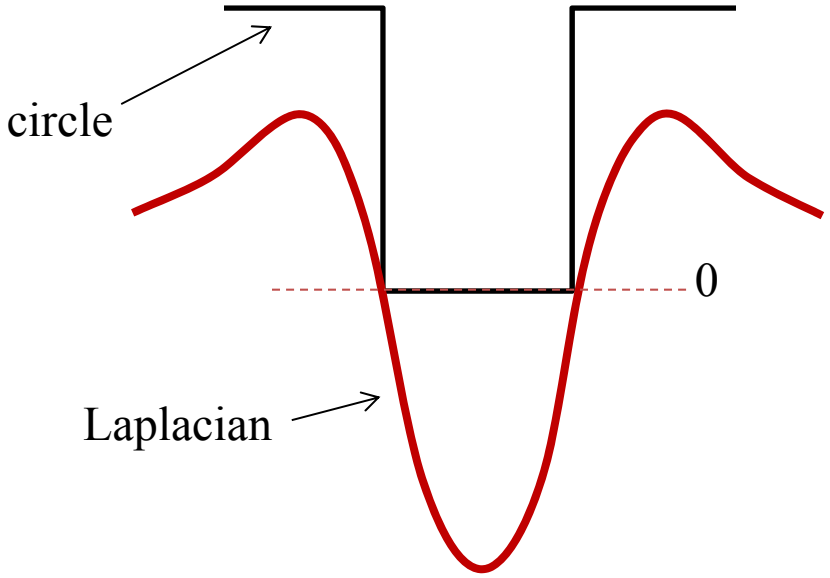
- At what scale does the Laplacian achieve a maximum response to a binary circle of radius r?
- To get maximum response, the zeros of the Laplacian have to be aligned with the circle
- The Laplacian is given by (up to scale):

$$(x^2 + y^2 - 2\sigma^2)\, e^{-(x^2+y^2)/2\sigma^2}$$

- Therefore, the maximum response occurs at $\sigma = r / \sqrt{2}$.

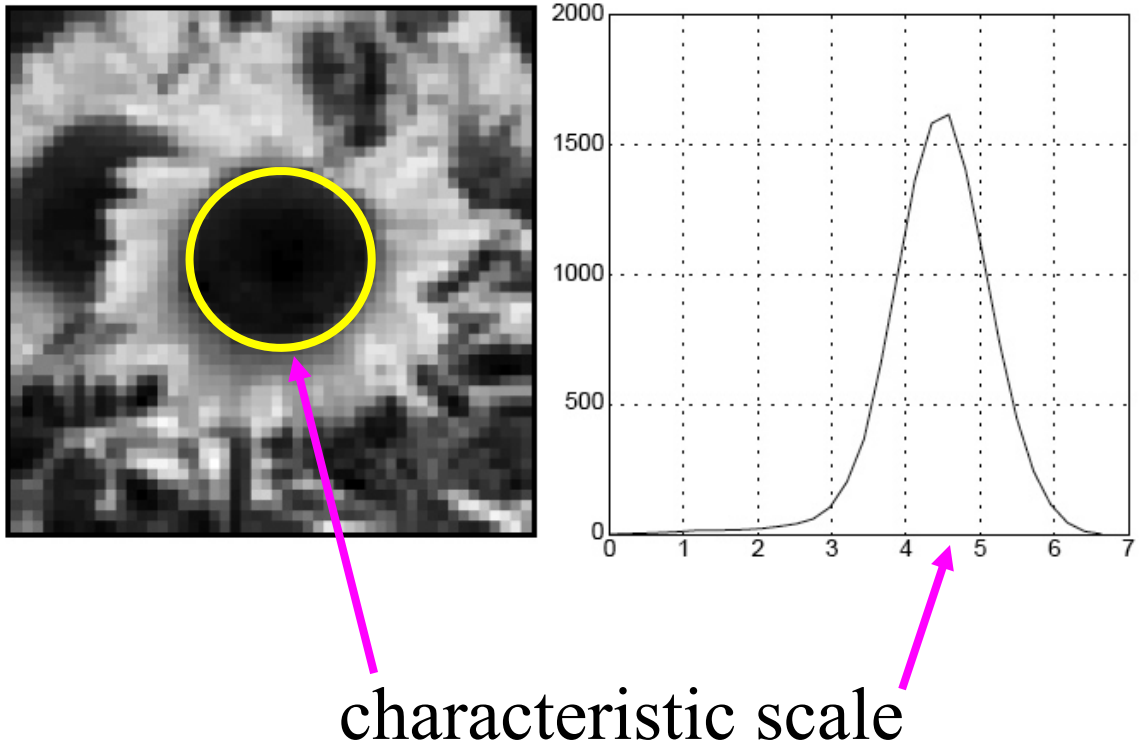


image



Characteristic scale

- We define the characteristic scale of a blob as the scale that produces peak of Laplacian response in the blob center



T. Lindeberg (1998). "Feature detection with automatic scale selection." *International Journal of Computer Vision* **30** (2): pp 77--116.

Scale-space blob detector

1. Convolve image with scale-normalized Laplacian at several scales

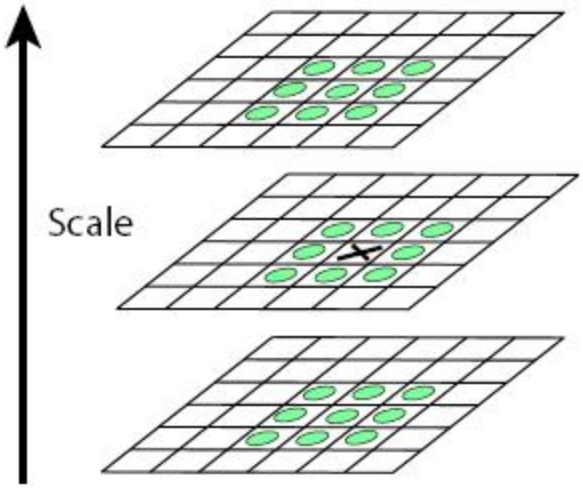
Scale-space blob detector: Example

Scale-space blob detector: Example

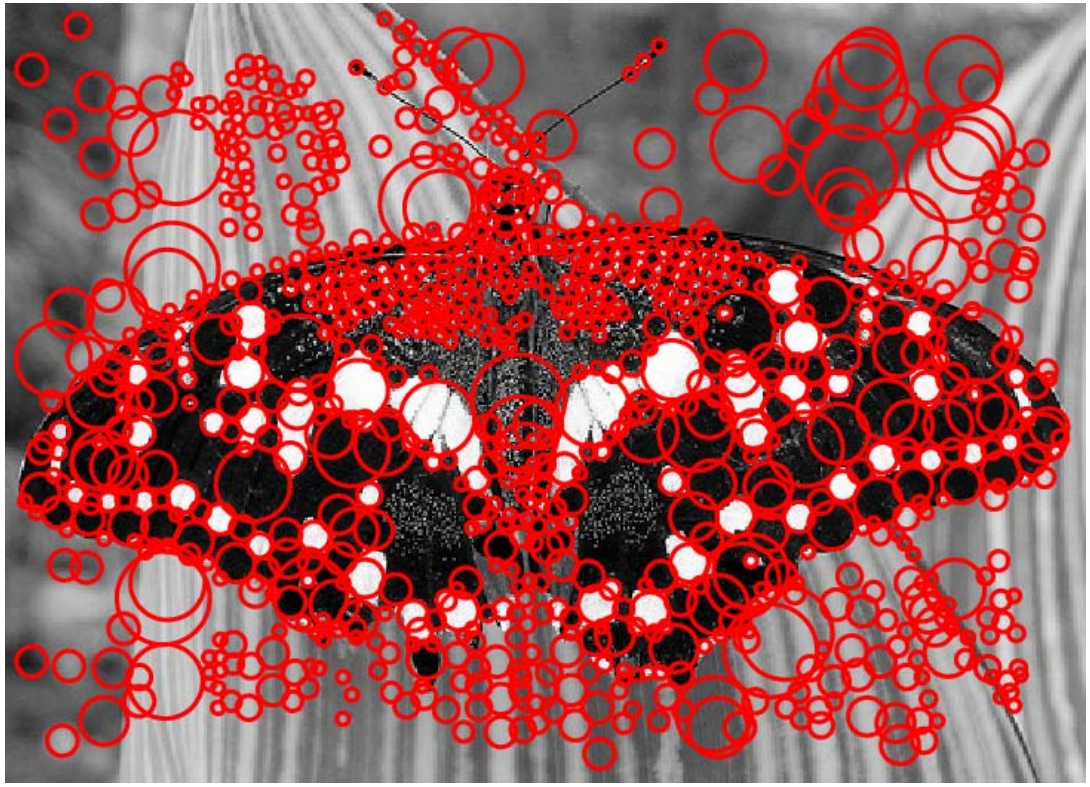
sigma = 11.9912

Scale-space blob detector

1. Convolve image with scale-normalized Laplacian at several scales
2. Find maxima of squared Laplacian response in scale-space



Scale-space blob detector: Example



Efficient implementation

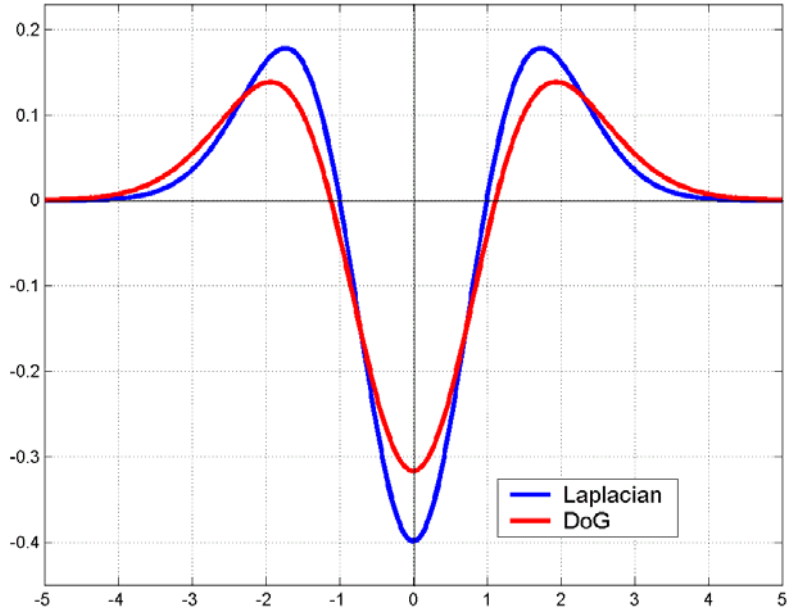
- Approximating the Laplacian with a difference of Gaussians:

$$L = \sigma^2 \left(G_{xx}(x, y, \sigma) + G_{yy}(x, y, \sigma) \right)$$

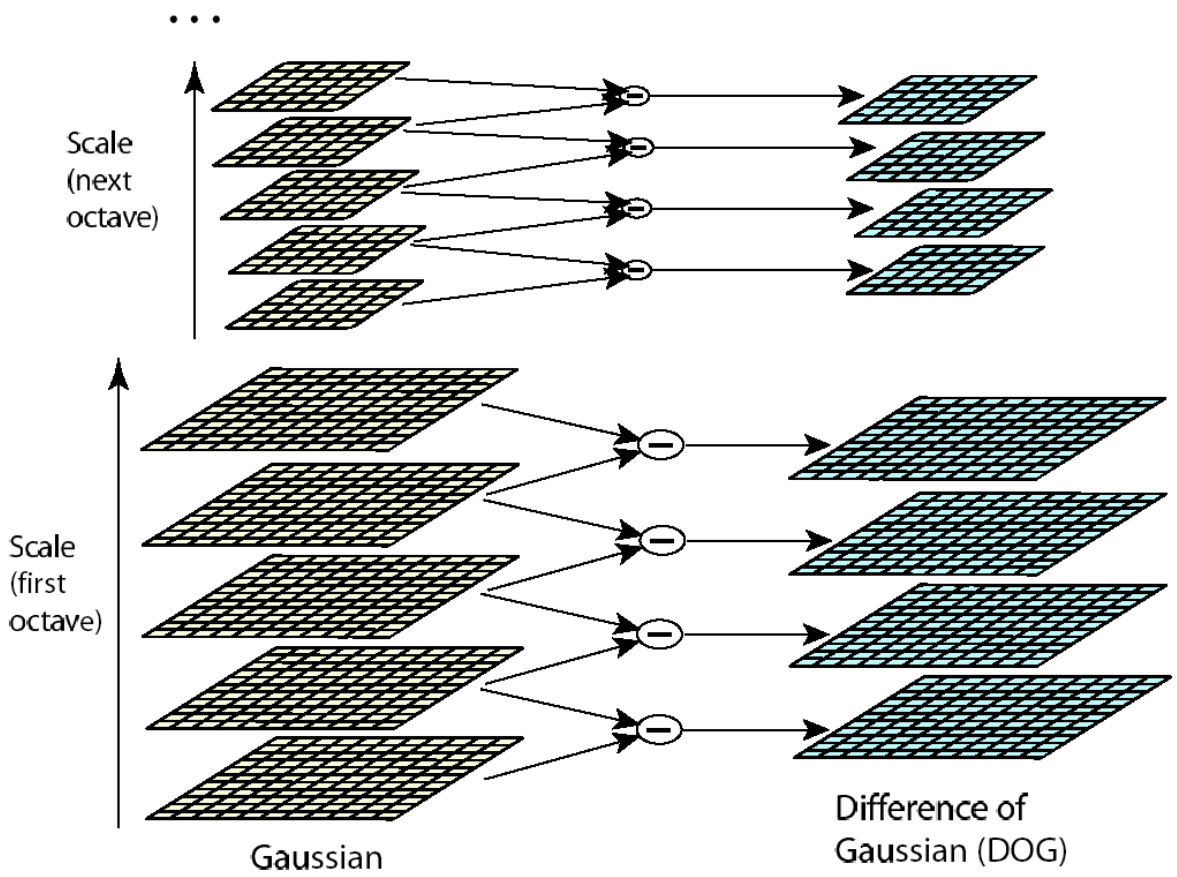
(Laplacian)

$$DoG = G(x, y, k\sigma) - G(x, y, \sigma)$$

(Difference of Gaussians)



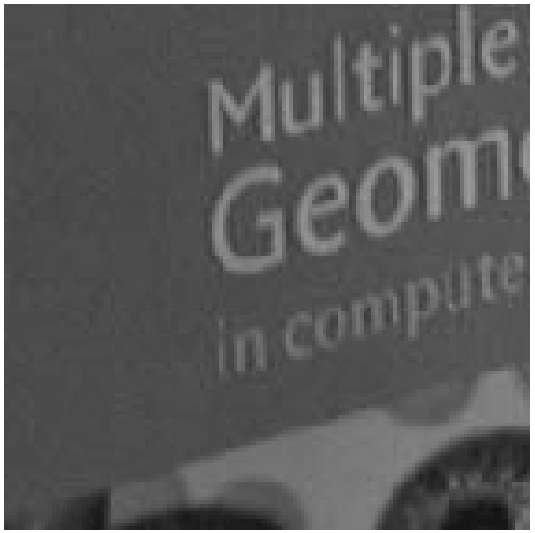
Efficient implementation



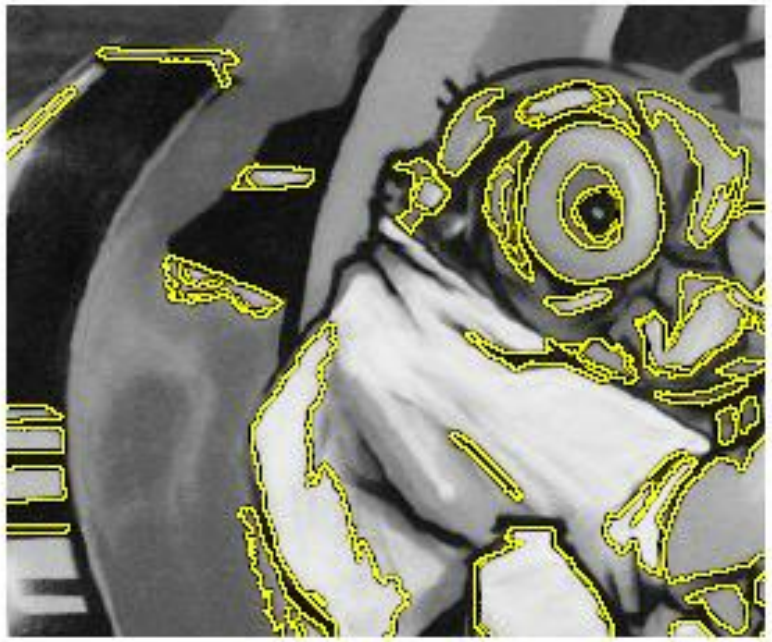
David G. Lowe. **"Distinctive image features from scale-invariant keypoints."** *IJCV* 60 (2), pp. 91-110, 2004.

Maximally Stable Extremal Regions [Matas '02]

- Based on Watershed segmentation algorithm
- Select regions that stay stable over a large parameter range



Example Results: MSER



Local Descriptors

- The ideal descriptor should be
  - Robust
  - Distinctive
  - Compact
  - Efficient

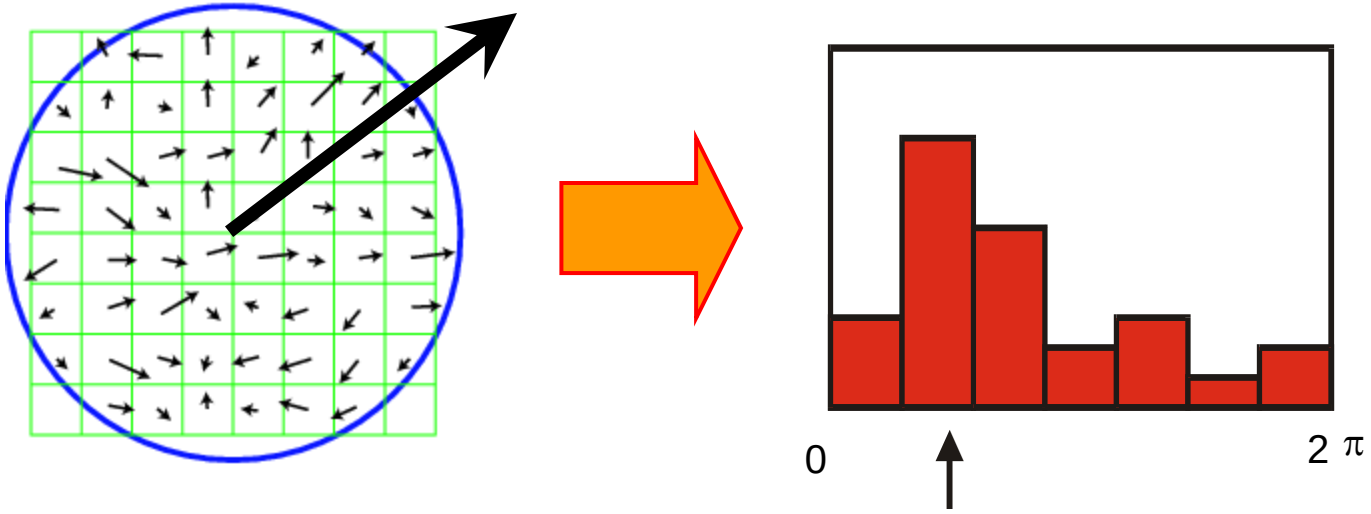
- Most available descriptors focus on edge/gradient information
  - Capture texture information
  - Color rarely used

From feature detection to feature description

- Scaled and rotated versions of the same neighborhood will give rise to blobs that are related by the same transformation
- What to do if we want to compare the appearance of these image regions?
  - *Normalization*: transform these regions into same-size circles
  - Problem: rotational ambiguity

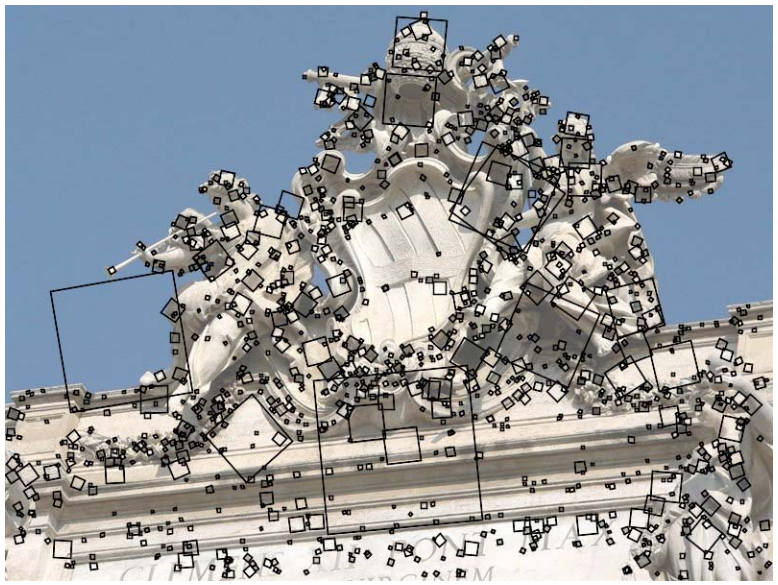
Eliminating rotation ambiguity

- To assign a unique orientation to circular image windows:
  - Create histogram of local gradient directions in the patch
  - Assign canonical orientation at peak of smoothed histogram



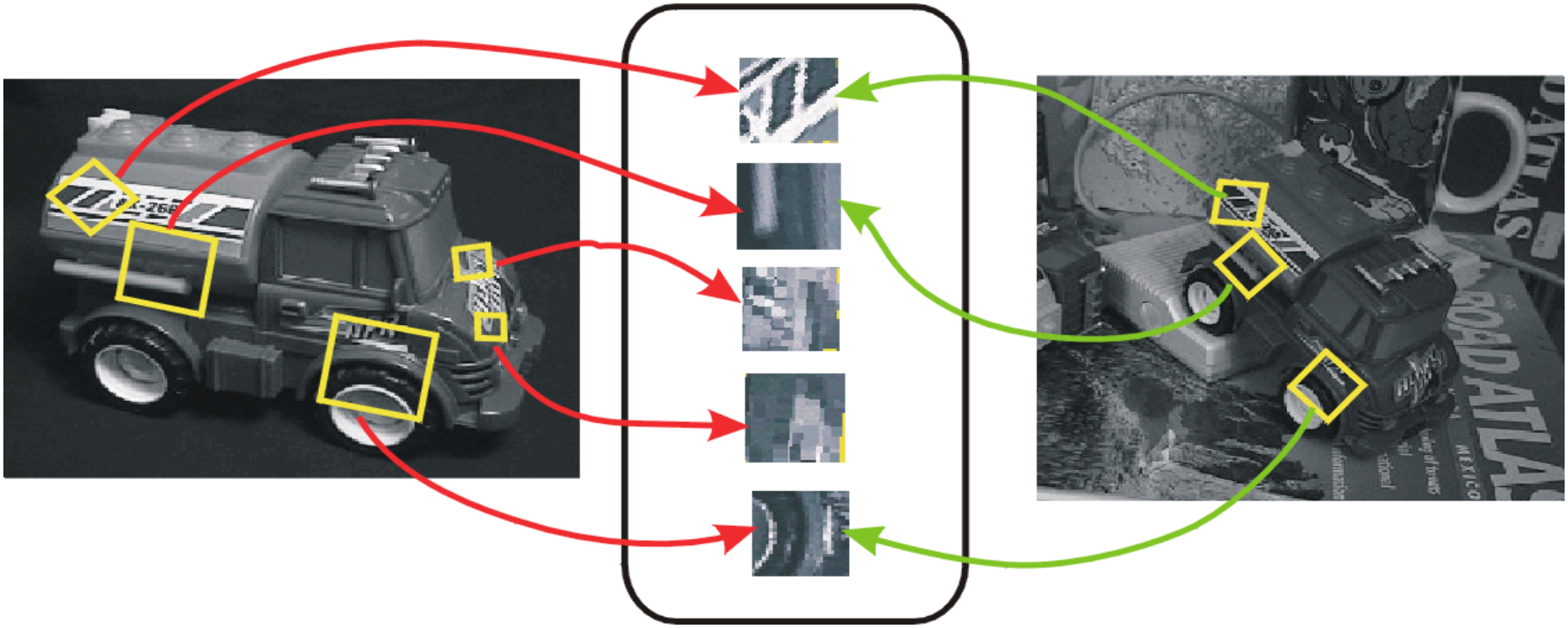
SIFT features

- Detected features with characteristic scales and orientations:



David G. Lowe. **"Distinctive image features from scale-invariant keypoints."** *IJCV* 60 (2), pp. 91-110, 2004.

From feature detection to feature description



- Detection is *covariant*:

  features(transform(image)) = transform(features(image))

- Description is *invariant*:

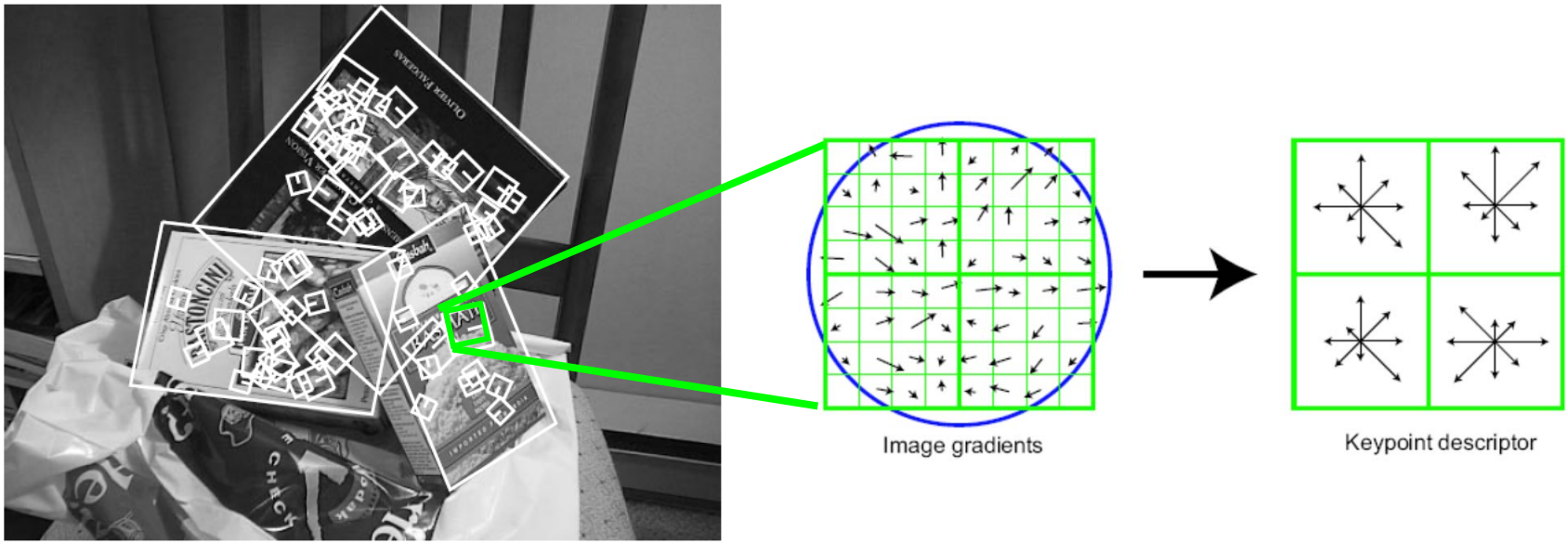
  features(transform(image)) = features(image)

Properties of SIFT

Extraordinarily robust detection and description technique

- Can handle changes in viewpoint
  - Up to about 60 degree out-of-plane rotation
- Can handle significant changes in illumination
  - Sometimes even day vs. night
- Fast and efficient—can run in real time
- Lots of code available

Local Descriptors: SIFT Descriptor



Histogram of oriented gradients

- **Captures important texture information**
- **Robust to small translations / affine deformations**

[Lowe, ICCV 1999]

Details of Lowe's SIFT algorithm

- Run DoG detector
  - Find maxima in location/scale space
  - Remove edge points (bad localization)
- Find all major orientations
  - Bin orientations into 36 bin histogram
    - Weight by gradient magnitude
    - Weight by distance to center (Gaussian-weighted mean)
  - Return orientations within 0.8 of peak of histogram
    - Use parabola for better orientation fit
- For each (x,y,scale,orientation), create descriptor:
  - Sample 16x16 gradient mag. and rel. orientation
  - Bin 4x4 samples into 4x4 histograms
  - Threshold values to max of 0.2, divide by L2 norm
  - Final descriptor: 4x4x8 normalized histograms

$$\mathbf{H} = \begin{bmatrix} D_{xx} & D_{xy} \\ D_{xy} & D_{yy} \end{bmatrix}$$

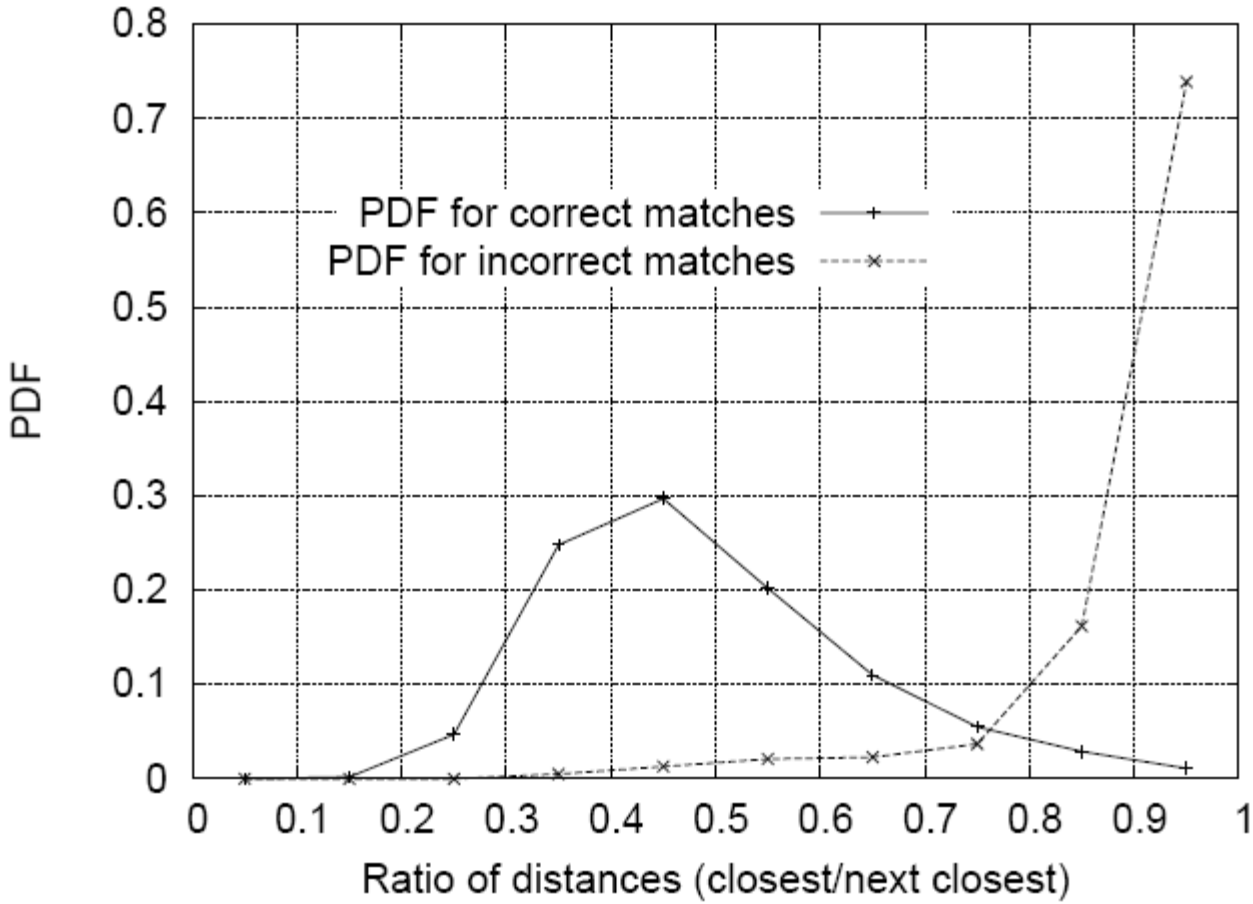
$$\frac{\mathrm{Tr}(\mathbf{H})^2}{\mathrm{Det}(\mathbf{H})} < \frac{(r+1)^2}{r}$$

r: eigenvalue ratio

Lowe IJCV 2004

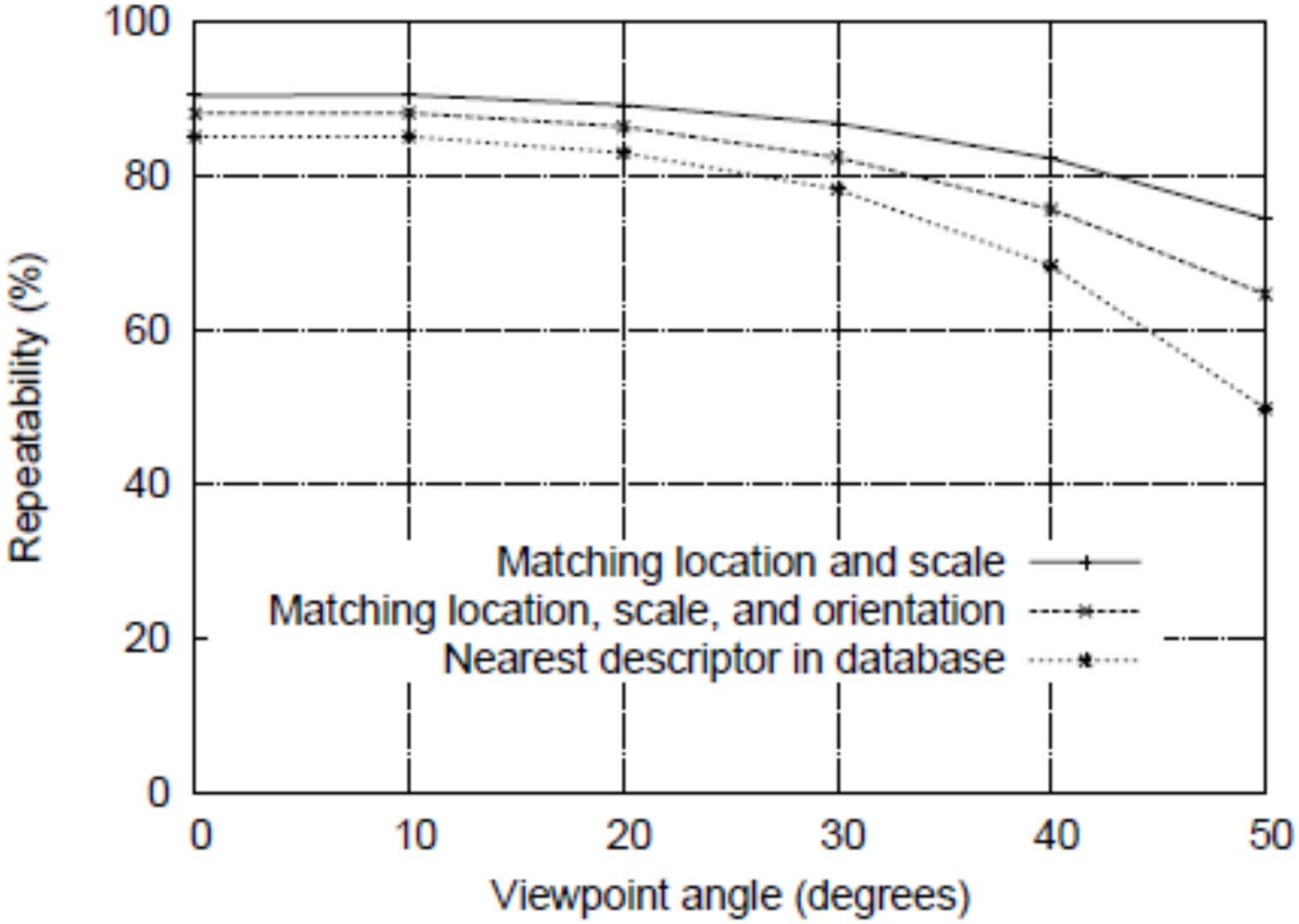
Matching SIFT Descriptors

- Nearest neighbor (Euclidean distance)
- Threshold ratio of nearest to 2nd nearest descriptor

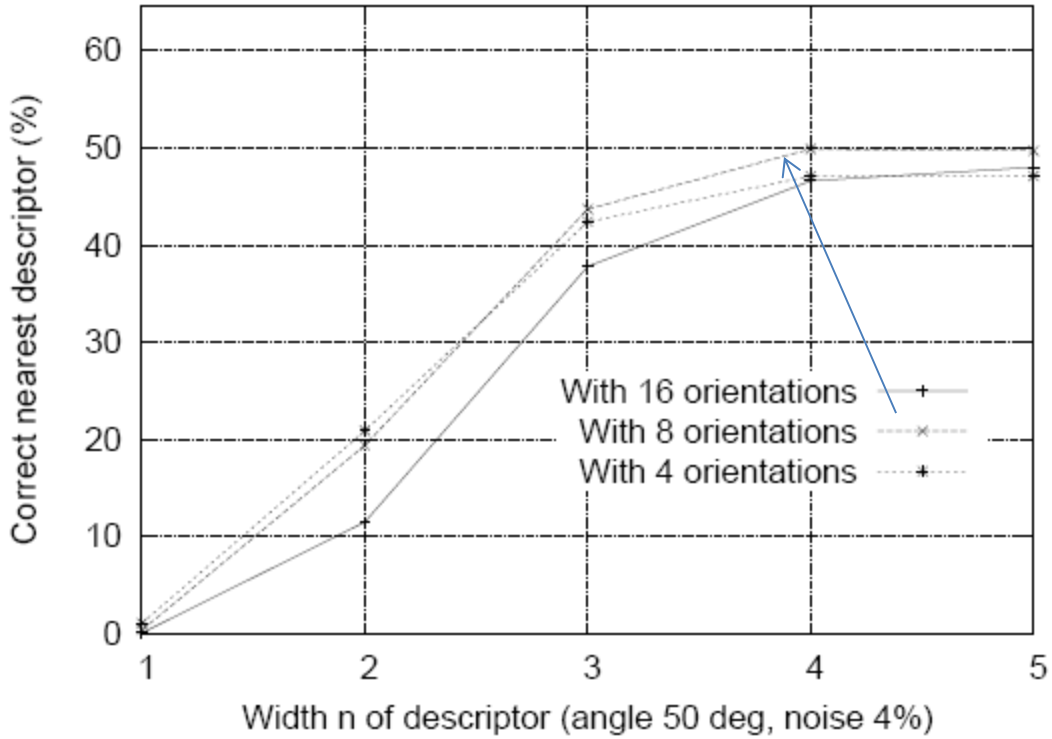


Lowe IJCV 2004

SIFT Repeatability

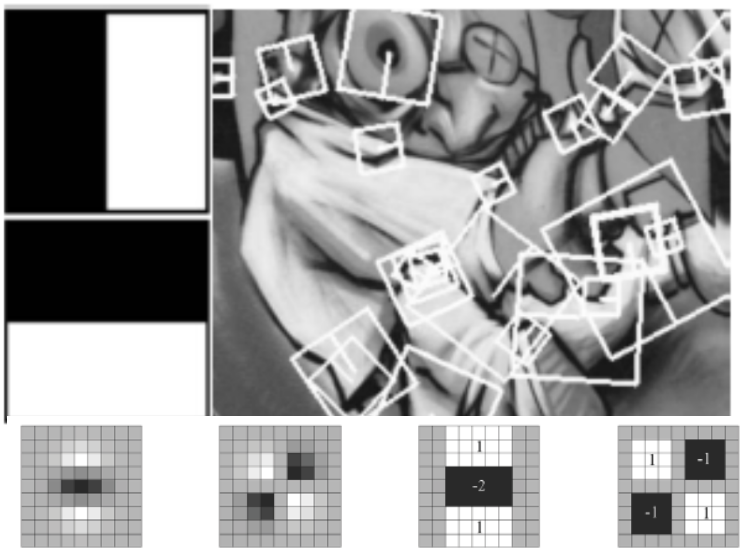


SIFT Repeatability



Lowe IJCV 2004

Local Descriptors: SURF

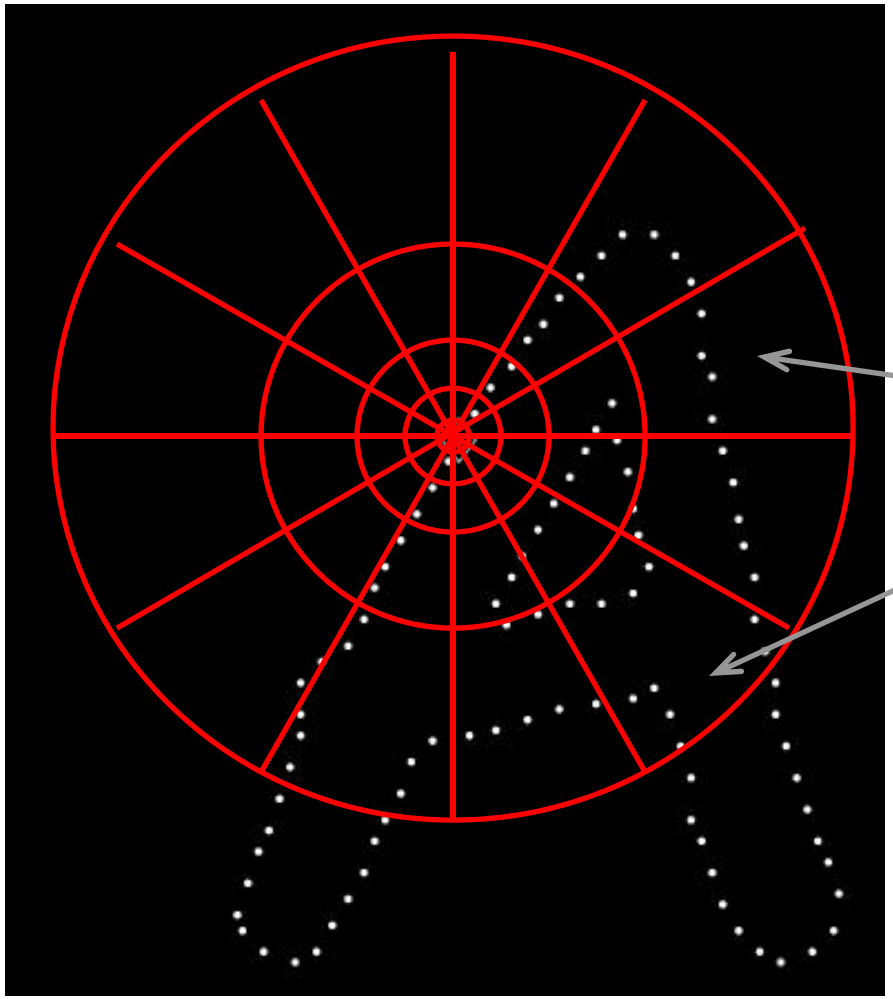


Fast approximation of SIFT idea

- Efficient computation by 2D box filters & integral images
  $\Rightarrow$ 6 times faster than SIFT
- Equivalent quality for object identification

[Bay, ECCV'06], [Cornelis, CVGPU'08]

Local Descriptors: Shape Context



Count the number of points inside each bin, e.g.:

Count = 4

⋮

Count = 10

Log-polar binning: more precision for nearby points, more flexibility for farther points.

Belongie & Malik, ICCV 2001

Choosing a detector

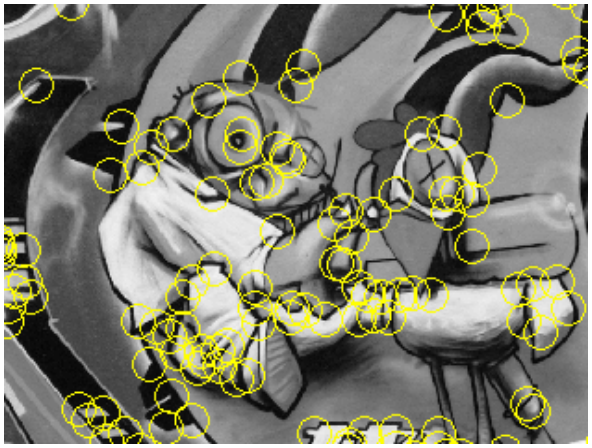
- What do you want it for?
  - Precise localization in x-y: Harris
  - Good localization in scale: Difference of Gaussian
  - Flexible region shape: MSER
- Best choice often application dependent
  - Harris-/Hessian-Laplace/DoG work well for many natural categories
  - MSER works well for buildings and printed things
- Why choose?
  - Get more points with more detectors
- There have been extensive evaluations/comparisons
  - All detectors/descriptors shown here work well

Choosing a descriptor

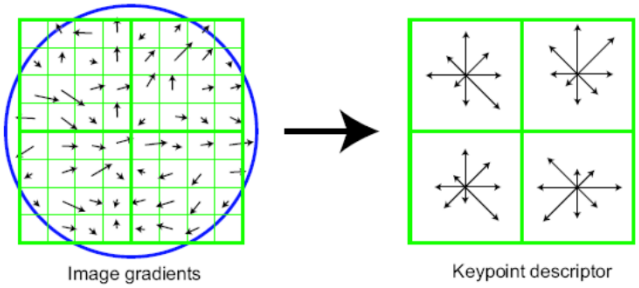
- Again, need not stick to one
- For object instance recognition or stitching, SIFT or variant is a good choice

Things to remember

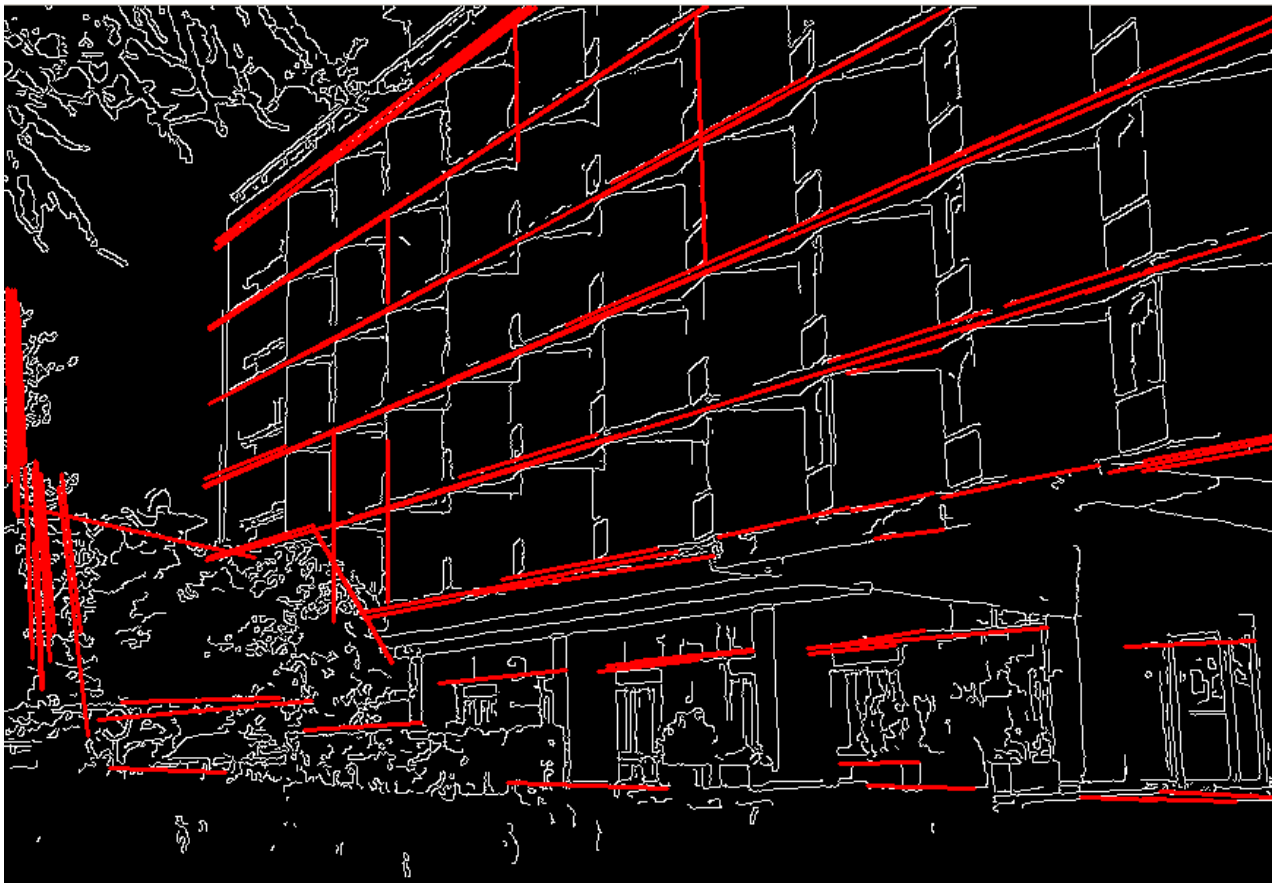
- Keypoint detection: repeatable and distinctive
  - Corners, blobs, stable regions
  - Harris, DoG



- Descriptors: robust and selective
  - spatial histograms of orientation
  - SIFT



Fitting

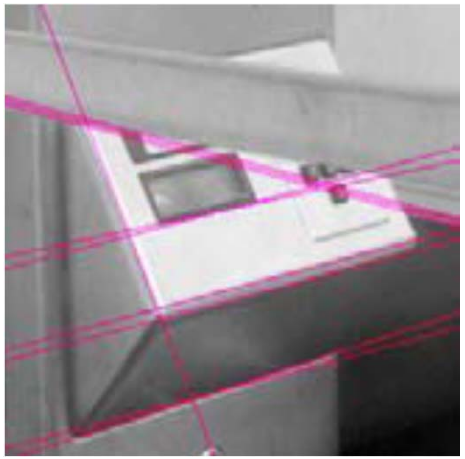


Fitting

- We've learned how to detect edges, corners, blobs. Now what?
- We would like to form a higher-level, more compact representation of the features in the image by grouping multiple features according to a simple model

Fitting

- Choose a parametric model to represent a set of features



simple model: lines

simple model: circles

complicated model: car

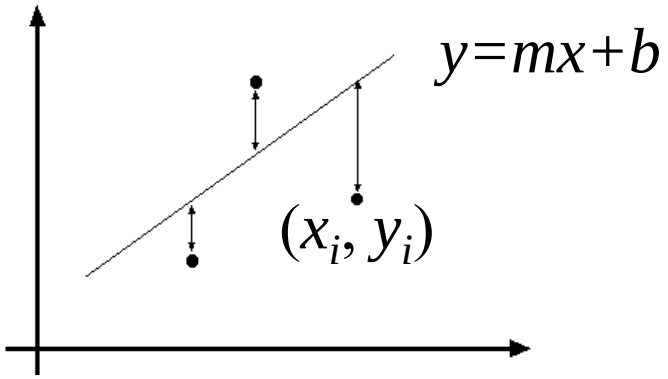
Source: K. Grauman

Fitting: Overview

- If we know which points belong to the line, how do we find the “optimal” line parameters?
  - Least squares
- What if there are outliers?
  - Robust fitting, RANSAC
- What if there are many lines?
  - Voting methods: RANSAC, Hough transform
- What if we’re not even sure it’s a line?
  - Model selection (not covered)

Least squares line fitting

- Data: $(x_1, y_1), \ldots, (x_n, y_n)$
- Line equation: $y_i = m x_i + b$
- Find (m, b) to minimize



$$E = \sum_{i=1}^{n} (y_i - m x_i - b)^2$$

$$E = \|Y - XB\|^2 \quad \text{where} \quad Y = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} \qquad X = \begin{bmatrix} x_1 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{bmatrix} \qquad B = \begin{bmatrix} m \\ b \end{bmatrix}$$

$$E = \|Y - XB\|^2 = (Y - XB)^T (Y - XB) = Y^T Y - 2(XB)^T Y + (XB)^T (XB)$$

$$\frac{dE}{dB} = 2X^T XB - 2X^T Y = 0$$

$$X^T XB = X^T Y$$

Pseudo-inverse solution

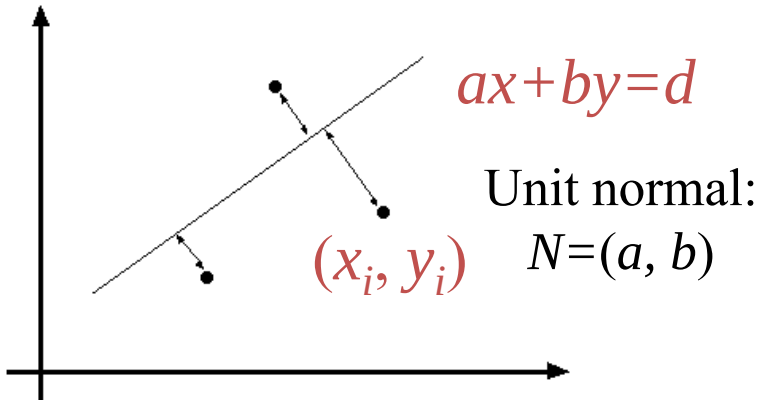
$$B = (X^T X)^{-1} X^T Y$$

Problem with “vertical” least squares

- Not rotation-invariant
- Fails completely for vertical lines

Total least squares

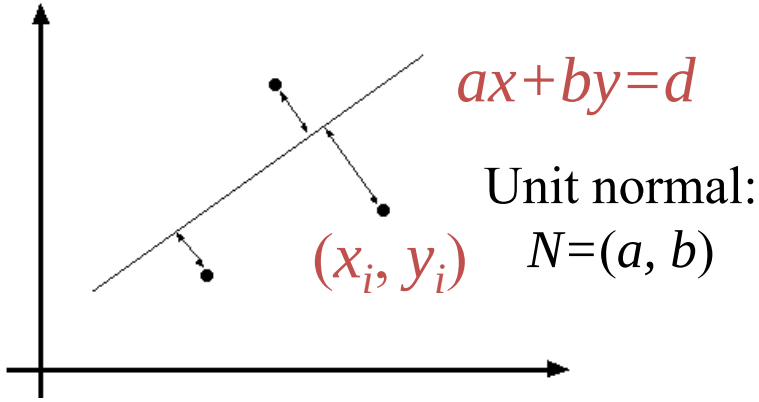
•Distance between point (x_i, y_i) and line $ax+by=d$ $(a^2+b^2=1)$: $|ax_i + by_i - d|$



Total least squares

- Distance between point (x_i, y_i) and line $ax+by=d$ $(a^2+b^2=1)$: $|ax_i + by_i - d|$
- Find (a, b, d) to minimize the sum of squared perpendicular distances

$$E = \sum_{i=1}^{n} (a x_i + b y_i - d)^2$$



Total least squares

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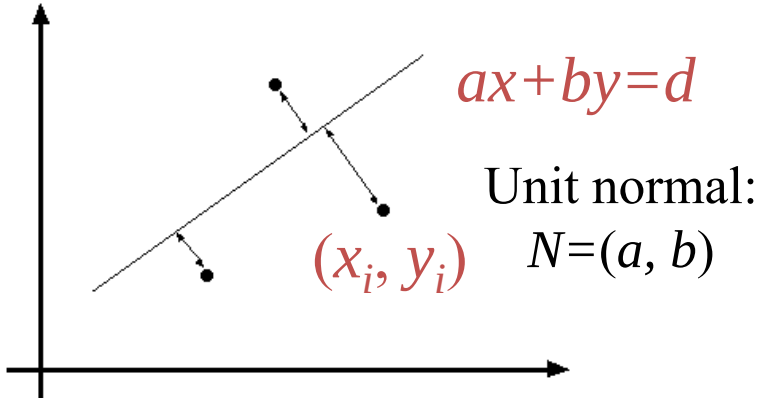


$$E = \sum_{i=1}^{n} (ax_i + by_i - d)^2$$

$$\frac{\partial E}{\partial d} = \sum_{i=1}^{n} -2(ax_i + by_i - d) = 0$$

$$d = \frac{a}{n}\sum_{i=1}^{n} x_i + \frac{b}{n}\sum_{i=1}^{n} y_i = a\bar{x} + b\bar{y}$$

$$E = \sum_{i=1}^{n} (a(x_i - \bar{x}) + b(y_i - \bar{y}))^2 = \left\| \begin{bmatrix} x_1 - \bar{x} & y_1 - \bar{y} \\ \vdots & \vdots \\ x_n - \bar{x} & y_n - \bar{y} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \right\|^2 = (UN)^T (UN)$$

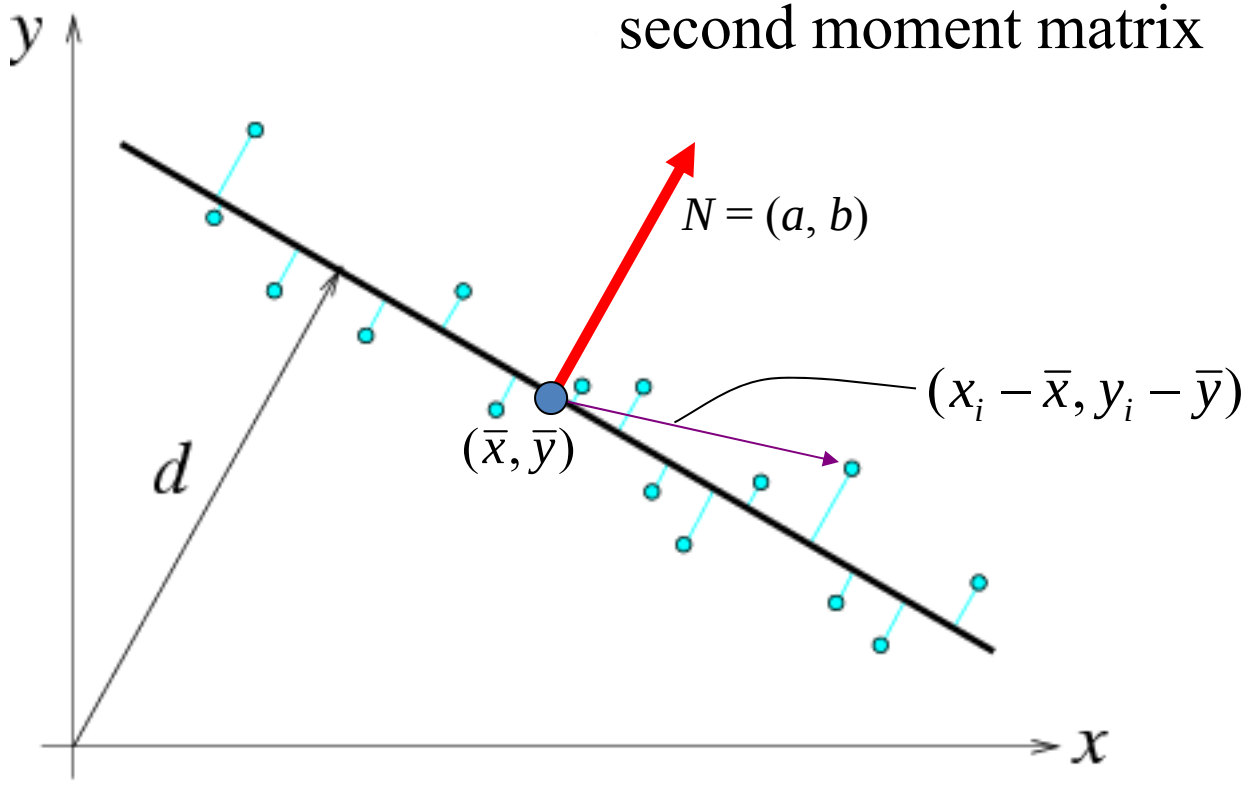
$$\frac{dE}{dN} = 2(U^T U)N = 0$$

Solution to $(U^T U)N = 0$, subject to $\|N\|^2 = 1$: eigenvector of $U^T U$ associated with the smallest eigenvalue (least squares solution to *homogeneous linear system* $UN = 0$)

Total least squares

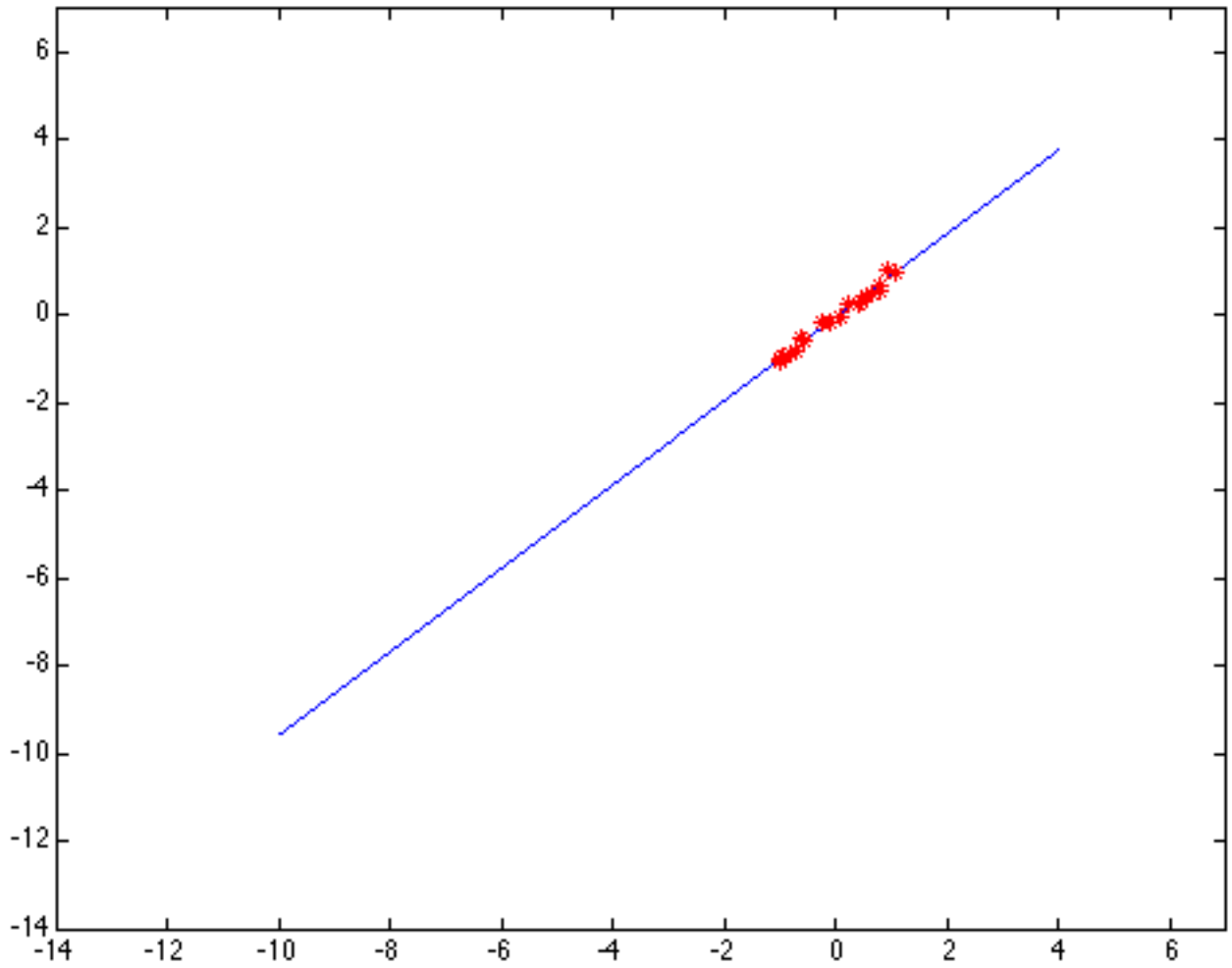
$$U = \begin{bmatrix} x_1 - \bar{x} & y_1 - \bar{y} \\ \vdots & \vdots \\ x_n - \bar{x} & y_n - \bar{y} \end{bmatrix} \qquad U^T U = \begin{bmatrix} \sum_{i=1}^{n} (x_i - \bar{x})^2 & \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y}) \\ \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y}) & \sum_{i=1}^{n} (y_i - \bar{y})^2 \end{bmatrix}$$

second moment matrix

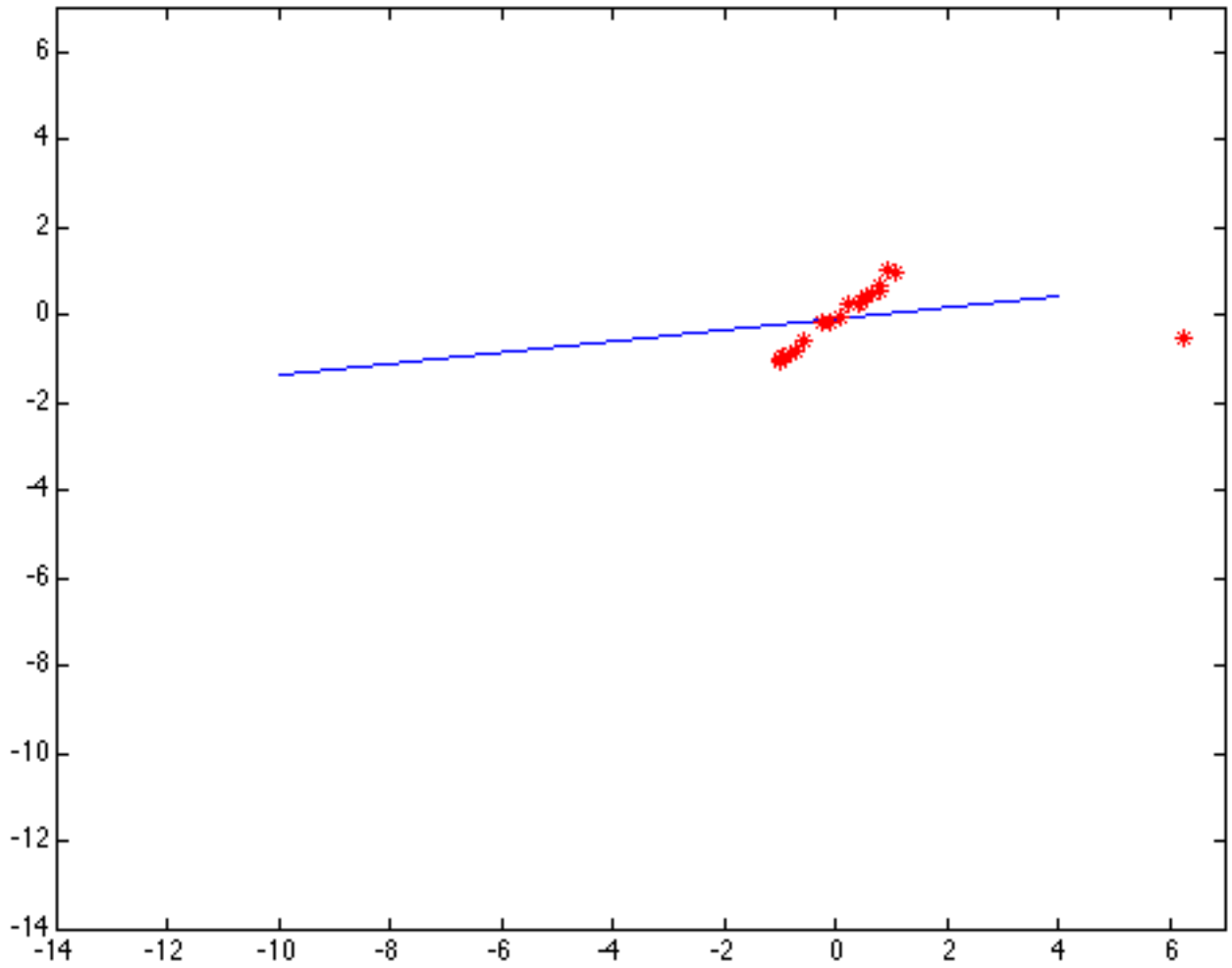


F&P (2nd ed.) sec. 22.1

Least squares: Robustness to noise



Least squares: Robustness to noise



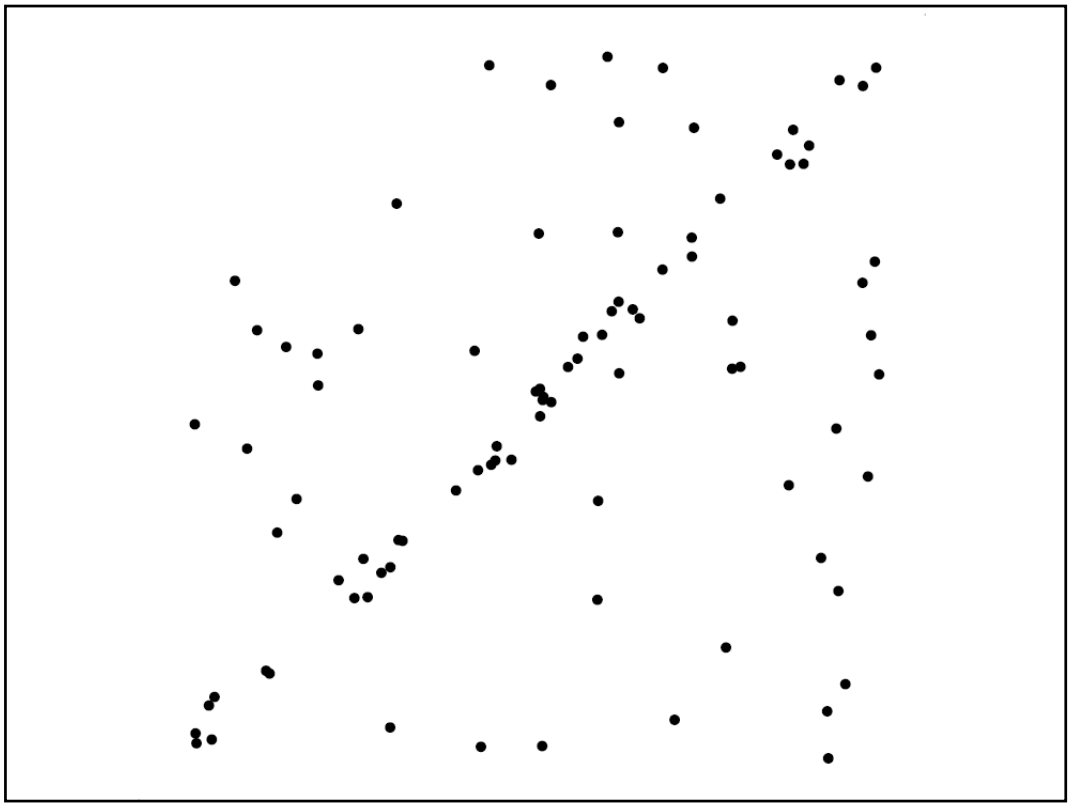
Problem: squared error heavily penalizes outliers

RANSAC

- Random sample consensus (RANSAC): Very general framework for model fitting in the presence of outliers
- Outline
  - Choose a small subset of points uniformly at random
  - Fit a model to that subset
  - Find all remaining points that are “close” to the model and reject the rest as outliers
  - Do this many times and choose the best model

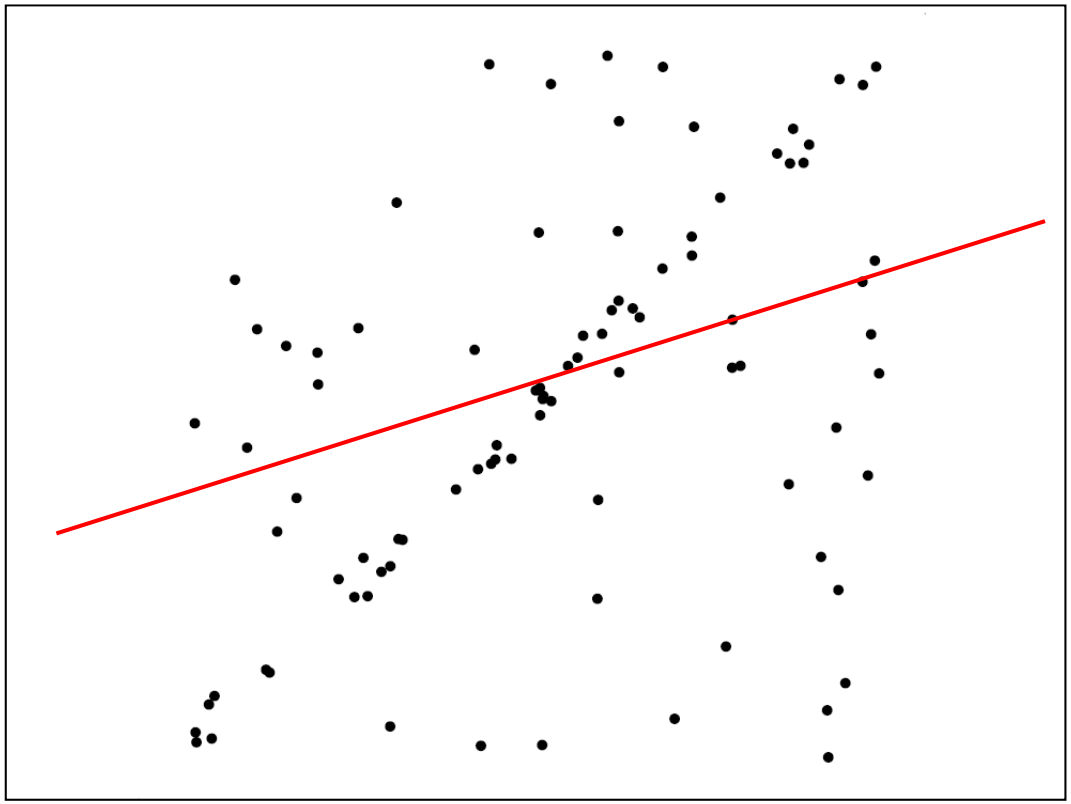
M. A. Fischler, R. C. Bolles. Random Sample Consensus: A Paradigm for Model Fitting with Applications to Image Analysis and Automated Cartography. Comm. of the ACM, Vol 24, pp 381-395, 1981.

RANSAC for line fitting example



Source: R. Raguram

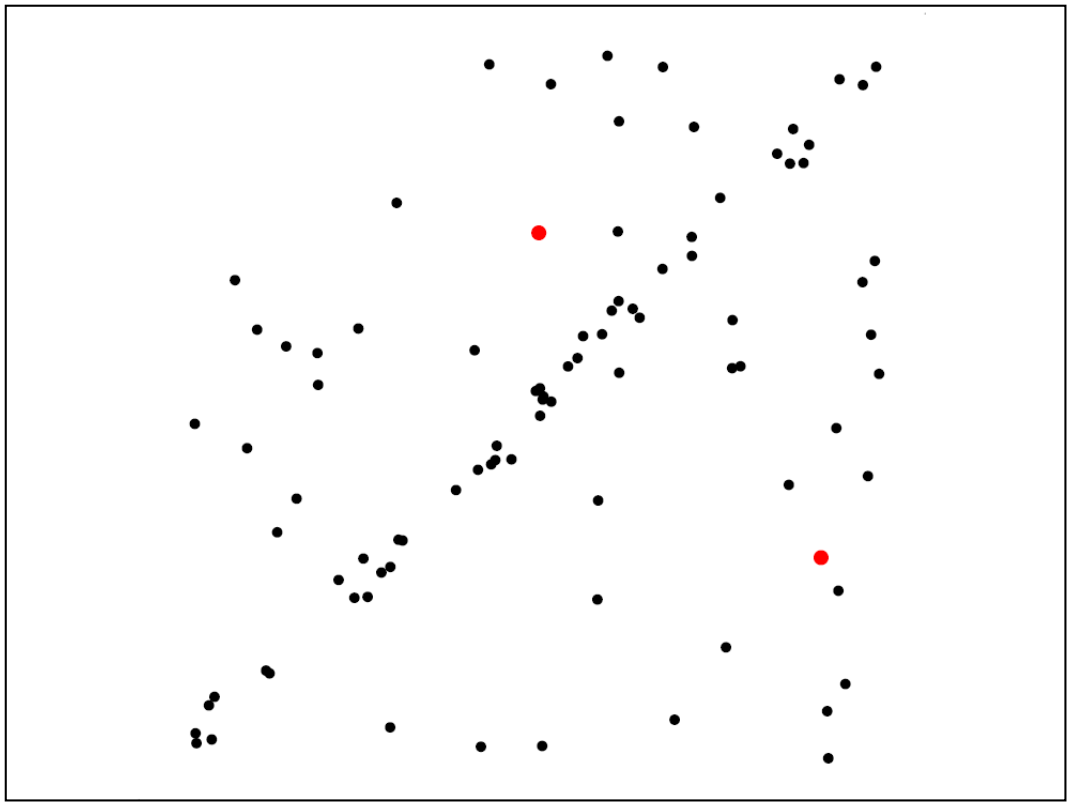
RANSAC for line fitting example



Least-squares fit

Source: R. Raguram

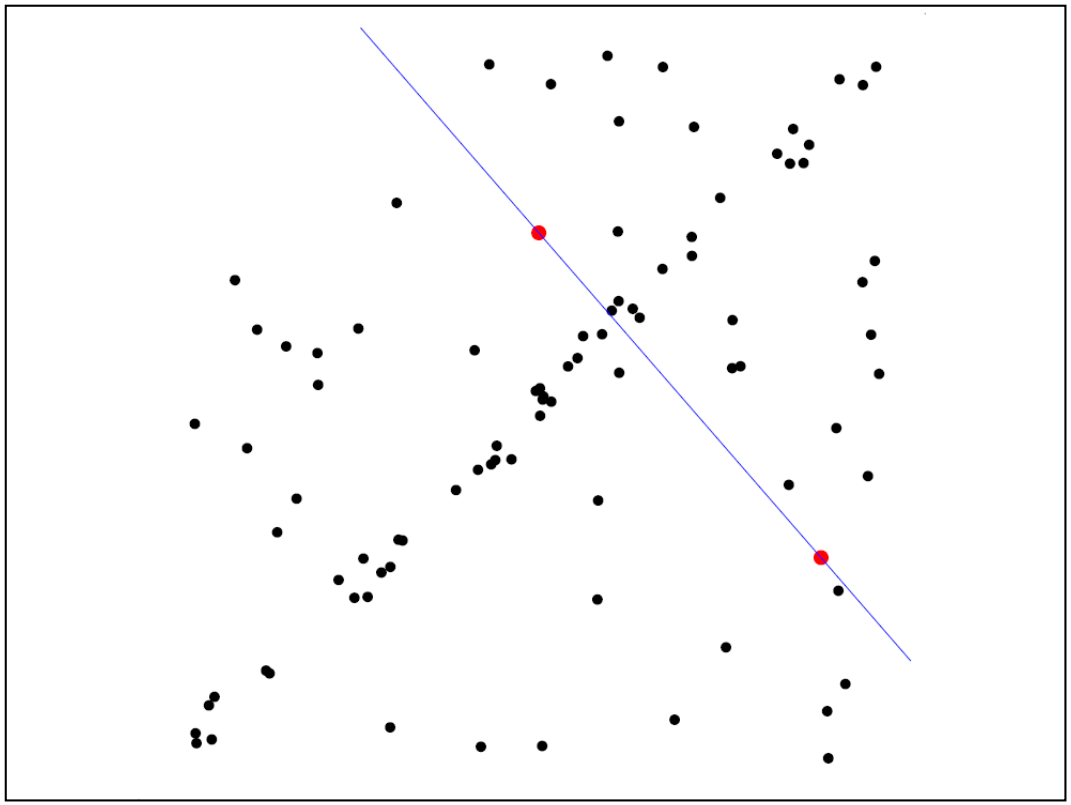
RANSAC for line fitting example



1. Randomly select minimal subset of points

Source: R. Raguram

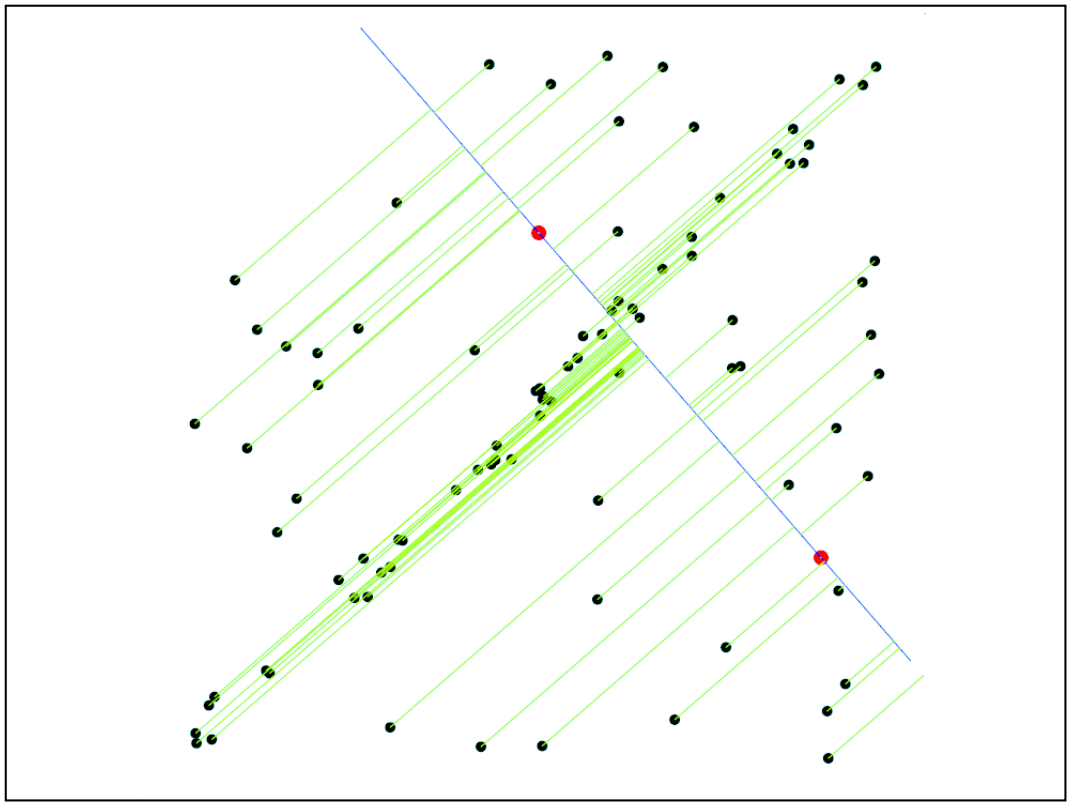
RANSAC for line fitting example



1. Randomly select minimal subset of points
2. Hypothesize a model

Source: R. Raguram

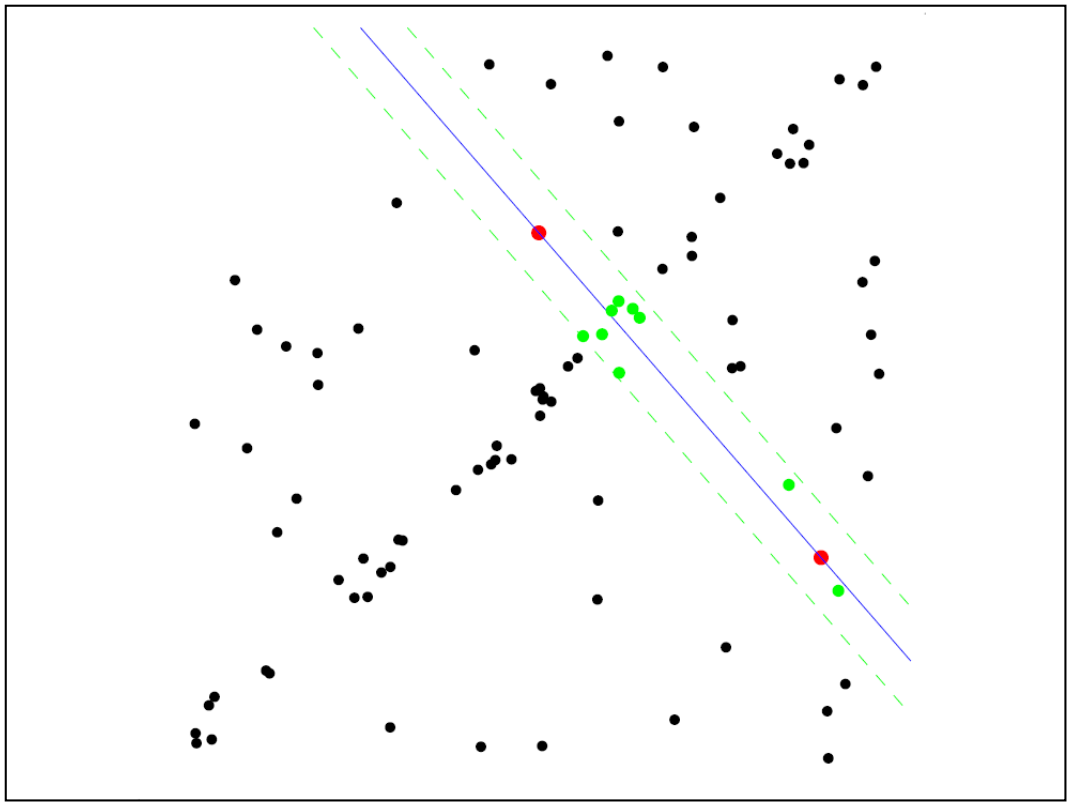
RANSAC for line fitting example



1. Randomly select minimal subset of points
2. Hypothesize a model
3. Compute error function

Source: R. Raguram

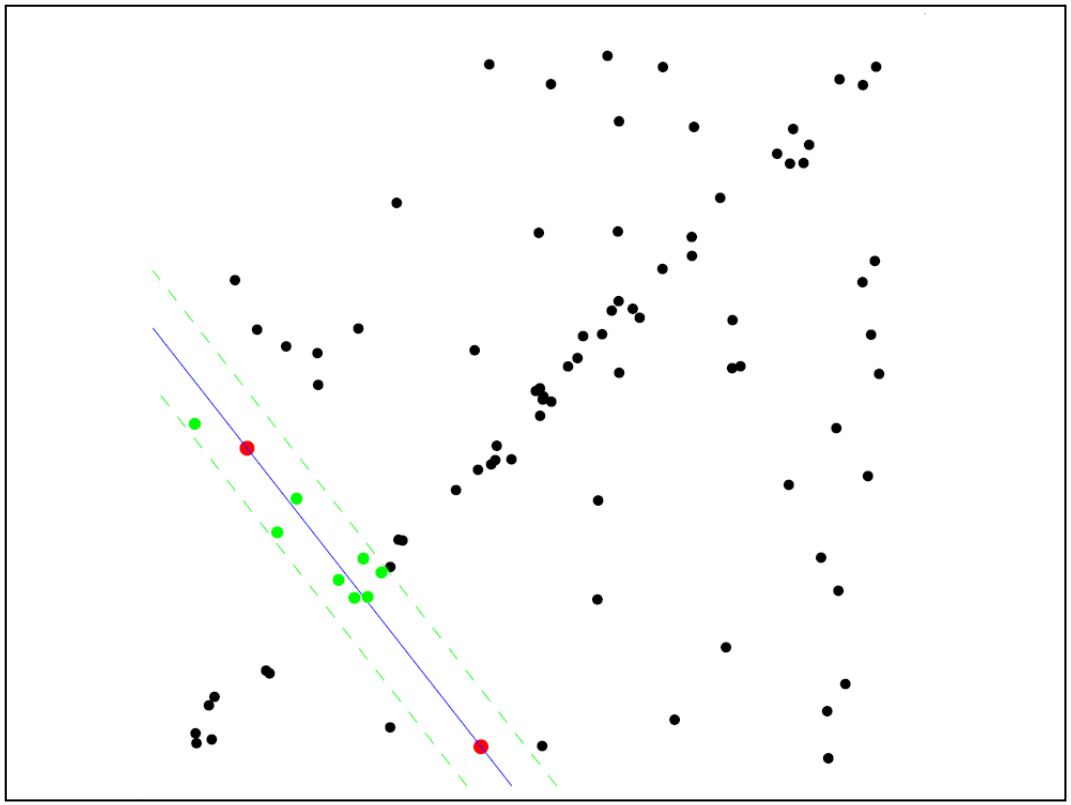
RANSAC for line fitting example



1. Randomly select minimal subset of points
2. Hypothesize a model
3. Compute error function
4. Select points consistent with model

Source: R. Raguram

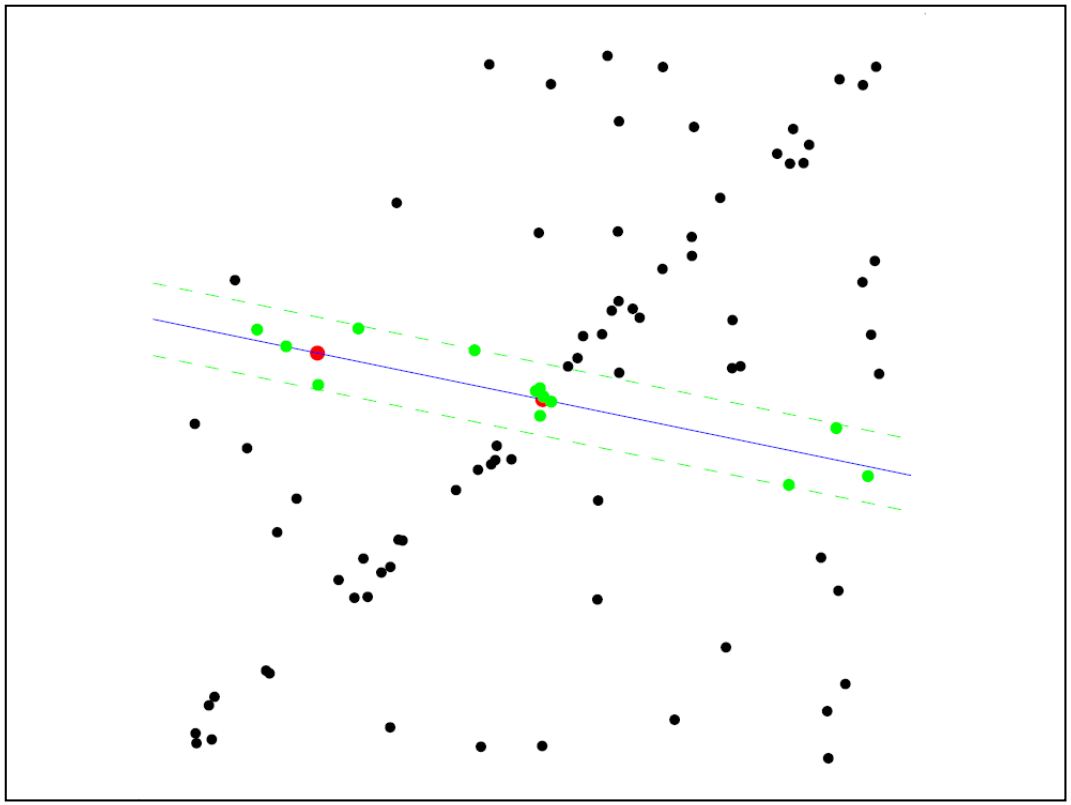
RANSAC for line fitting example



1. Randomly select minimal subset of points
2. Hypothesize a model
3. Compute error function
4. Select points consistent with model
5. Repeat *hypothesize-and-verify* loop

Source: R. Raguram

RANSAC for line fitting example

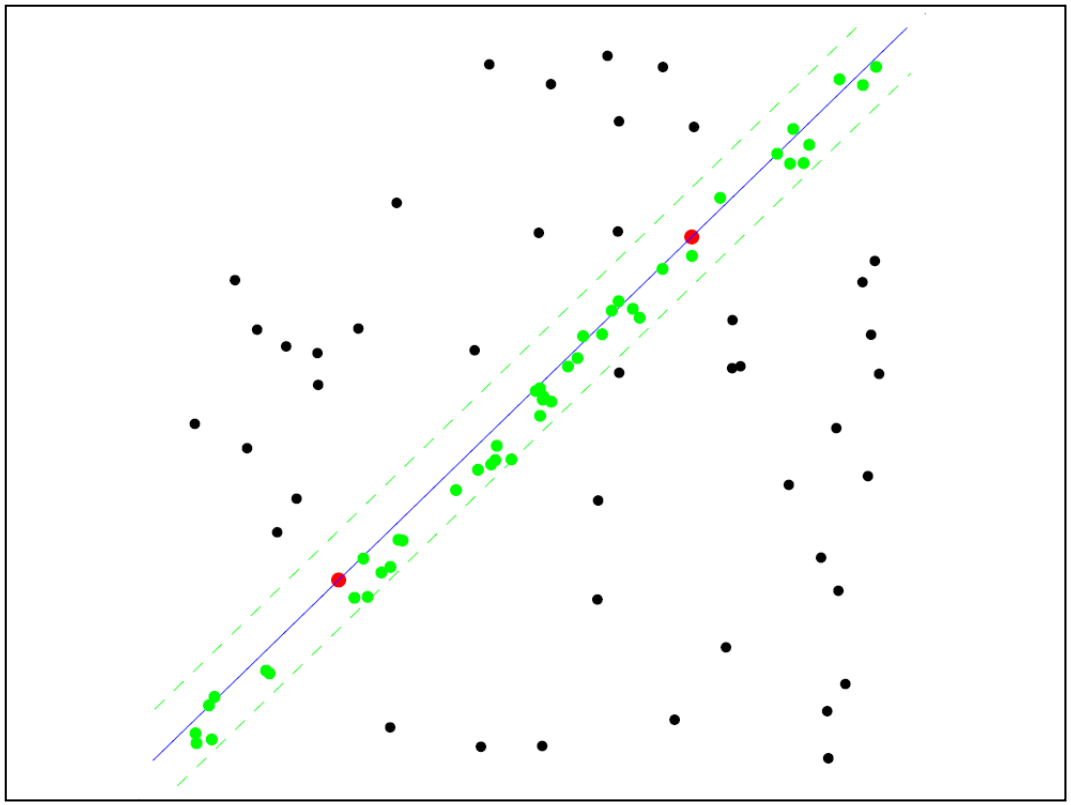


1. Randomly select minimal subset of points
2. Hypothesize a model
3. Compute error function
4. Select points consistent with model
5. Repeat *hypothesize-and-verify* loop

Source: R. Raguram

RANSAC for line fitting example

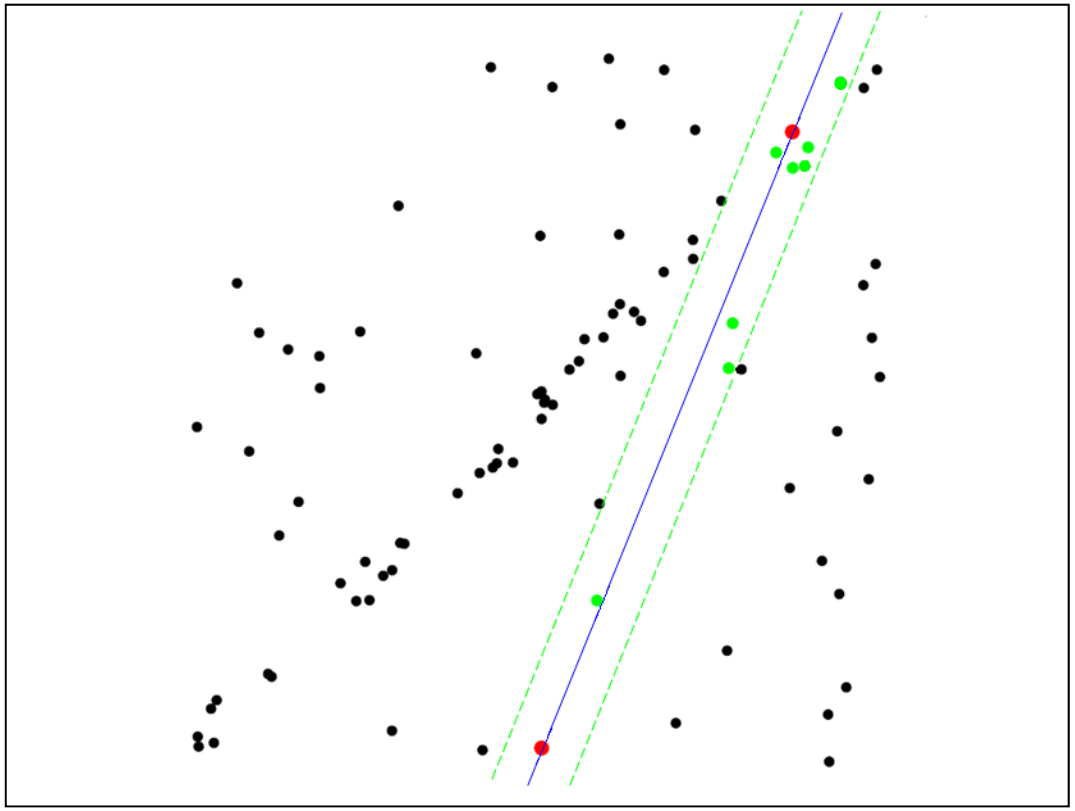
Uncontaminated sample



1. Randomly select minimal subset of points
2. Hypothesize a model
3. Compute error function
4. Select points consistent with model
5. Repeat *hypothesize-and-verify* loop

Source: R. Raguram

RANSAC for line fitting example



1. Randomly select minimal subset of points
2. Hypothesize a model
3. Compute error function
4. Select points consistent with model
5. Repeat *hypothesize-and-verify* loop

Source: R. Raguram

RANSAC for line fitting

- Repeat ***N*** times:
- Draw ***s*** points uniformly at random
- Fit line to these ***s*** points
- Find *inliers* to this line among the remaining points (i.e., points whose distance from the line is less than ***t***)
- If there are ***d*** or more inliers, accept the line and refit using all inliers

Choosing the parameters

- Initial number of points ***s***
  - Typically minimum number needed to fit the model
- Distance threshold ***t***
  - Choose ***t*** so probability for inlier is *p* (e.g. 0.95)
  - Zero-mean Gaussian noise with std. dev. σ: $t^2=3.84\sigma^2$
- Number of samples ***N***
  - Choose ***N*** so that, with probability *p*, at least one random sample is free from outliers (e.g. p=0.99) (outlier ratio: *e*)

Source: M. Pollefeys

Choosing the parameters

- Initial number of points ***s***
  - Typically minimum number needed to fit the model
- Number of samples ***N***
  - Choose ***N*** so that, with probability *p*, at least one random sample is free from outliers (e.g. *p*=0.99) (outlier ratio: *e*)

$$\left(1-\left(1-e\right)^{s}\right)^{N}=1-p$$

$$N=\log\left(1-p\right)/\log\left(1-\left(1-e\right)^{s}\right)$$

	proportion of outliers *e*						
s	5%	10%	20%	25%	30%	40%	50%
2	2	3	5	6	7	11	17
3	3	4	7	9	11	19	35
4	3	5	9	13	17	34	72
5	4	6	12	17	26	57	146
6	4	7	16	24	37	97	293
7	4	8	20	33	54	163	588
8	5	9	26	44	78	272	1177

Source: M. Pollefeys

Choosing the parameters

- Initial number of points ***s***
  - Typically minimum number needed to fit the model
- Distance threshold ***t***
  - Choose ***t*** so probability for inlier is *p* (e.g. 0.95)
  - Zero-mean Gaussian noise with std. dev. σ: $t^2=3.84\sigma^2$
- Number of samples ***N***
  - Choose ***N*** so that, with probability *p*, at least one random sample is free from outliers (e.g. p=0.99) (outlier ratio: *e*)
- Consensus set size ***d***
  - Should match expected inlier ratio

Source: M. Pollefeys

Adaptively determining the number of samples

- Outlier ratio *e* is often unknown a priori, so pick worst case, e.g. 50%, and adapt if more inliers are found, e.g. 80% would yield *e*=0.2
- Adaptive procedure:
  - *N*=∞, *sample_count* =0
  - While *N* >*sample_count*
    - Choose a sample and count the number of inliers
    - If inlier ratio is highest of any found so far, set e = 1 – (number of inliers)/(total number of points)
    - Recompute *N* from *e:*

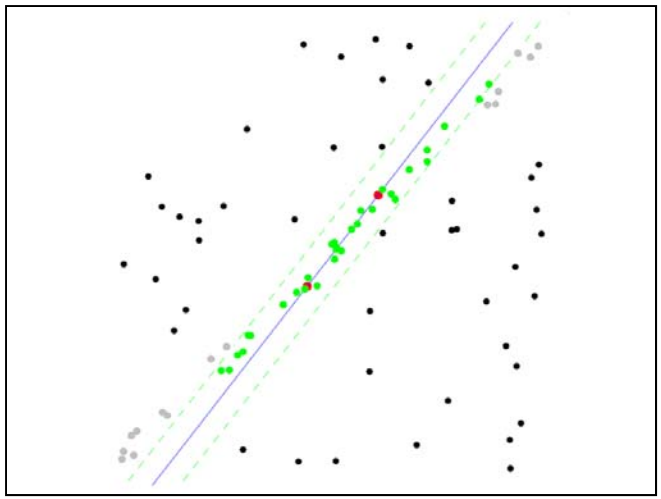
$$N = \log(1-p) / \log\left(1-(1-e)^s\right)$$

    - Increment the *sample_count* by 1

Source: M. Pollefeys

RANSAC pros and cons

- Pros
  - Simple and general
  - Applicable to many different problems
  - Often works well in practice
- Cons
  - Computational time grows quickly with fraction of outliers and number of parameters
  - Not as good for getting multiple fits (though one solution is to remove inliers after each fit and repeat)
  - Sensitivity to threshold t



Slide Credits

- This set of sides contains contributions kindly made available by the following authors
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  - Svetlana Lazebnik
  - Kristen Grauman
  - Bastian Leibe
  - David Lowe
  - Marc Pollefeys
  - Rahul Raguram
  - Steve Seitz
  - Noah Snavely