CS 558: Computer Vision
3rd Set of Notes

Instructor: Philippos Mordohai
Webpage: www.cs.stevens.edu/~mordohai
E-mail: Philippos.Mordohai@stevens.edu
Office: Lieb 215

Overview

- Denoising
  - Based on slides by S. Lazebnik
- Edge detection
  - Based on slides by S. Lazebnik and D. Hoiem
- Feature extraction: Corners
  - Based on slides by S. Lazebnik
- Sampling images
  - Based on slides by D. Hoiem

Image denoising

- How can we reduce noise in a photograph?

Moving average

- Let's replace each pixel with a *weighted* average of its neighborhood
- The weights are called the *filter kernel*
- What are the weights for the average of a 3x3 neighborhood?

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

"box filter"

Source: D. Lowe

Noise

Original

Salt and pepper noise

Impulse noise

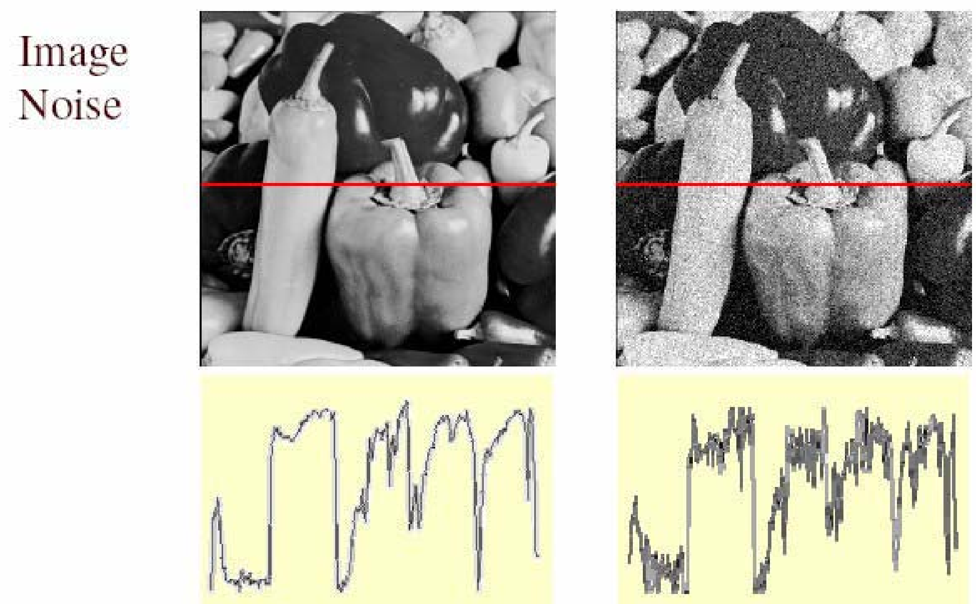
Gaussian noise

- **Salt and pepper noise**: contains random occurrences of black and white pixels
- **Impulse noise:** contains random occurrences of white pixels
- **Gaussian noise**: variations in intensity drawn from a Gaussian normal distribution

Source: S. Seitz

Gaussian noise

- Mathematical model: sum of many independent factors
- Good for small standard deviations
- Assumption: independent, zero-mean noise

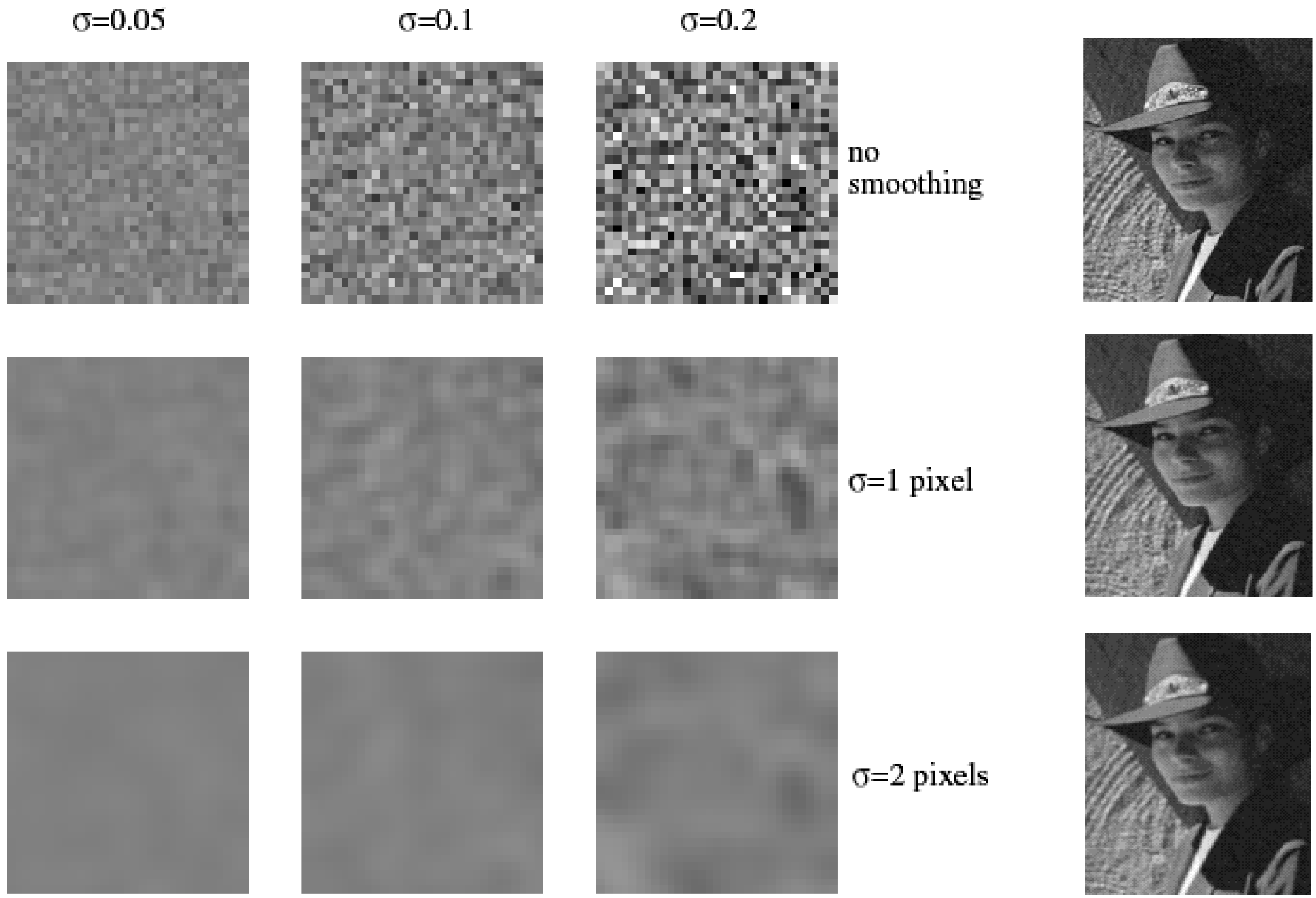


$$f(x,y) = \overbrace{\tilde{f}(x,y)}^{\text{Ideal Image}} + \overbrace{\eta(x,y)}^{\text{Noise process}}$$

Gaussian i.i.d. ("white") noise:
$\eta(x,y) \sim \mathcal{N}(\mu,\sigma)$

Source: M. Hebert

Reducing Gaussian noise



Smoothing with larger standard deviations suppresses noise, but also blurs the image

Reducing salt-and-pepper noise

3x3

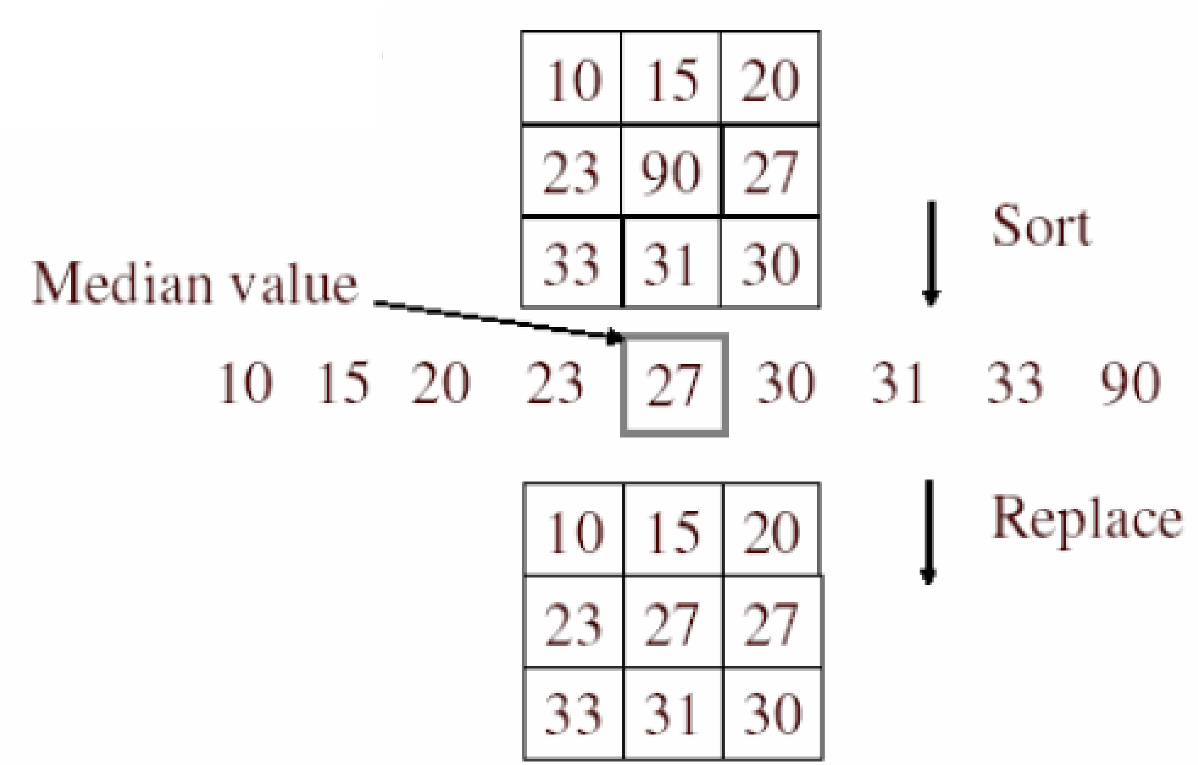
5x5

7x7

- What's wrong with the results?

Alternative idea: Median filtering

- A **median filter** operates over a window by selecting the median intensity in the window

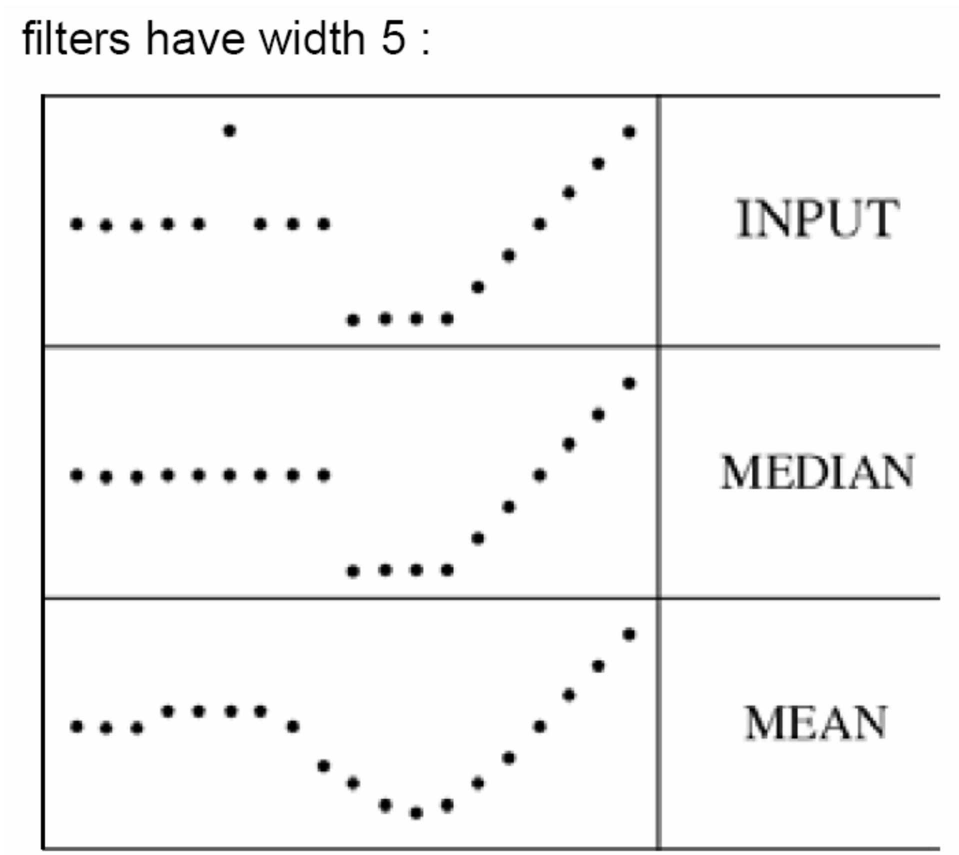


- Is median filtering linear?

Source: K. Grauman

Median filter

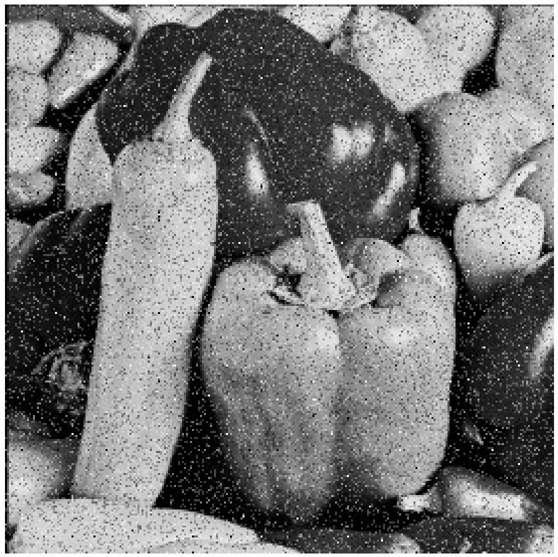
- What advantage does median filtering have over Gaussian filtering?
  - Robustness to outliers

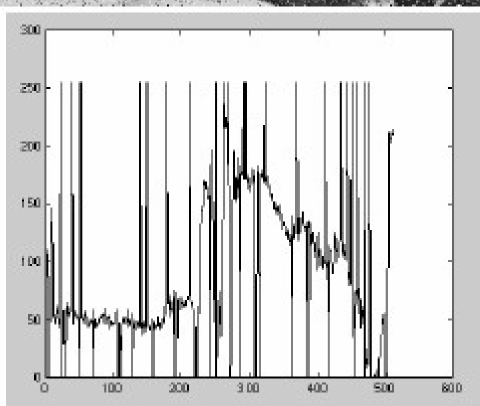


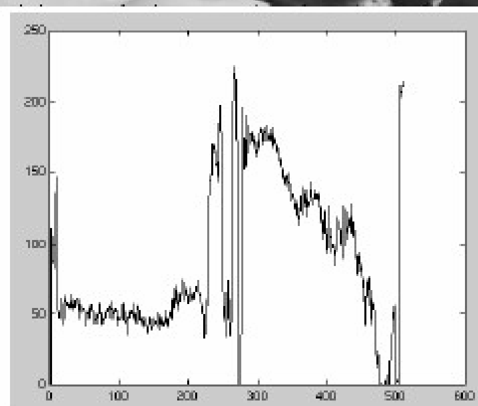
Source: K. Grauman

Median filter

Salt-and-pepper noise | Median filtered







- MATLAB: medfilt2(image, [h w])

Source: M. Hebert

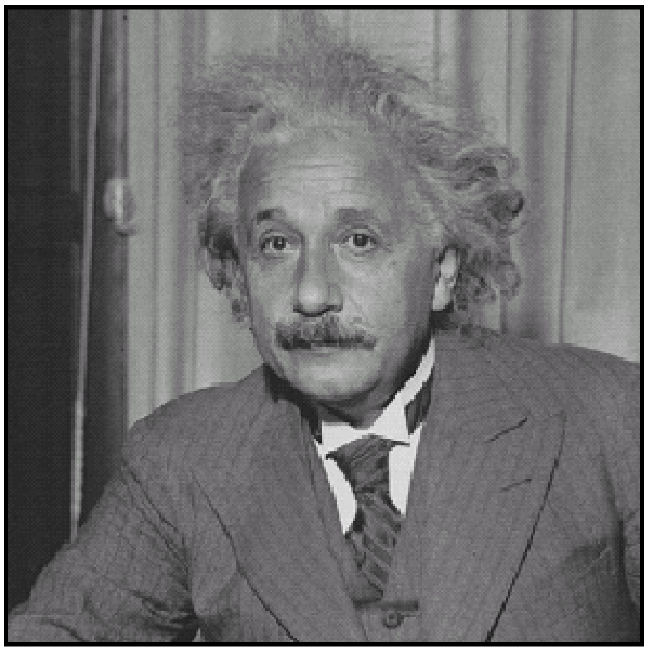
Gaussian vs. median filtering

3x3 5x5 7x7

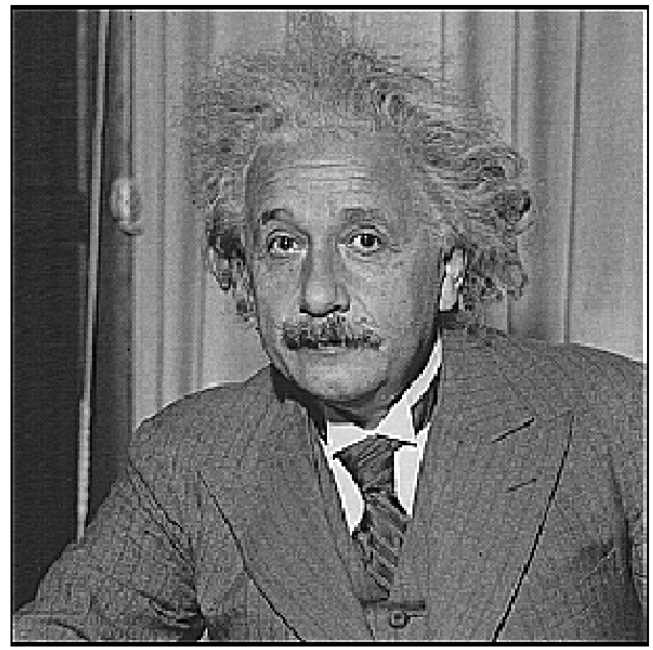
Gaussian

Median

Sharpening revisited



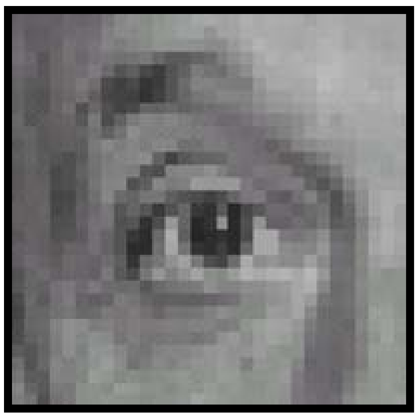
before



after

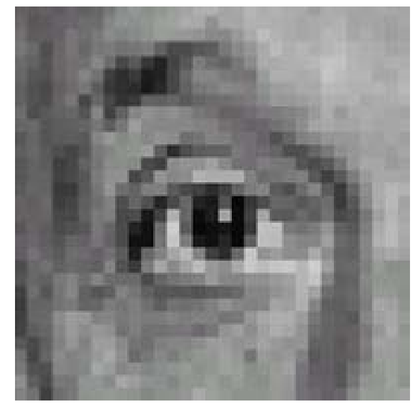
Source: D. Lowe

Sharpening filter



Original

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$



Sharpening filter

- Accentuates differences with local average

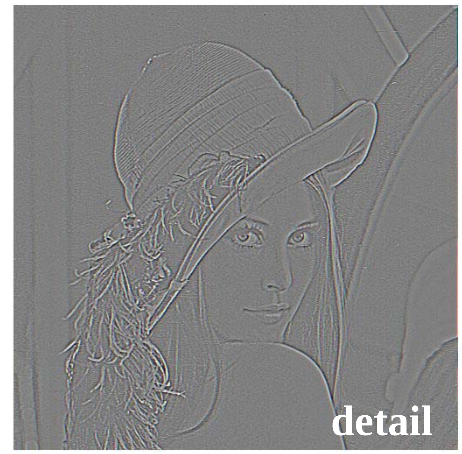
Source: D. Lowe

Sharpening revisited

- What does blurring take away?

$-$

$=$



Let’s add it back:

$+ \alpha$

$=$

Edge detection

Winter in Kraków photographed by Marcin Ryczek

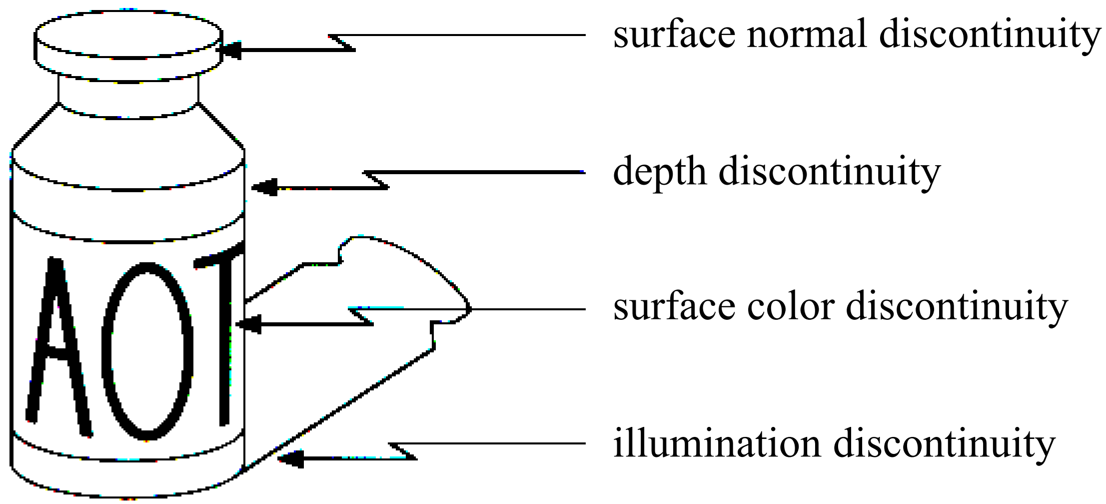
Edge detection

- **Goal:** Identify sudden changes (discontinuities) in an image
  - Intuitively, most semantic and shape information from the image can be encoded in the edges
  - More compact than pixels
- **Ideal:** artist's line drawing (but artist is also using object-level knowledge)

Source: D. Lowe

Origin of edges

- Edges are caused by a variety of factors:



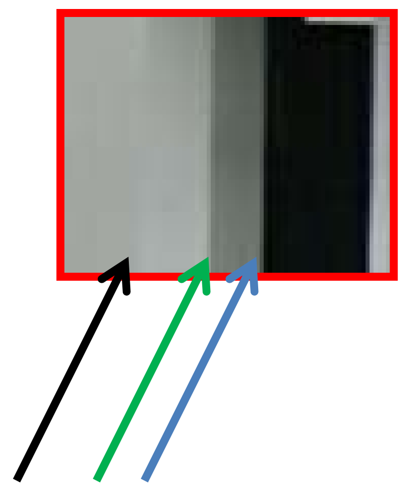
Source: Steve Seitz

Why finding edges is important

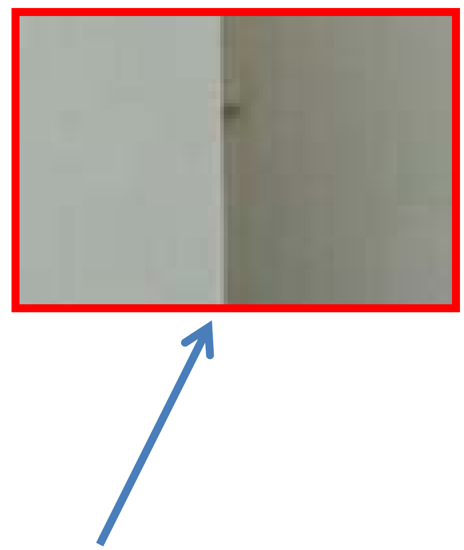
- Group pixels into objects or parts
- Cues for 3D shape
- Guiding interactive image editing

Closeup of edges

Closeup of edges



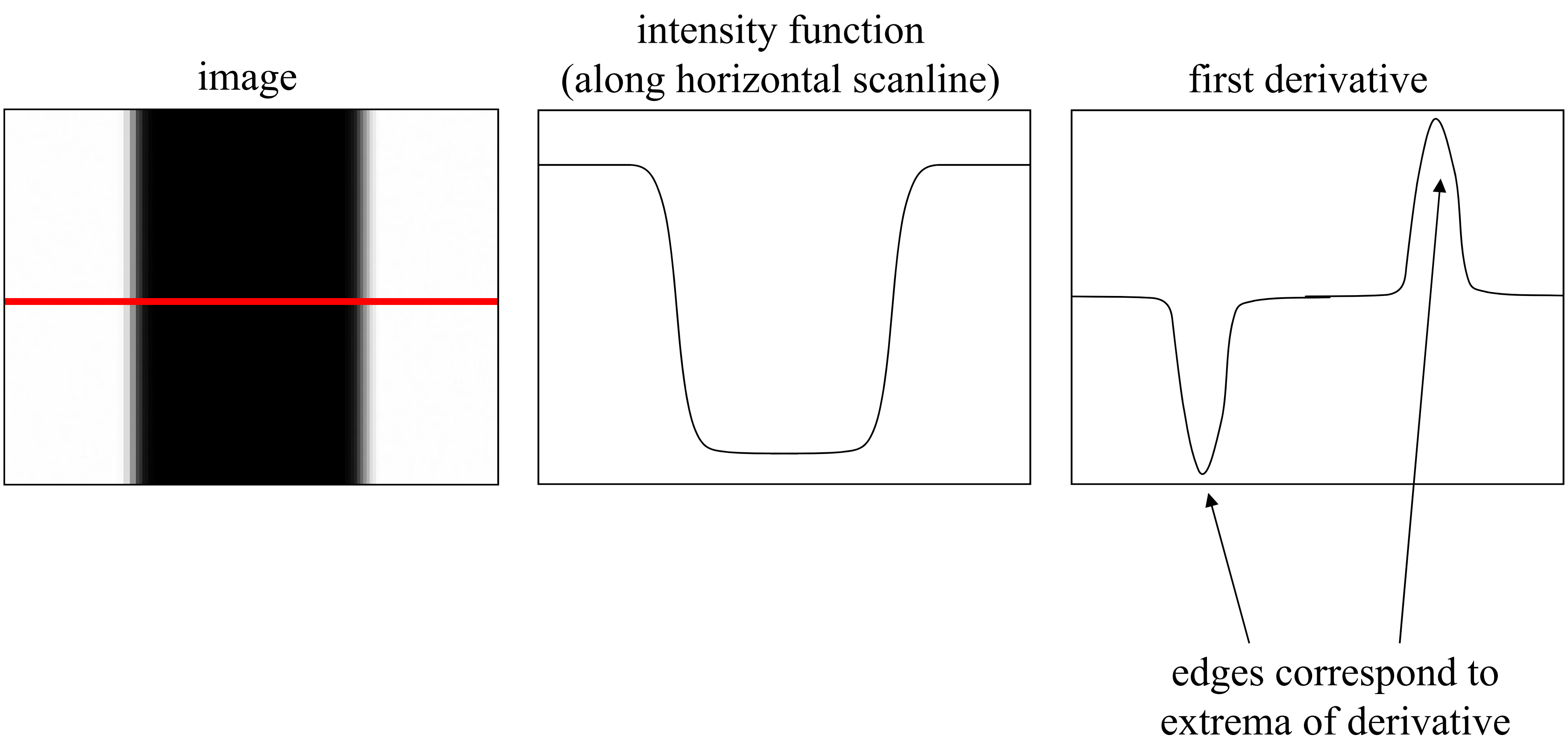
Closeup of edges



Closeup of edges

Edge detection

- An edge is a place of rapid change in the image intensity function



Derivatives with convolution

For 2D function f(x,y), the partial derivative is:

$$\frac{\partial f(x,y)}{\partial x} = \lim_{\varepsilon \to 0} \frac{f(x+\varepsilon, y) - f(x,y)}{\varepsilon}$$

For discrete data, we can approximate using finite differences:

$$\frac{\partial f(x,y)}{\partial x} \approx \frac{f(x+1, y) - f(x,y)}{1}$$

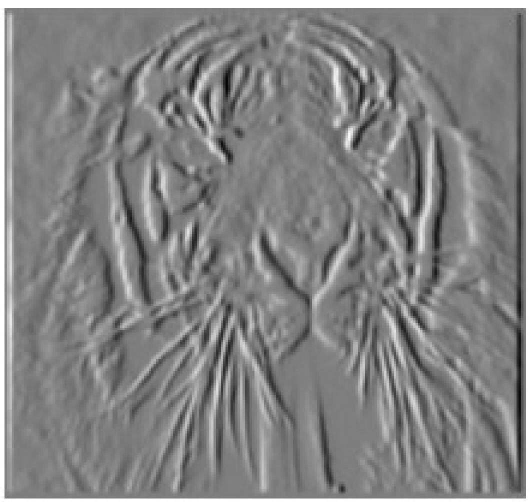
To implement the above as convolution, what would be the associated filter?

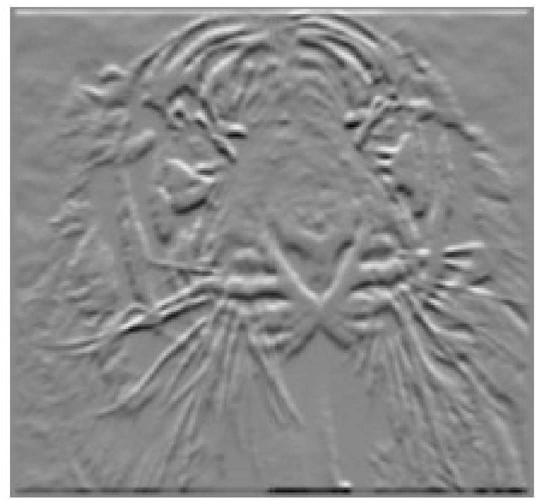
Source: K. Grauman

Partial derivatives of an image

$$\frac{\partial f(x,y)}{\partial x}$$

$$\frac{\partial f(x,y)}{\partial y}$$





$$\begin{bmatrix} -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 \\ 1 \end{bmatrix} \text{ or } \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Which shows changes with respect to x?

Finite difference filters

- Other approximations of derivative filters exist:

Prewitt: $M_x = \begin{array}{|c|c|c|}\hline -1 & 0 & 1 \\ \hline -1 & 0 & 1 \\ \hline -1 & 0 & 1 \\ \hline \end{array}$; $M_y = \begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 0 & 0 & 0 \\ \hline -1 & -1 & -1 \\ \hline \end{array}$

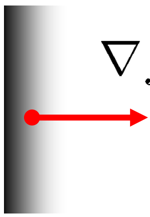
Sobel: $M_x = \begin{array}{|c|c|c|}\hline -1 & 0 & 1 \\ \hline -2 & 0 & 2 \\ \hline -1 & 0 & 1 \\ \hline \end{array}$; $M_y = \begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 0 & 0 & 0 \\ \hline -1 & -2 & -1 \\ \hline \end{array}$

Roberts: $M_x = \begin{array}{|c|c|}\hline 0 & 1 \\ \hline -1 & 0 \\ \hline \end{array}$; $M_y = \begin{array}{|c|c|}\hline 1 & 0 \\ \hline 0 & -1 \\ \hline \end{array}$

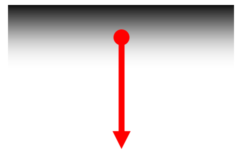
Source: K. Grauman

Image gradient

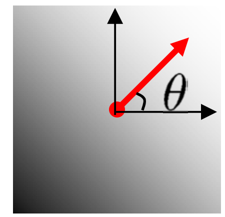
- The gradient of an image: $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$



$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$



$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$



$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$

The gradient points in the direction of most rapid increase in intensity

- How does this direction relate to the direction of the edge?

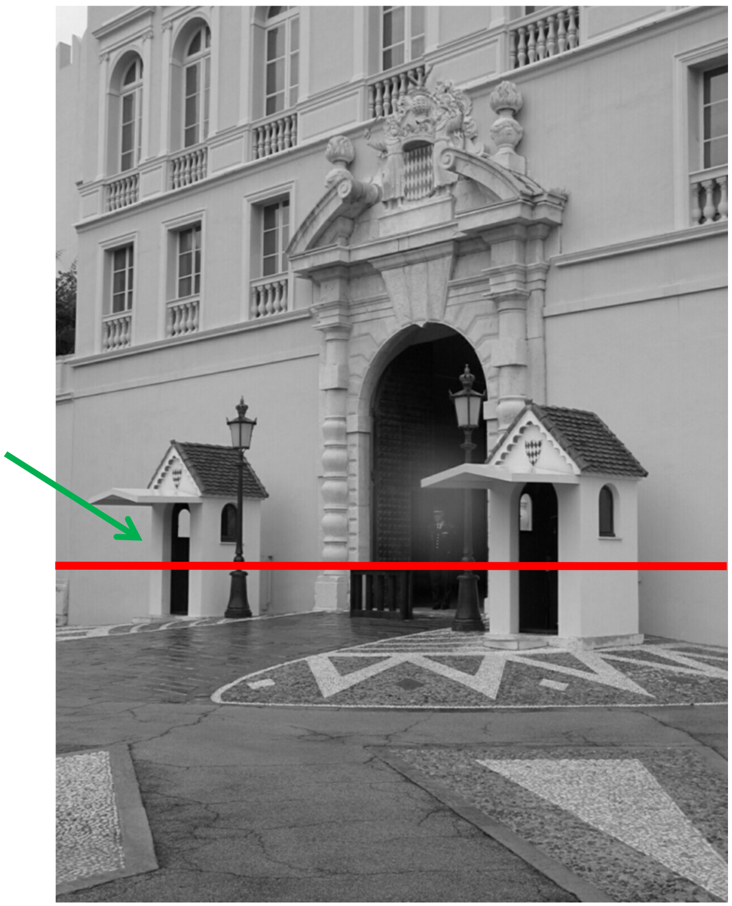
The gradient direction is given by $\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$

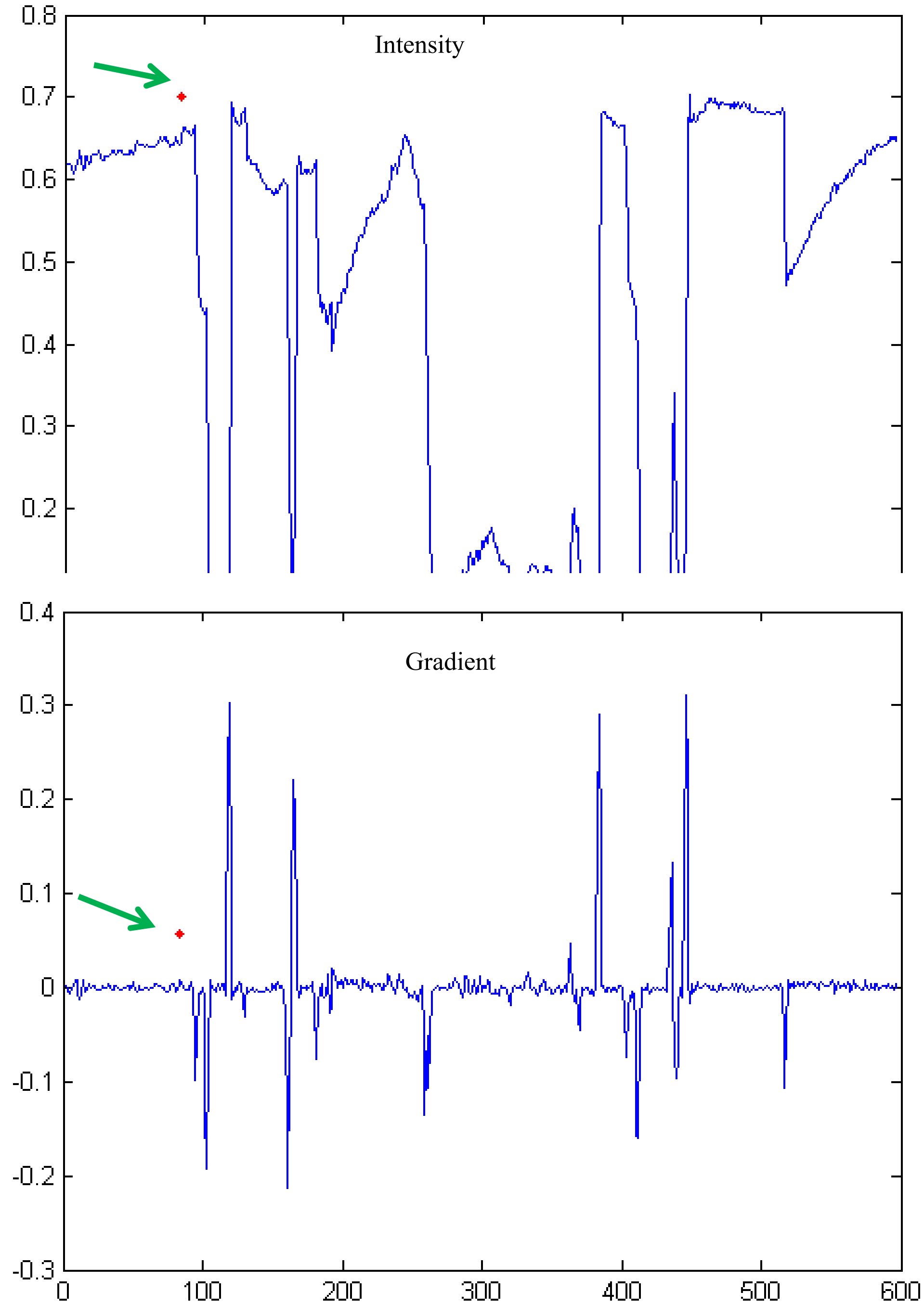
The edge strength is given by the gradient magnitude

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

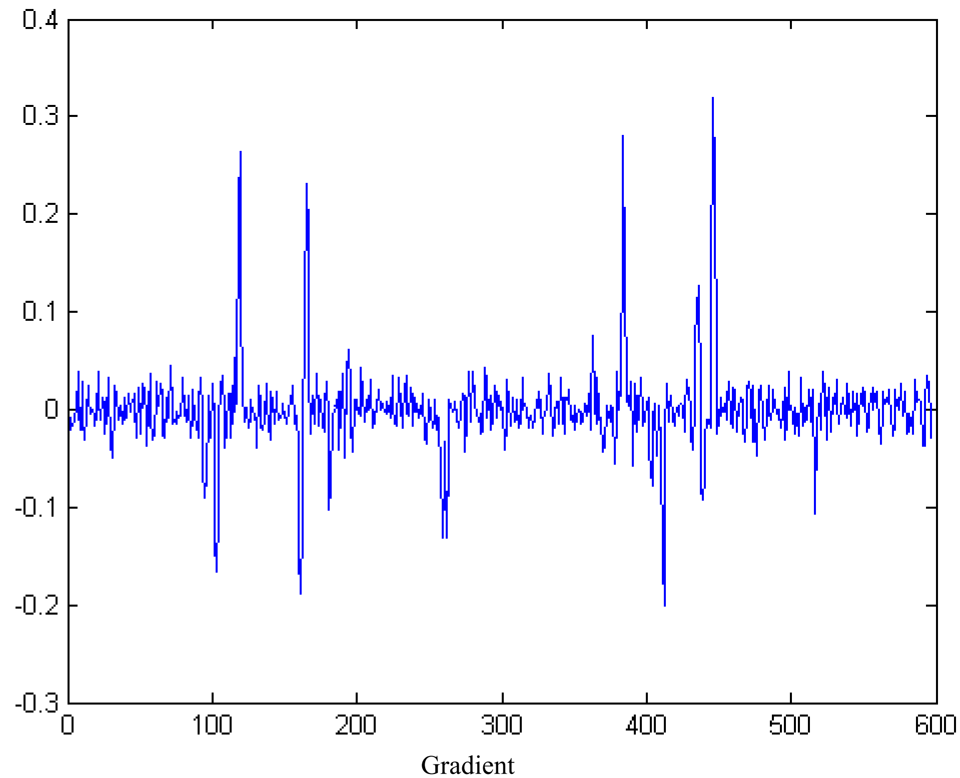
Source: Steve Seitz

Intensity profile



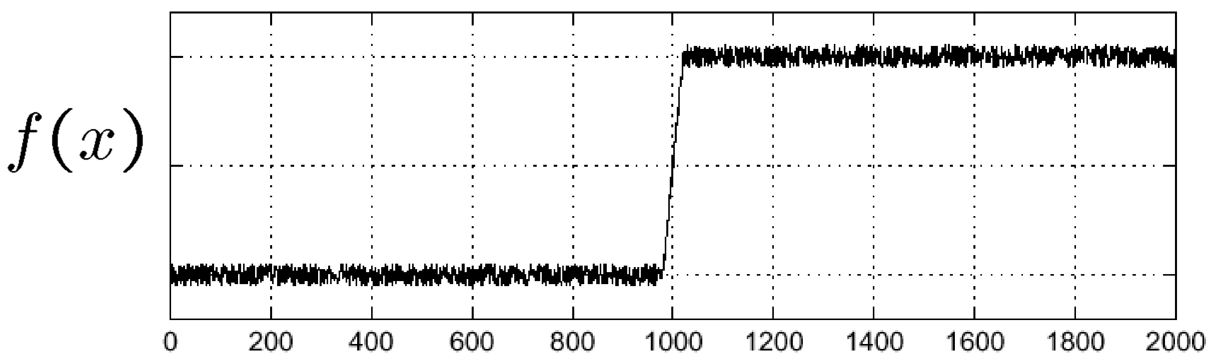


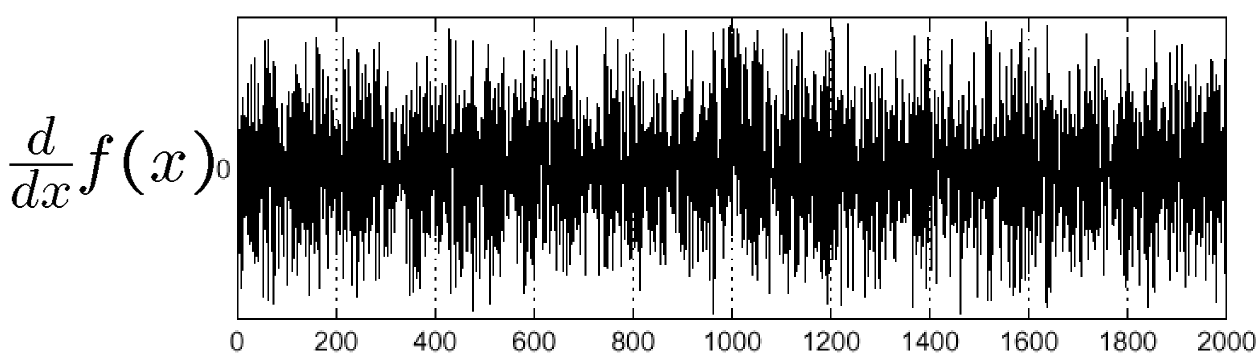
With a little Gaussian noise



Effects of noise

- Consider a single row or column of the image





Where is the edge?

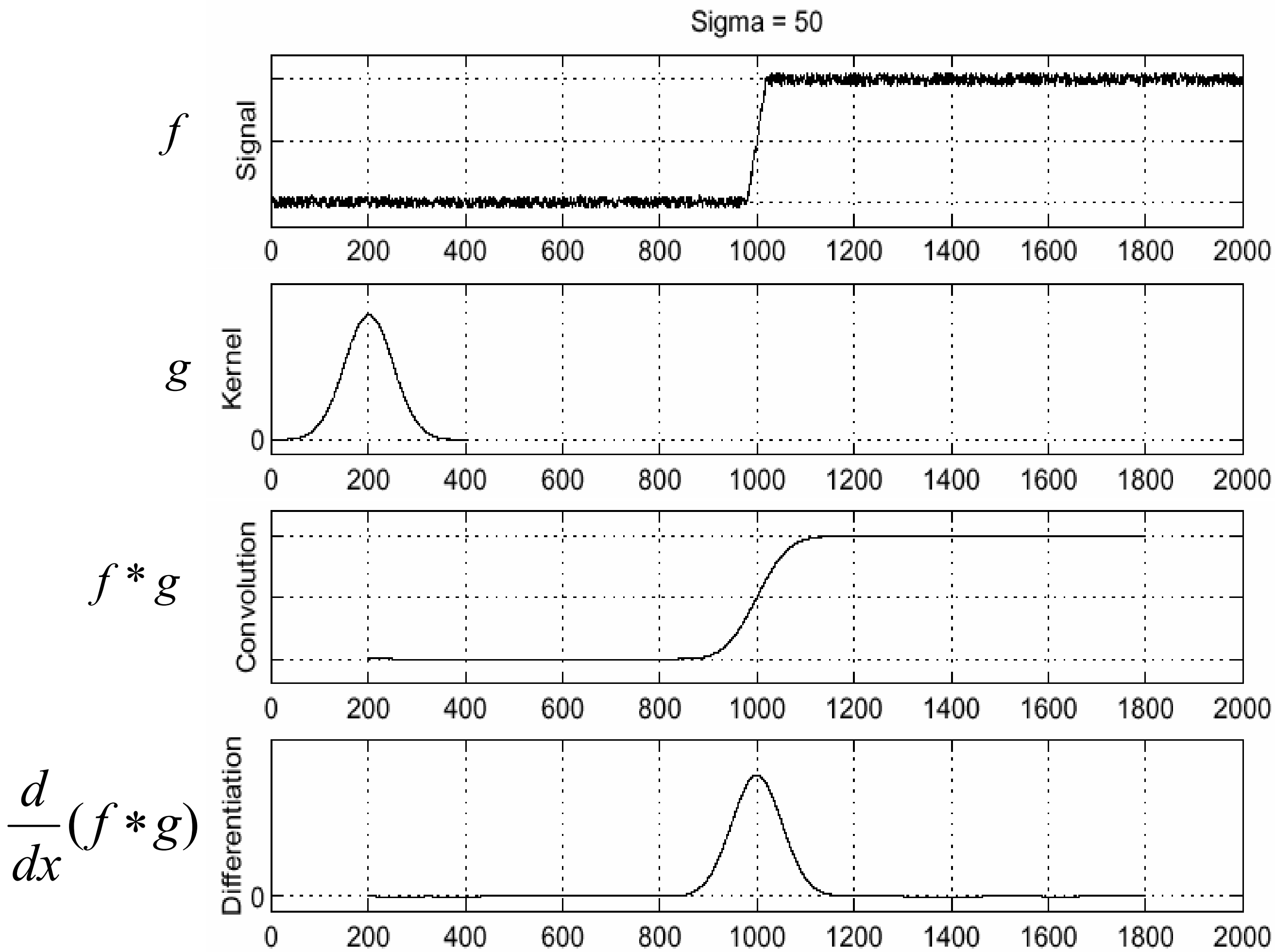
Source: S. Seitz

Effects of noise

- Difference filters respond strongly to noise
  - Image noise results in pixels that look very different from their neighbors
  - Generally, the larger the noise the stronger the response
- What can we do about it?

Source: D. Forsyth

Solution: smooth first

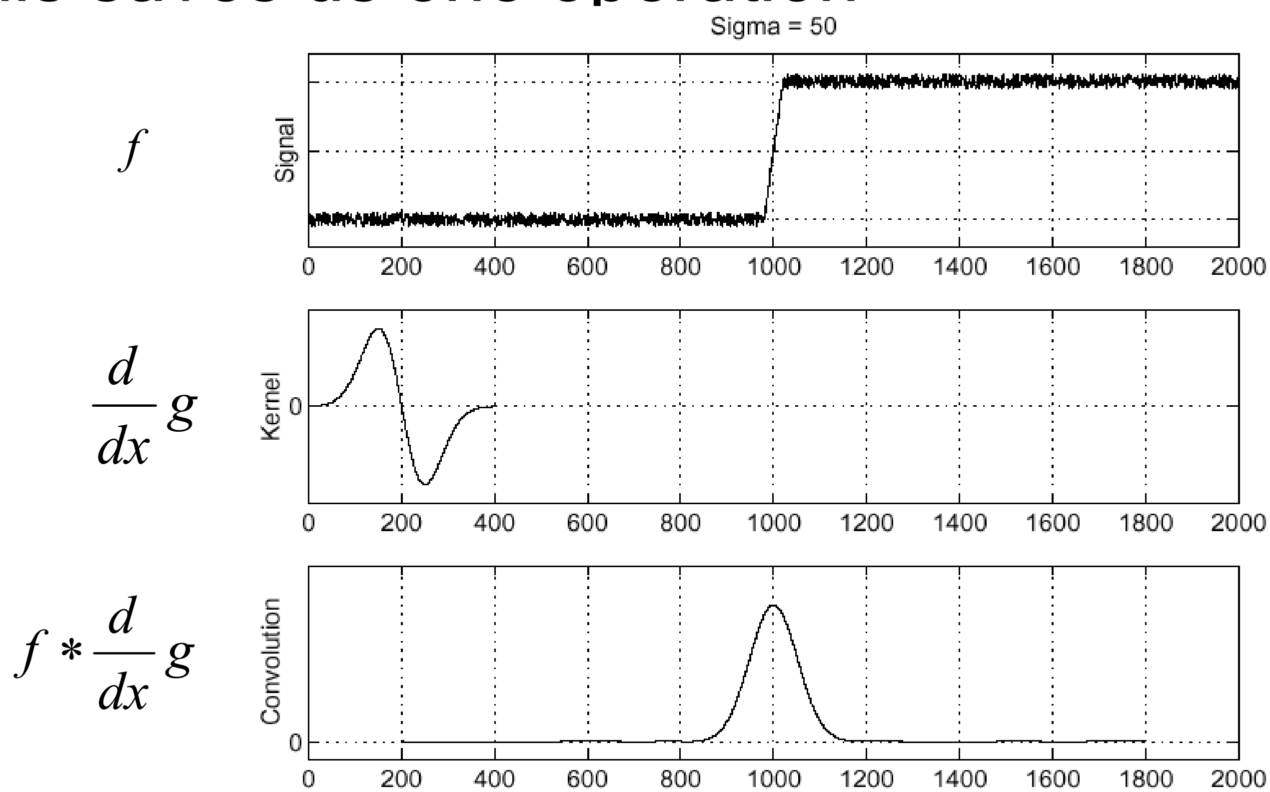


- To find edges, look for peaks in $\frac{d}{dx}(f * g)$

Source: S. Seitz

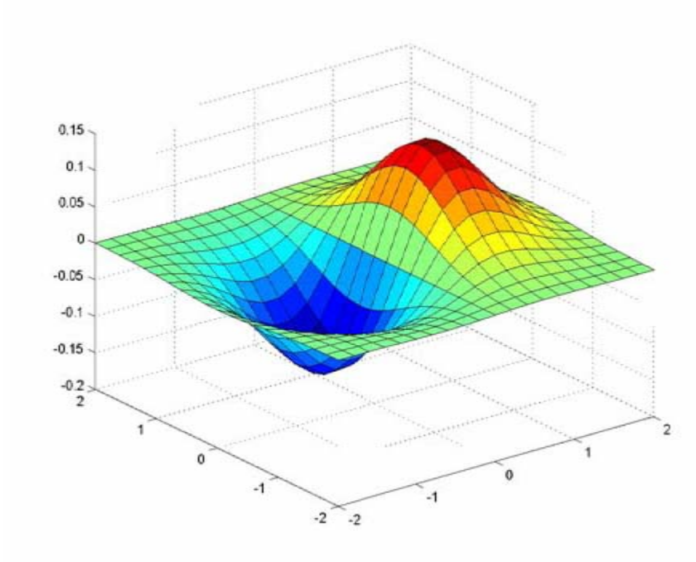
Derivative theorem of convolution

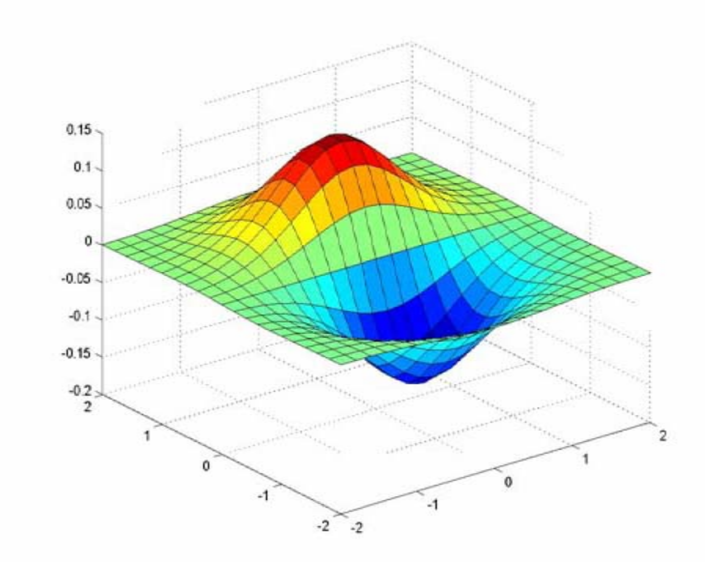
- Differentiation is convolution, and convolution is associative: $$\frac{d}{dx}(f*g)=f*\frac{d}{dx}g$$
- This saves us one operation

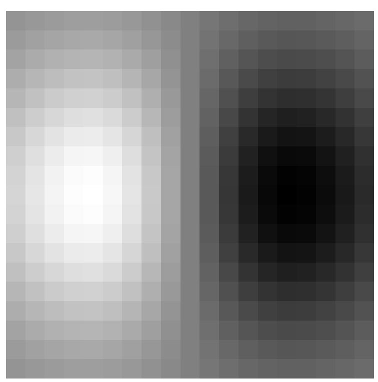


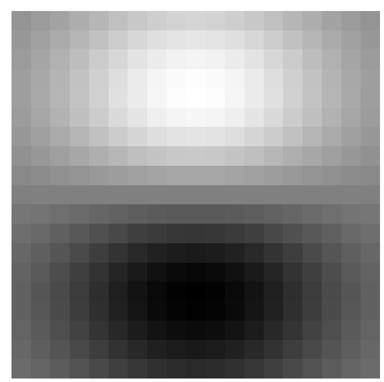
Source: S. Seitz

Derivative of Gaussian filters



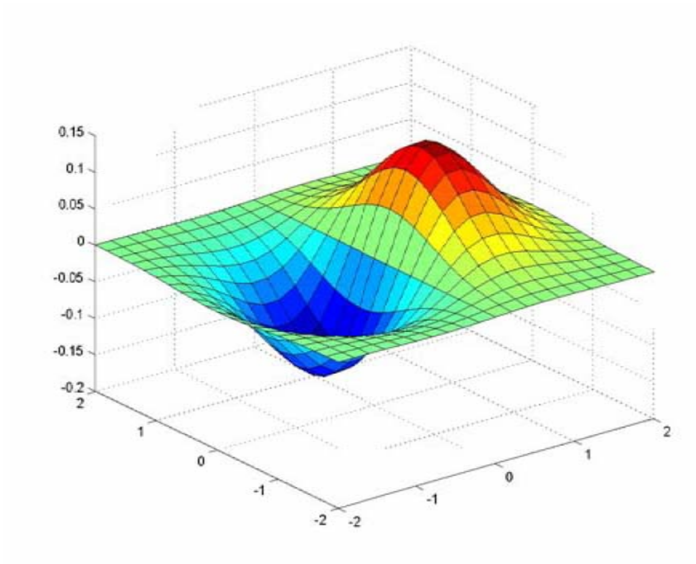


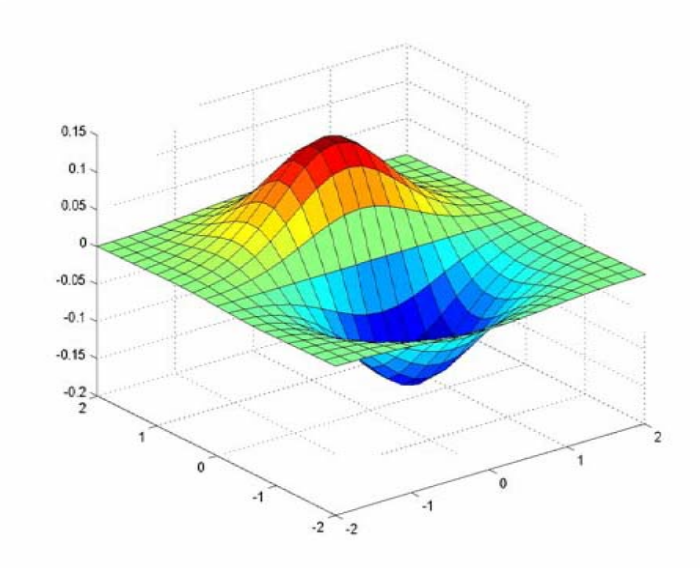


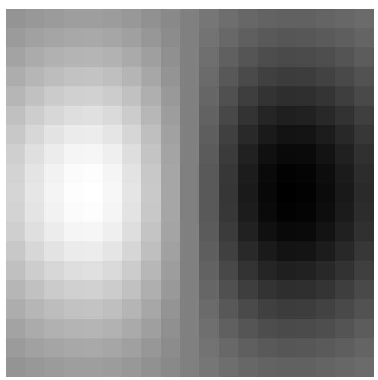


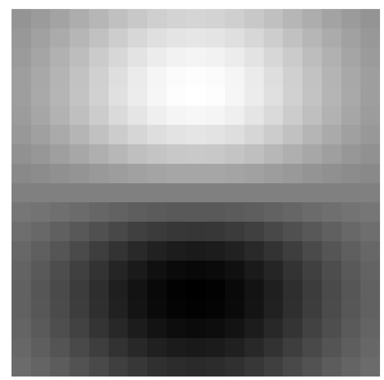
- Which one finds horizontal/vertical edges?

Derivative of Gaussian filters









- Are these filters separable?

Recall: Separability of the Gaussian filter

$$G_\sigma(x,y) = \frac{1}{2\pi\sigma^2} \exp^{-\frac{x^2+y^2}{2\sigma^2}}$$

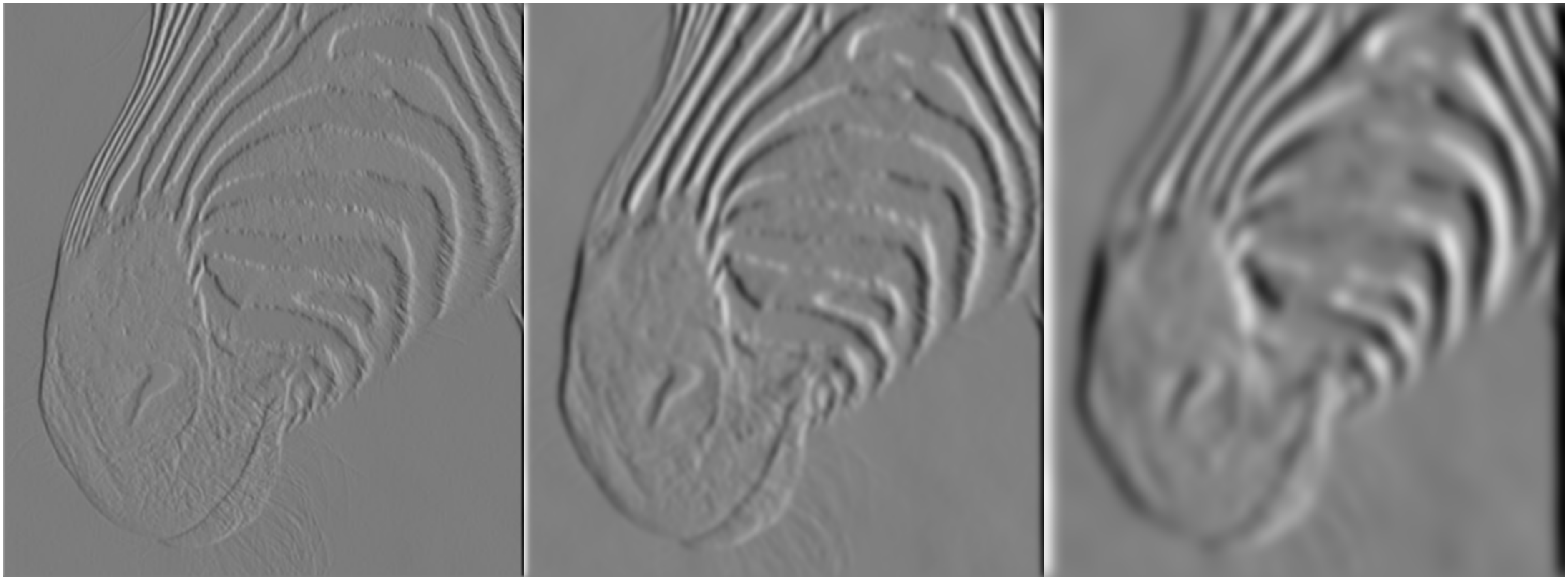
$$= \left(\frac{1}{\sqrt{2\pi}\sigma} \exp^{-\frac{x^2}{2\sigma^2}}\right) \left(\frac{1}{\sqrt{2\pi}\sigma} \exp^{-\frac{y^2}{2\sigma^2}}\right)$$

The 2D Gaussian can be expressed as the product of two functions, one a function of x and the other a function of y

In this case, the two functions are the (identical) 1D Gaussian

Source: D. Lowe

Scale of Gaussian derivative filter



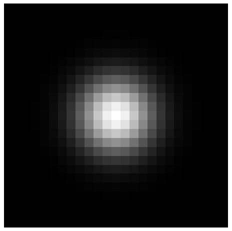
1 pixel | 3 pixels | 7 pixels

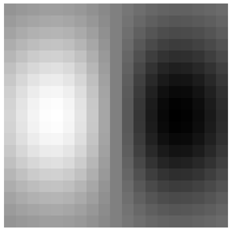
- Smoothed derivative removes noise, but blurs edge. Also finds edges at different "scales"

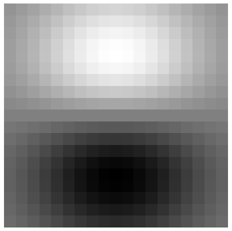
Source: D. Forsyth

Review: Smoothing vs. derivative filters

- Smoothing filters
  - Gaussian: remove "high-frequency" components; "low-pass" filter
  - Can the values of a smoothing filter be negative?
  - What should the values sum to?
    - **One:** constant regions are not affected by the filter







- Derivative filters
  - Derivatives of Gaussian
  - Can the values of a derivative filter be negative?
  - What should the values sum to?
    - **Zero:** no response in constant regions
    - High absolute value at points of high contrast

The Canny edge detector

1. Filter image with derivative of Gaussian
2. Find magnitude and orientation of gradient
3. **Non-maximum suppression**:
   – Thin wide "ridges" down to single pixel width
4. **Linking and thresholding** (**hysteresis**):
   – Define two thresholds: low and high
   – Use the high threshold to start edge curves and the low threshold to continue them

The Canny edge detector

original image

Slide credit: Steve Seitz

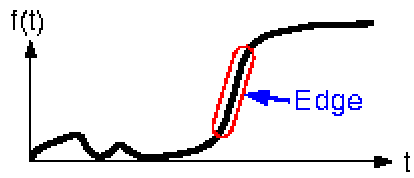
The Canny edge detector

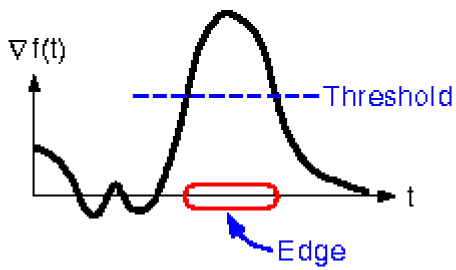
norm of the gradient

The Canny edge detector

thresholding

The Canny edge detector

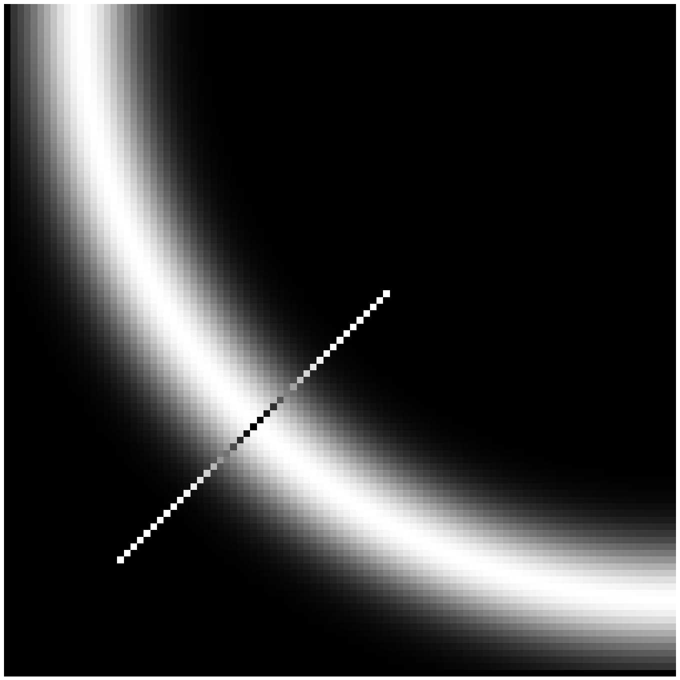


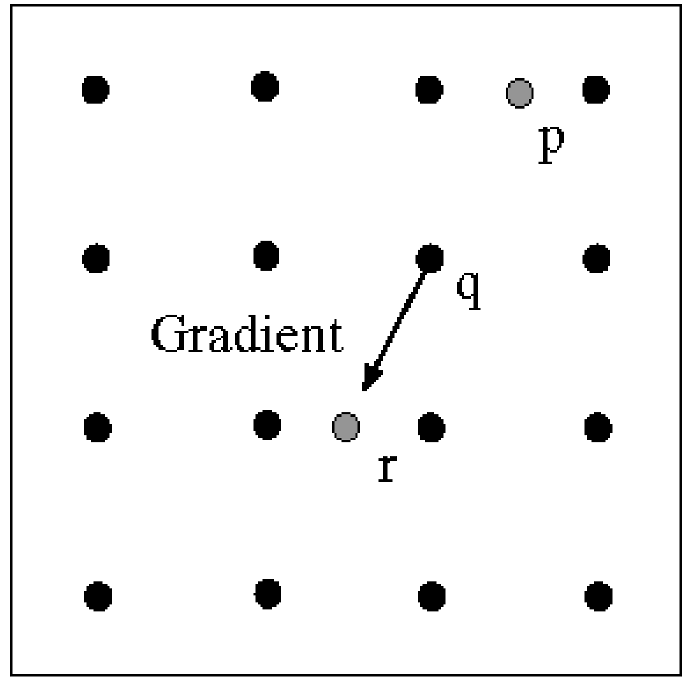


How to turn these thick regions of the gradient into curves?

thresholding

Non-maximum suppression





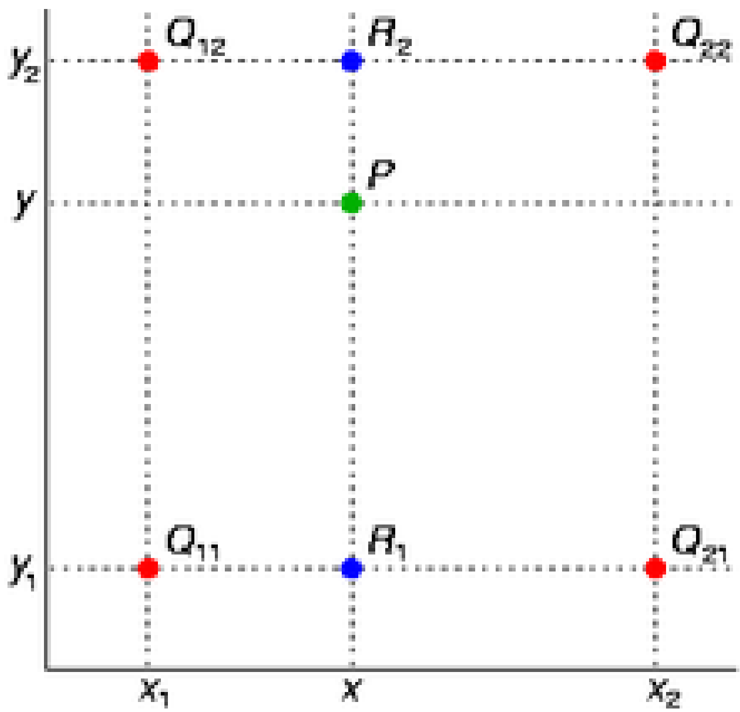
Check if pixel is local maximum along gradient direction, select single max across width of the edge

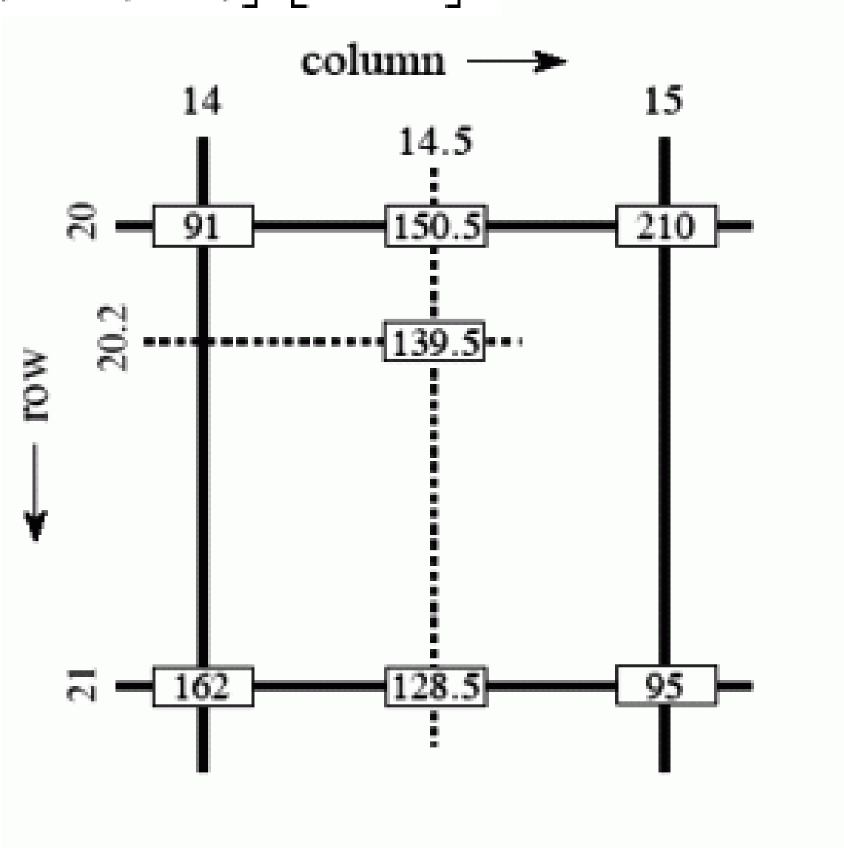
- requires checking interpolated pixels p and r

Source: D. Forsyth

Bilinear Interpolation

$$f(x,y) \approx \begin{bmatrix}1-x & x\end{bmatrix} \begin{bmatrix} f(0,0) & f(0,1) \\ f(1,0) & f(1,1) \end{bmatrix} \begin{bmatrix} 1-y \\ y \end{bmatrix}.$$





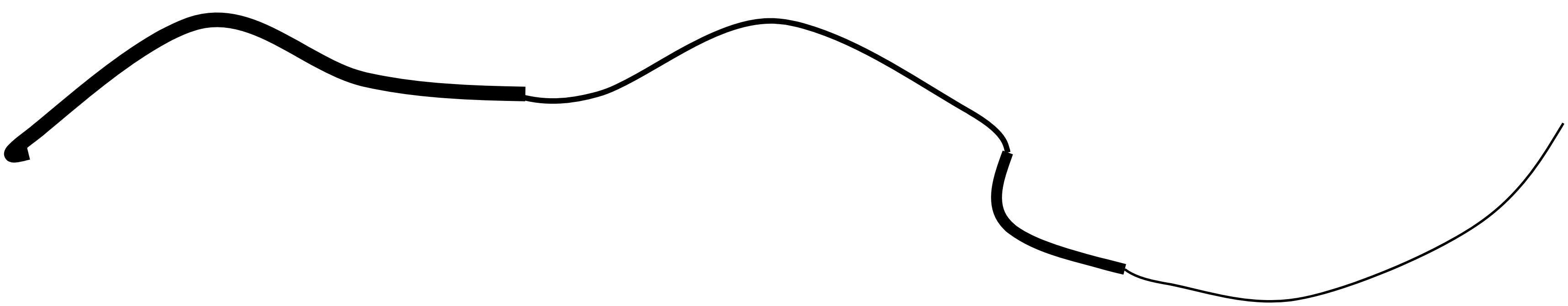
The Canny edge detector

Thinning (non-maximum suppression)

Problem: pixels along this edge didn't survive the thresholding

Hysteresis thresholding

- Use a high threshold to start edge curves, and a low threshold to continue them



Source: Steve Seitz

Hysteresis thresholding

- Threshold at low/high levels to get weak/strong edge pixels
- Trace connected components, starting from strong edge pixels

Recap: Canny edge detector

1. Compute x and y gradient images
2. Find magnitude and orientation of gradient
3. **Non-maximum suppression**:
   – Thin wide "ridges" down to single pixel width
4. **Linking and thresholding** (**hysteresis**):
   – Define two thresholds: low and high
   – Use the high threshold to start edge curves and the low threshold to continue them

- MATLAB: `edge(image, 'canny');`

J. Canny, ***A Computational Approach To Edge Detection***, IEEE Trans. Pattern Analysis and Machine Intelligence, 8:679-714, 1986.

Effect of σ (Gaussian kernel spread/size)

original | Canny with $\sigma = 1$ | Canny with $\sigma = 2$

The choice of σ depends on desired behavior

- large σ detects large scale edges
- small σ detects fine features

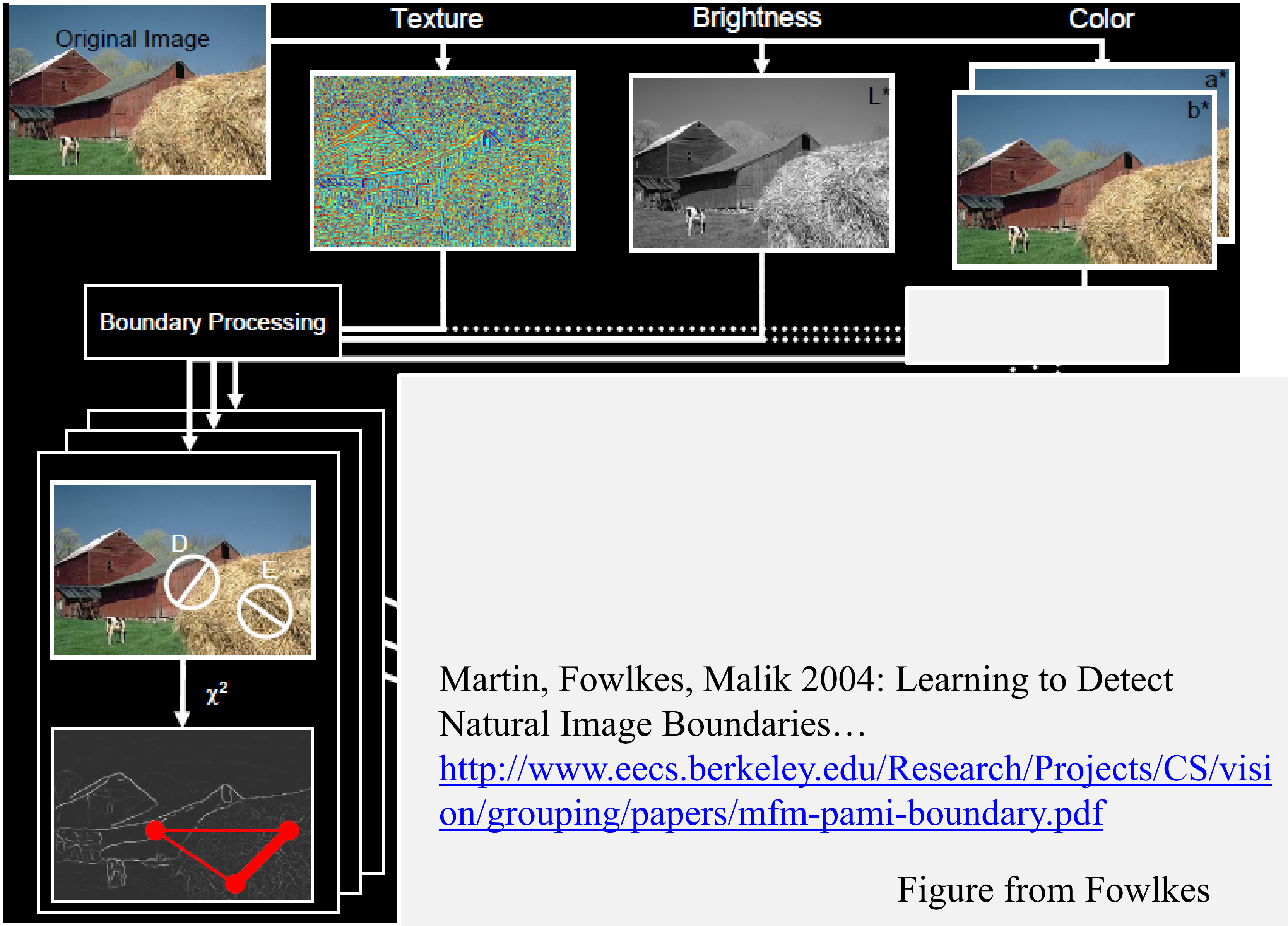
Source: S. Seitz

Learning to detect boundaries

image | human segmentation | gradient magnitude

- Berkeley segmentation database:
  http://www.eecs.berkeley.edu/Research/Projects/CS/vision/grouping/segbench/

pB boundary detector



Martin, Fowlkes, Malik 2004: Learning to Detect Natural Image Boundaries…
http://www.eecs.berkeley.edu/Research/Projects/CS/vision/grouping/papers/mfm-pami-boundary.pdf

Figure from Fowlkes

pB Boundary Detector

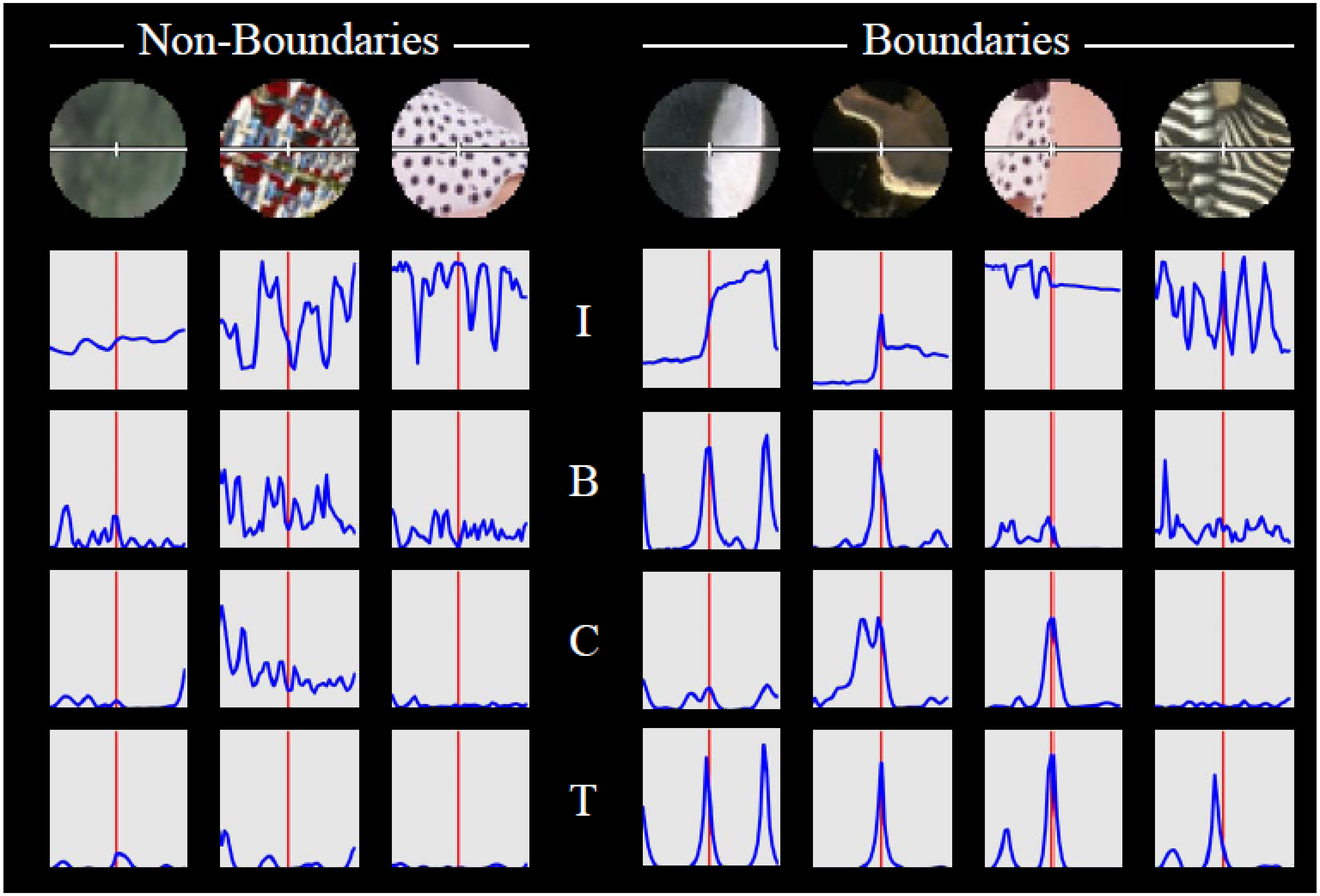


Figure from Fowlkes

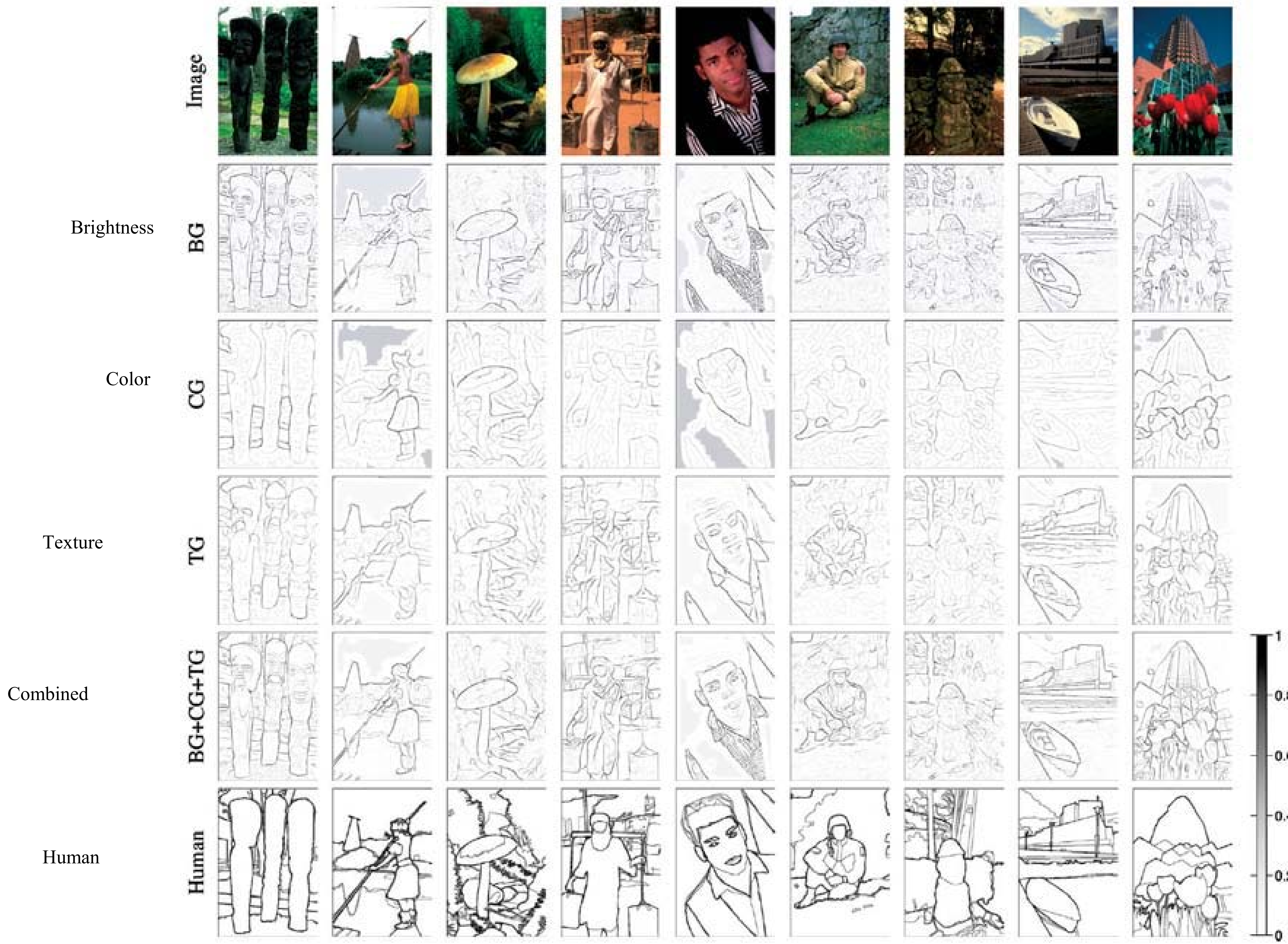
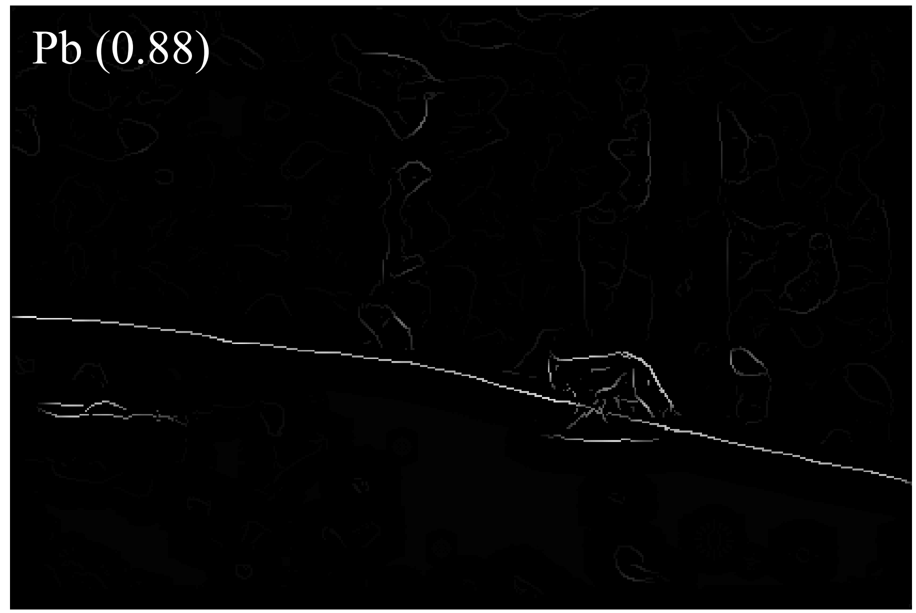
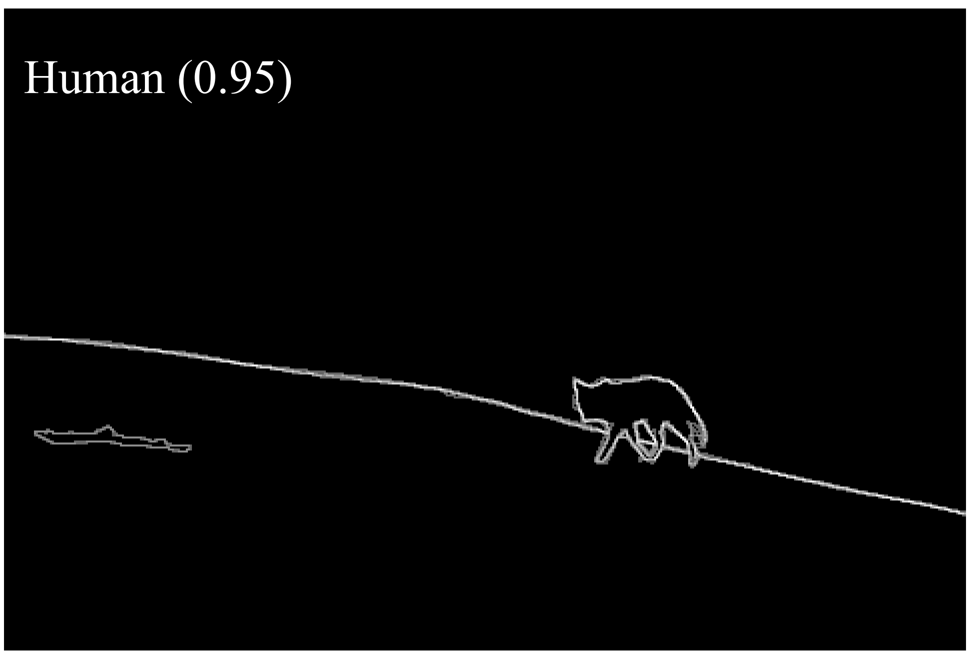


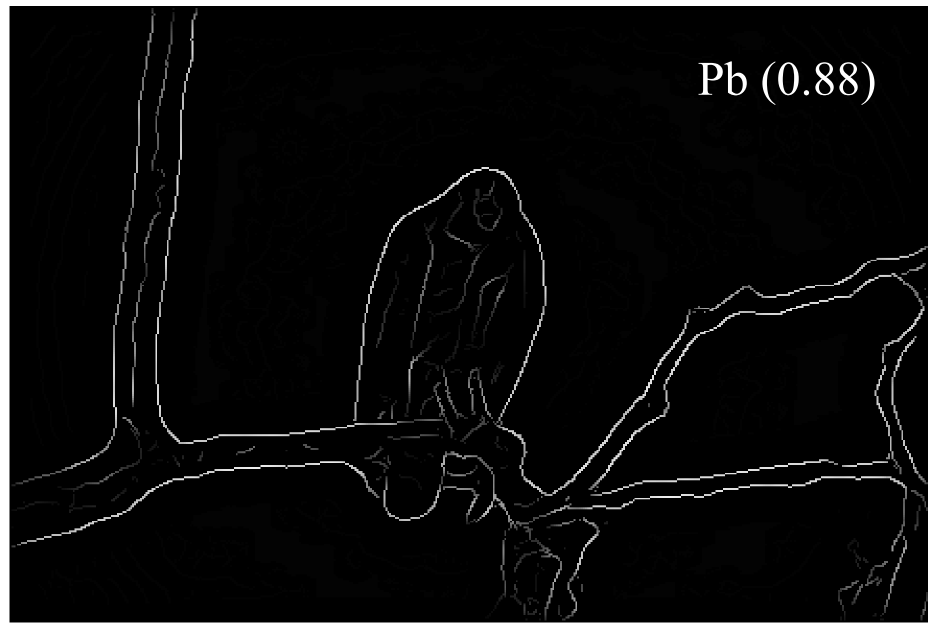
Image
Brightness
BG
Color
CG
Texture
TG
Combined
BG+CG+TG
Human
1
0.8
0.6
0.4
0.2
0

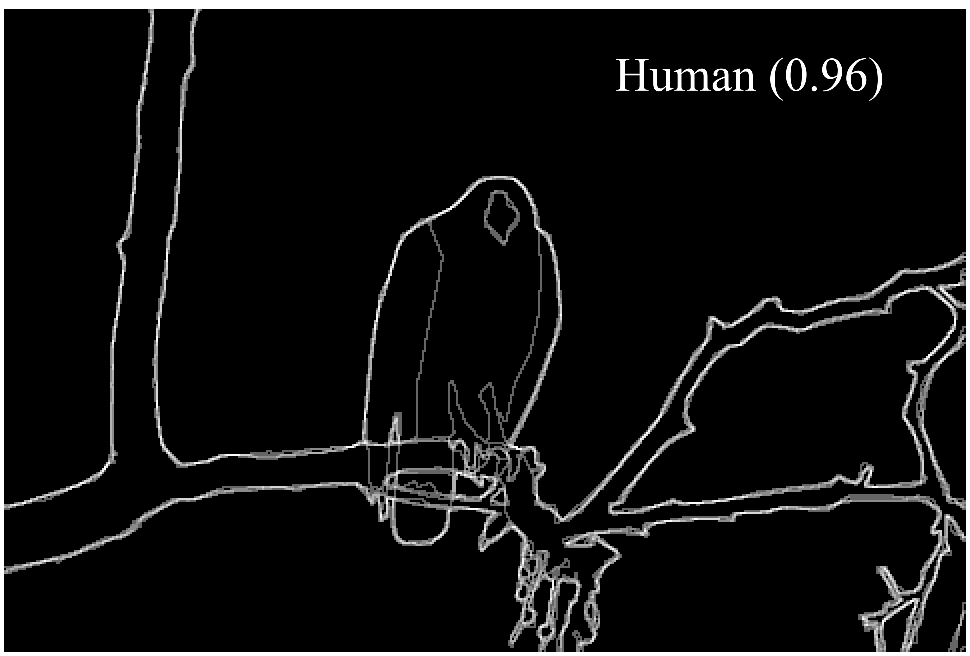
Results

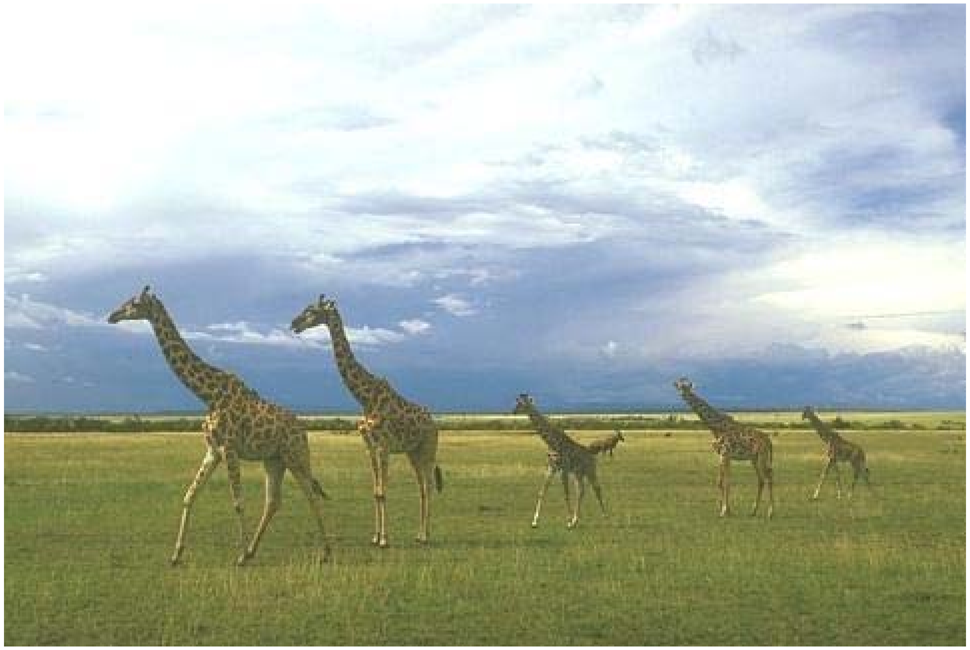


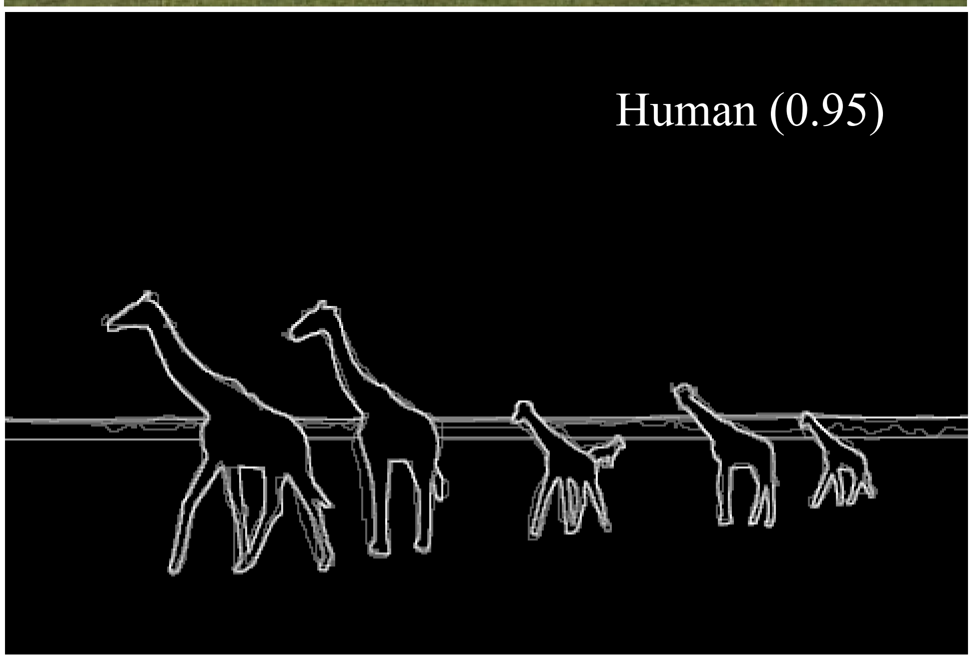


Results

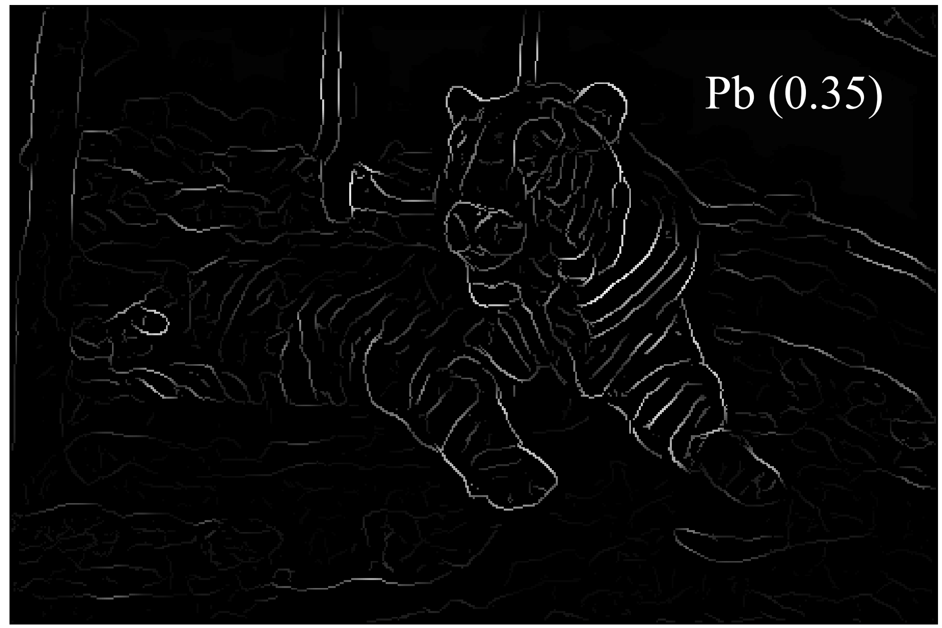







Human (0.95)


Pb (0.63)



For more:
http://www.eecs.berkeley.edu/Research/Projects/CS/vision/bsds/bench/html/108082-color.html

Feature extraction: Corners

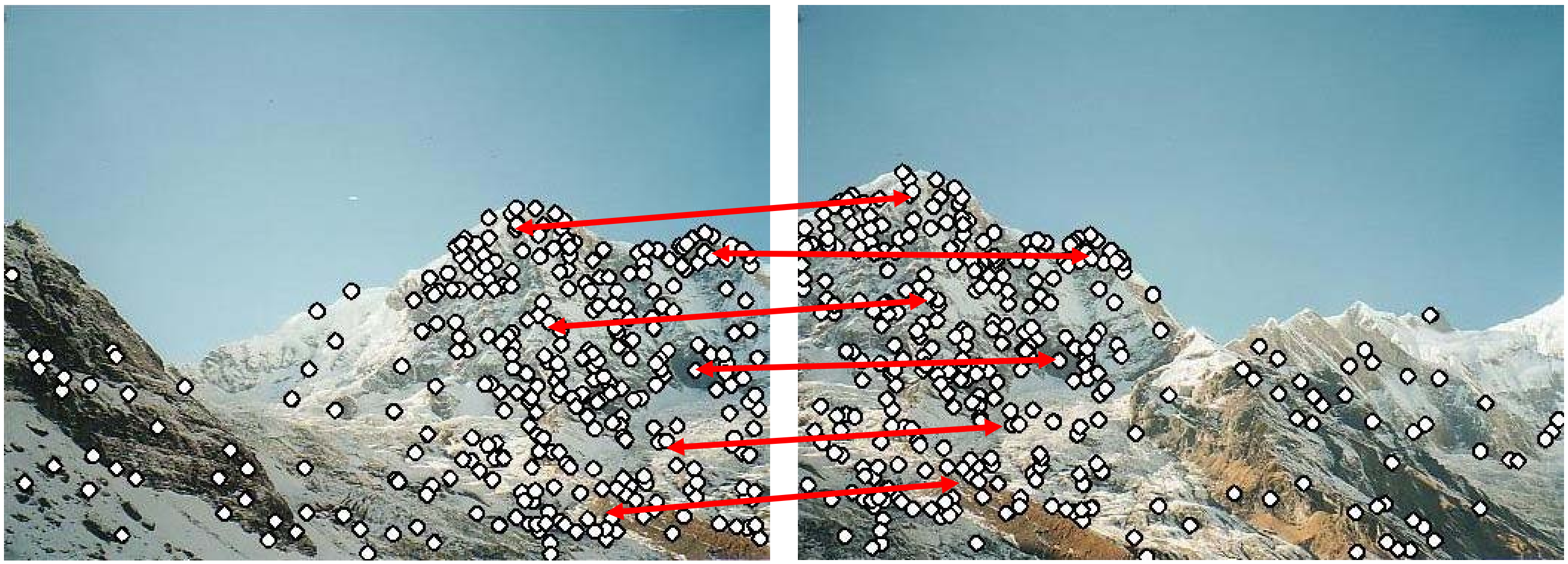
Why extract features?

- Motivation: panorama stitching
  - We have two images – how do we combine them?

Why extract features?

- Motivation: panorama stitching
  - We have two images – how do we combine them?



Step 1: extract features

Step 2: match features

Why extract features?

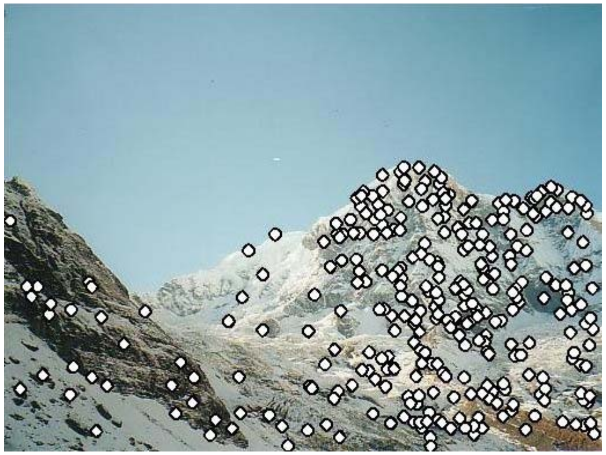
- Motivation: panorama stitching
  - We have two images – how do we combine them?

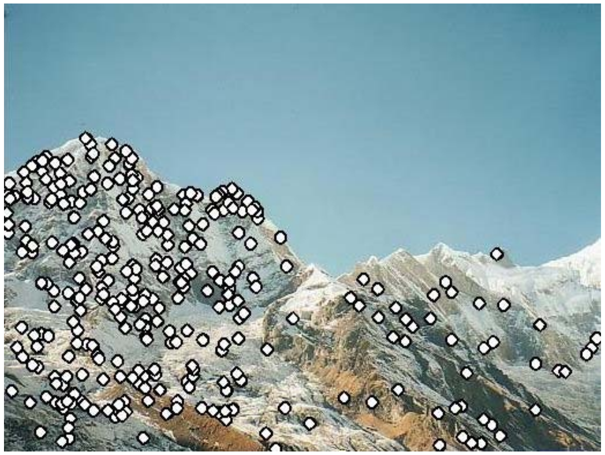
Step 1: extract features

Step 2: match features

Step 3: align images

Characteristics of good features



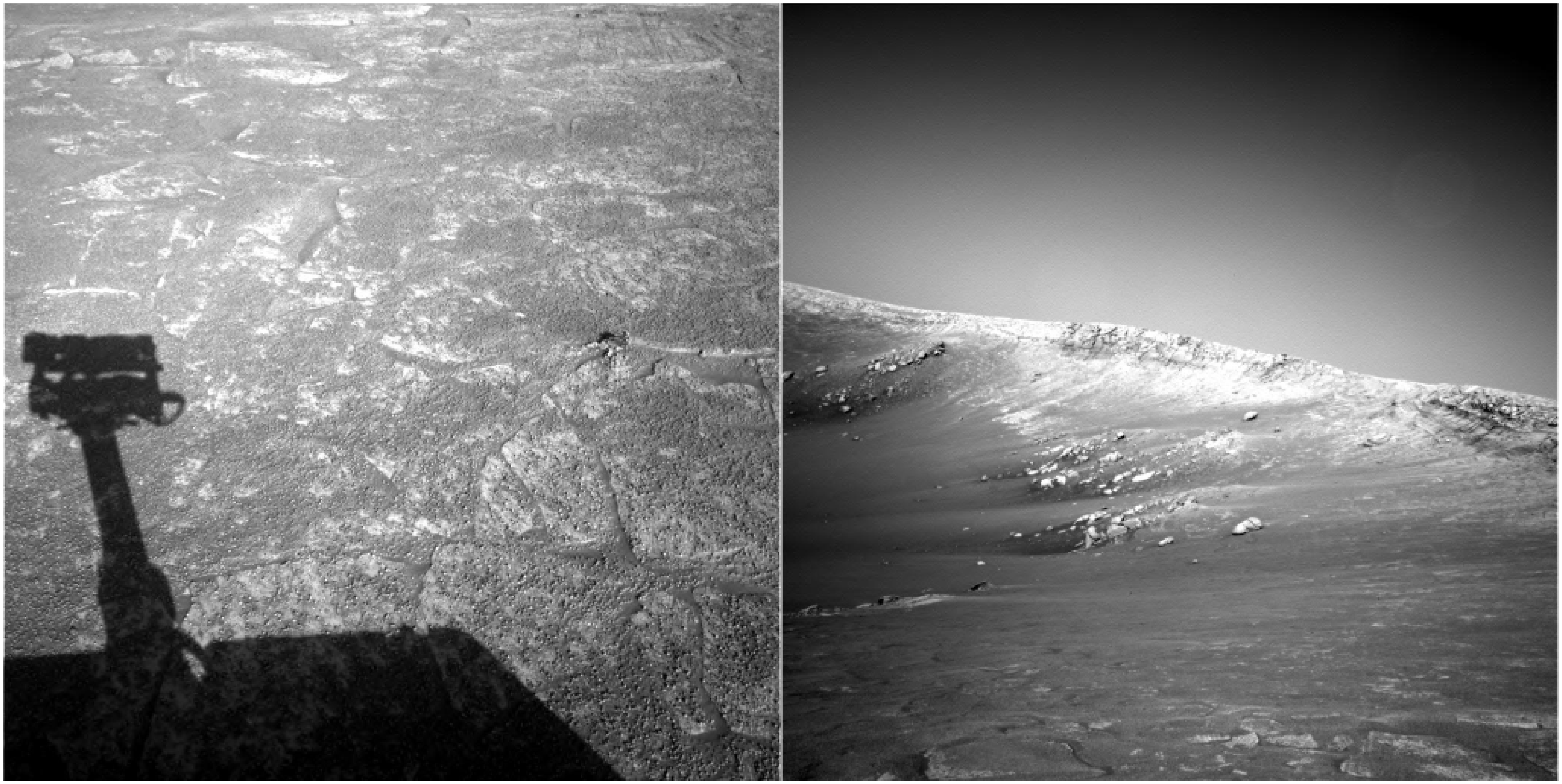


- Repeatability
  - The same feature can be found in several images despite geometric and photometric transformations
- Saliency
  - Each feature is distinctive
- Compactness and efficiency
  - Many fewer features than image pixels
- Locality
  - A feature occupies a relatively small area of the image; robust to clutter and occlusion

Applications

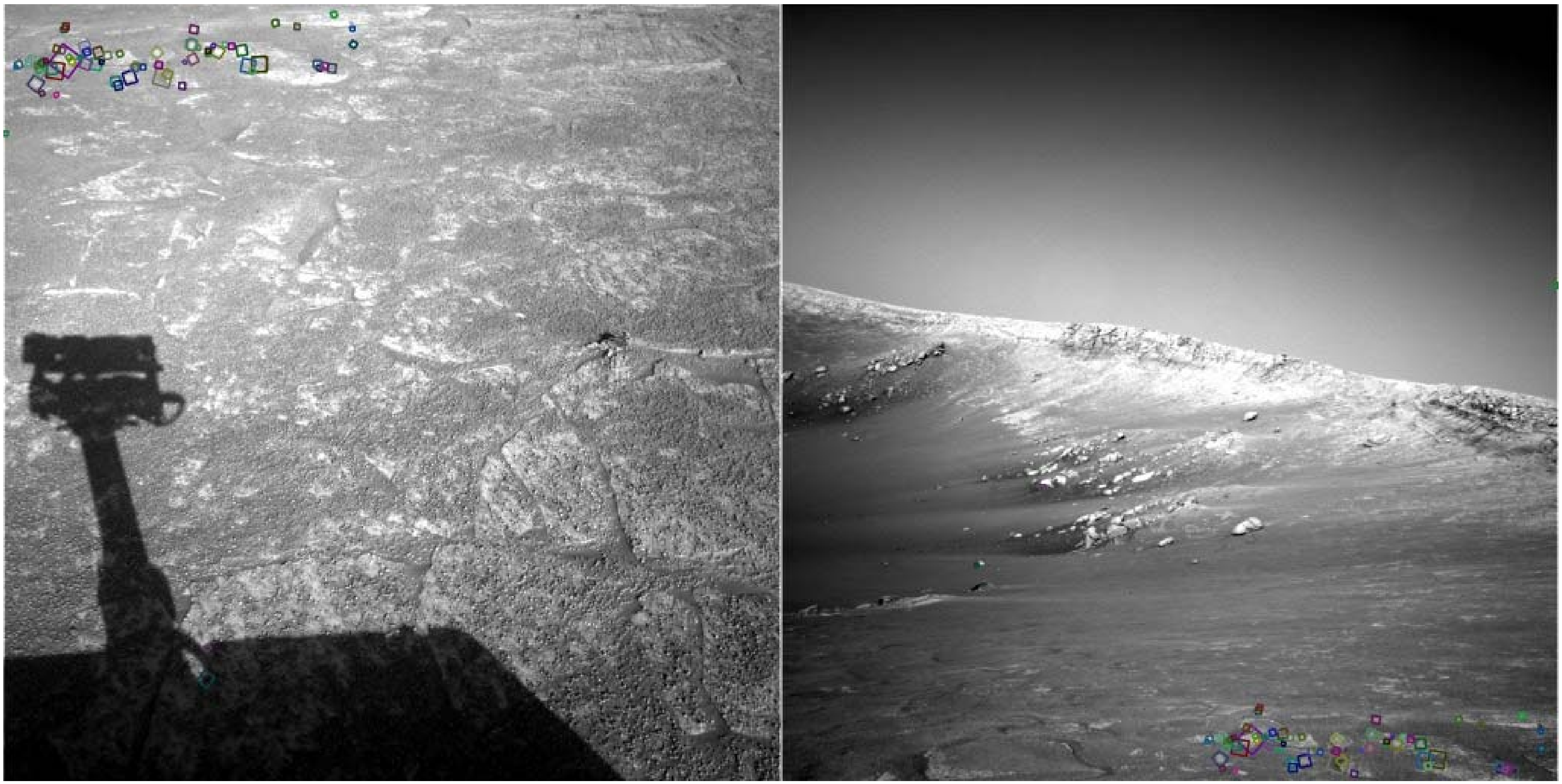
- Feature points are used for:
  - Image alignment
  - 3D reconstruction
  - Motion tracking
  - Robot navigation
  - Indexing and database retrieval
  - Object recognition

A hard feature matching problem



NASA Mars Rover images

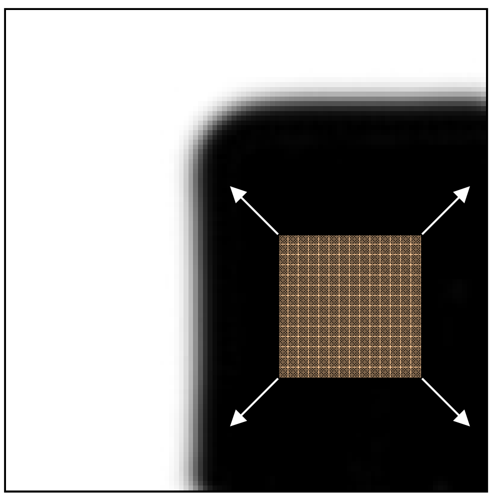
Answer below (look for tiny colored squares…)



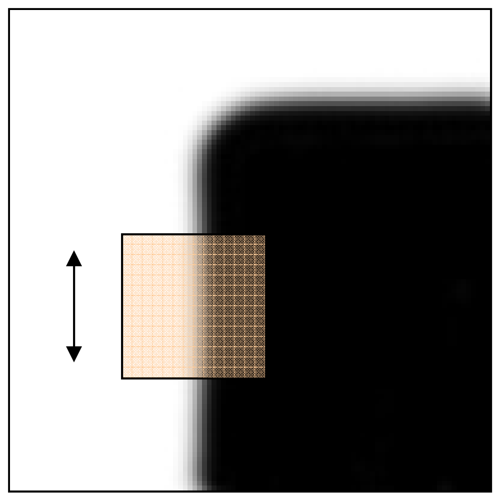
NASA Mars Rover images
with SIFT feature matches
Figure by Noah Snavely

Corner Detection: Basic Idea

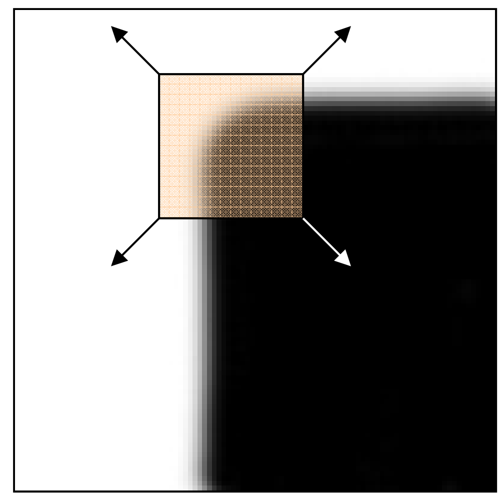
- We should easily recognize the point by looking through a small window
- Shifting a window in *any direction* should give *a large change* in intensity



"flat" region:
no change in
all directions



"edge":
no change
along the edge
direction



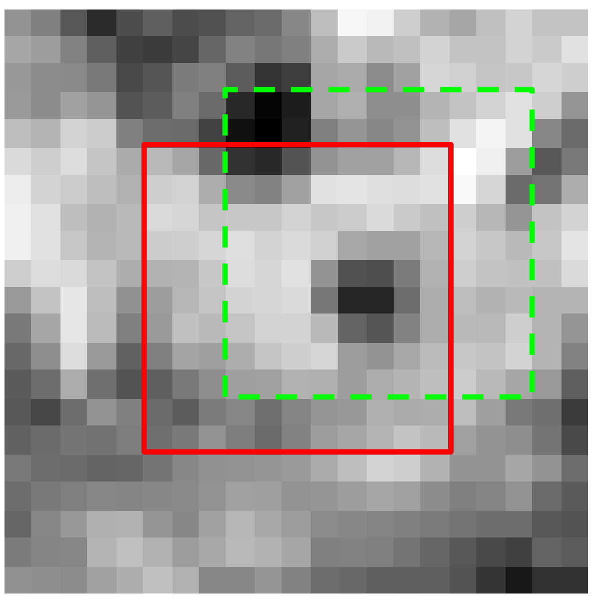
"corner":
significant
change in all
directions

Corner Detection: Mathematics

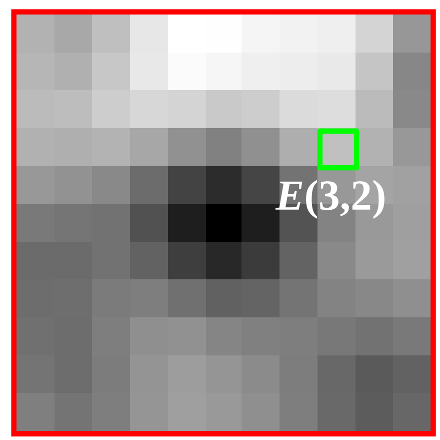
Change in appearance of window W for the shift $[u,v]$:

$$E(u,v) = \sum_{(x,y)\in W} [I(x+u, y+v) - I(x,y)]^2$$

$I(x, y)$



$E(u, v)$

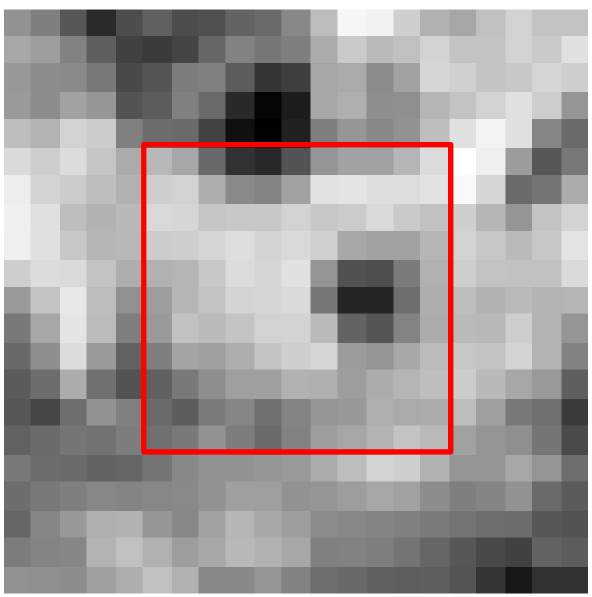


Corner Detection: Mathematics

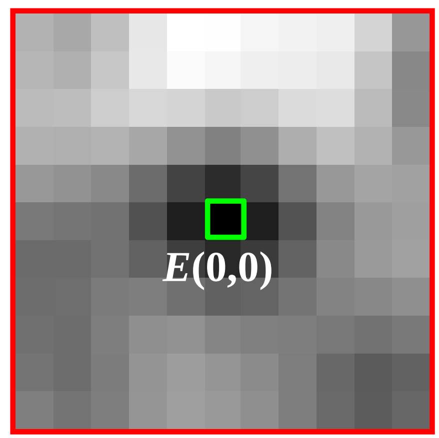
Change in appearance of window W for the shift $[u,v]$:

$$E(u,v) = \sum_{(x,y)\in W} [I(x+u, y+v) - I(x,y)]^2$$

$I(x, y)$



$E(u, v)$



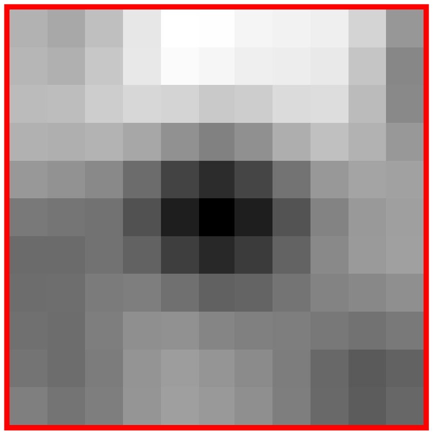
Corner Detection: Mathematics

Change in appearance of window W for the shift $[u,v]$:

$$E(u,v) = \sum_{(x,y)\in W} [I(x+u, y+v) - I(x,y)]^2$$

We want to find out how this function behaves for small shifts

$E(u, v)$



Corner Detection: Mathematics

- First-order Taylor approximation for small motions [*u*, *v*]:

$$I(x+u, y+v) \approx I(x, y) + I_x u + I_y v$$

- Let's plug this into $E(u,v)$:

$$E(u,v) = \sum_{(x,y)\in W} [I(x+u, y+v) - I(x,y)]^2$$

$$\approx \sum_{(x,y)\in W} [I(x,y) + I_x u + I_y v - I(x,y)]^2$$

$$= \sum_{(x,y)\in W} [I_x u + I_y v]^2 = \sum_{(x,y)\in W} I_x^2 u^2 + 2 I_x I_y uv + I_y^2 v^2$$

Corner Detection: Mathematics

The quadratic approximation can be written as

$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix}$$

where M is a *second moment matrix* computed from image derivatives:

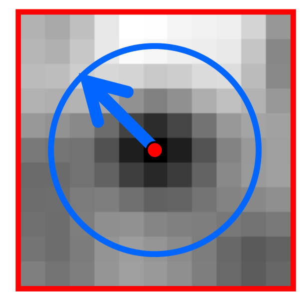
$$M = \begin{bmatrix} \sum_{x,y} I_x^2 & \sum_{x,y} I_x I_y \\ \sum_{x,y} I_x I_y & \sum_{x,y} I_y^2 \end{bmatrix}$$

(the sums are over all the pixels in the window W)

Interpreting the second moment matrix

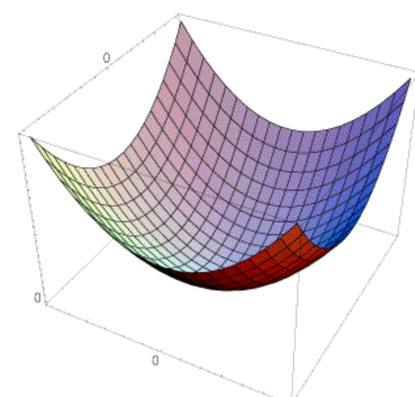
- The surface $E(u,v)$ is locally approximated by a quadratic form. Let's try to understand its shape.
  - Specifically, in which directions does it have the smallest/greatest change?

$E(u,v)$



$$E(u,v) \approx [u \;\; v] \; M \begin{bmatrix} u \\ v \end{bmatrix}$$

$$M = \begin{bmatrix} \sum_{x,y} I_x^2 & \sum_{x,y} I_x I_y \\ \sum_{x,y} I_x I_y & \sum_{x,y} I_y^2 \end{bmatrix}$$



Interpreting the second moment matrix

First, consider the axis-aligned case (gradients are either horizontal or vertical)

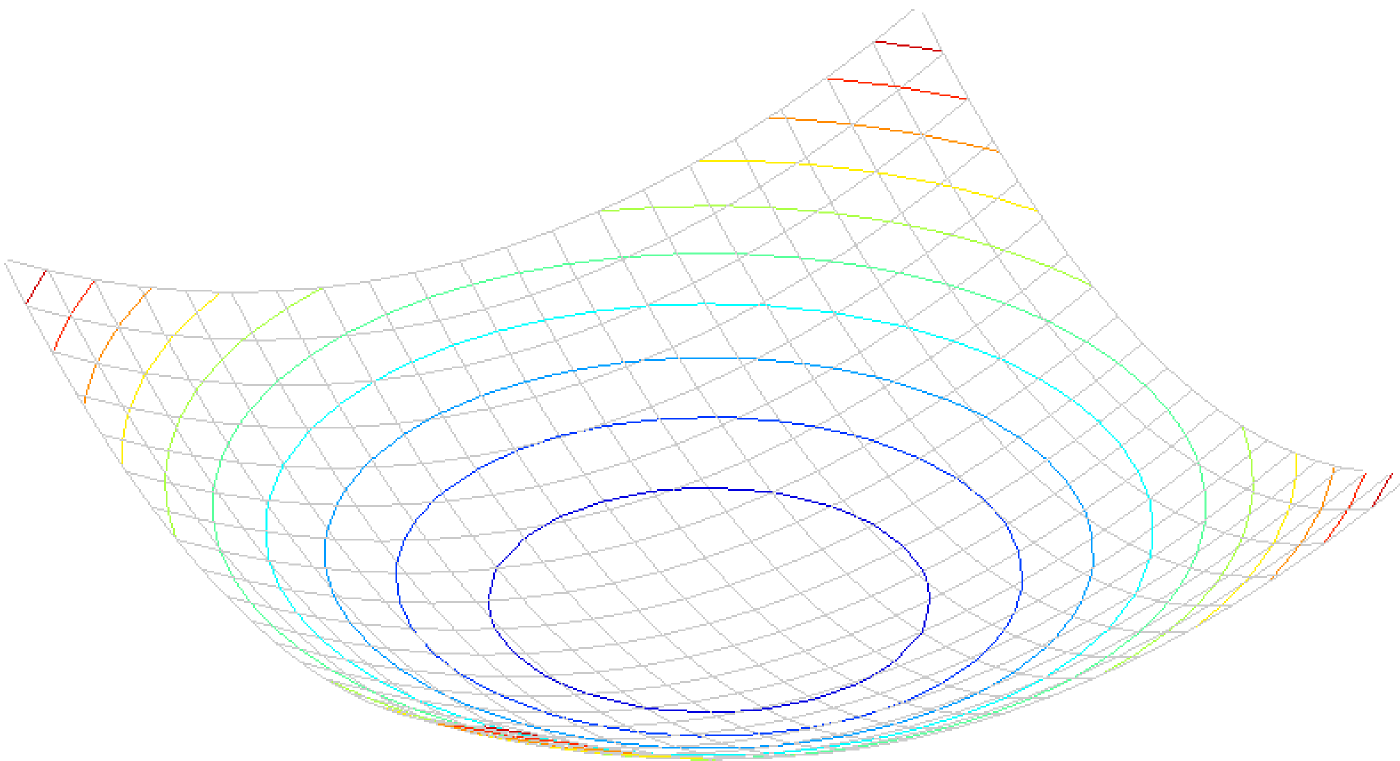
$$M = \begin{bmatrix} \sum_{x,y} I_x^2 & \sum_{x,y} I_x I_y \\ \sum_{x,y} I_x I_y & \sum_{x,y} I_y^2 \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

If either a or b is close to 0, then this is **not** a corner, so look for locations where both are large.

Interpreting the second moment matrix

Consider a horizontal "slice" of $E(u, v)$: $$[u \ \ v] \ M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$$

This is the equation of an ellipse.



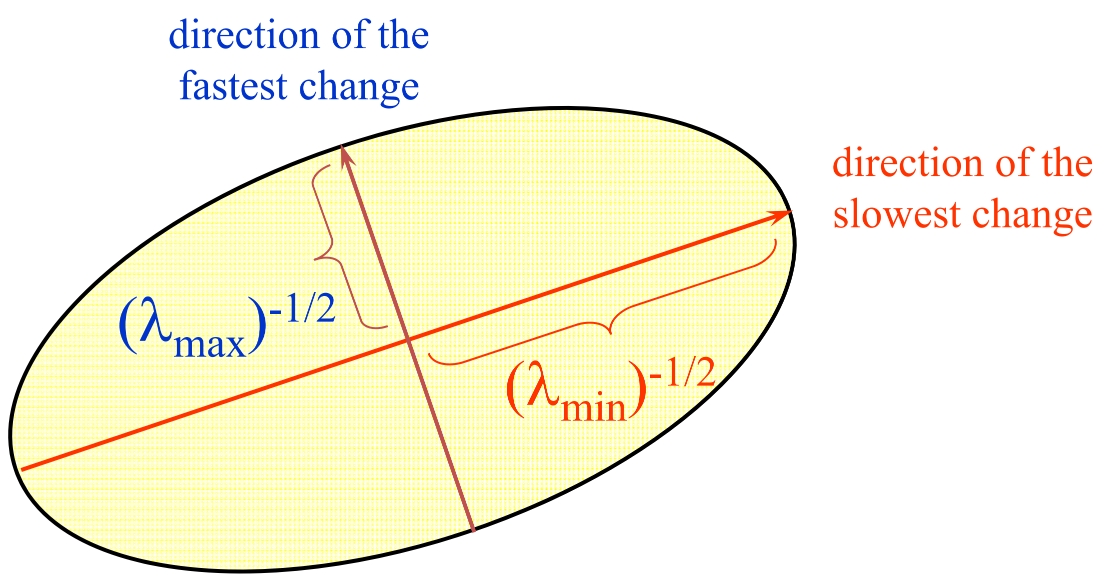
Interpreting the second moment matrix

Consider a horizontal "slice" of $E(u, v)$: $$[u \; v] \; M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$$

This is the equation of an ellipse.

Diagonalization of M: $$M = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$

The axis lengths of the ellipse are determined by the eigenvalues and the orientation is determined by R



Quick Eigenvalue/Eigenvector Review

The **eigenvectors** of a matrix A are the vectors **x** that satisfy:

$$Ax = \lambda x$$

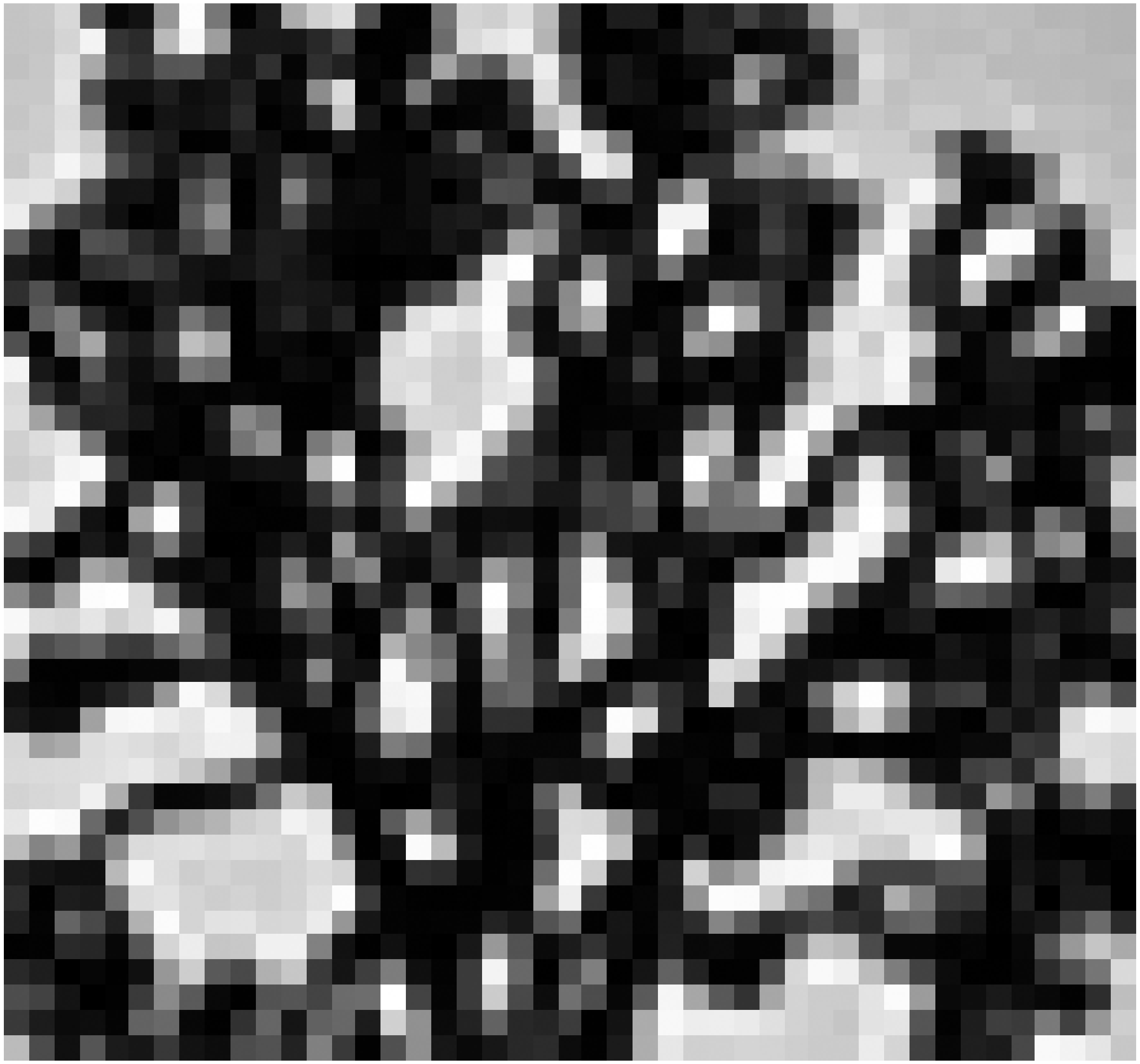
The scalar λ is the **eigenvalue** corresponding to **x**

- The eigenvalues are found by solving: $$det(A - \lambda I) = 0$$
- In our case, ***A = H*** is a 2x2 matrix, so we have $det \begin{bmatrix} h_{11} - \lambda & h_{12} \\ h_{21} & h_{22} - \lambda \end{bmatrix} = 0$
- The solution: $$\lambda_{\pm} = \frac{1}{2}\left[(h_{11} + h_{22}) \pm \sqrt{4h_{12}h_{21} + (h_{11} - h_{22})^2}\right]$$

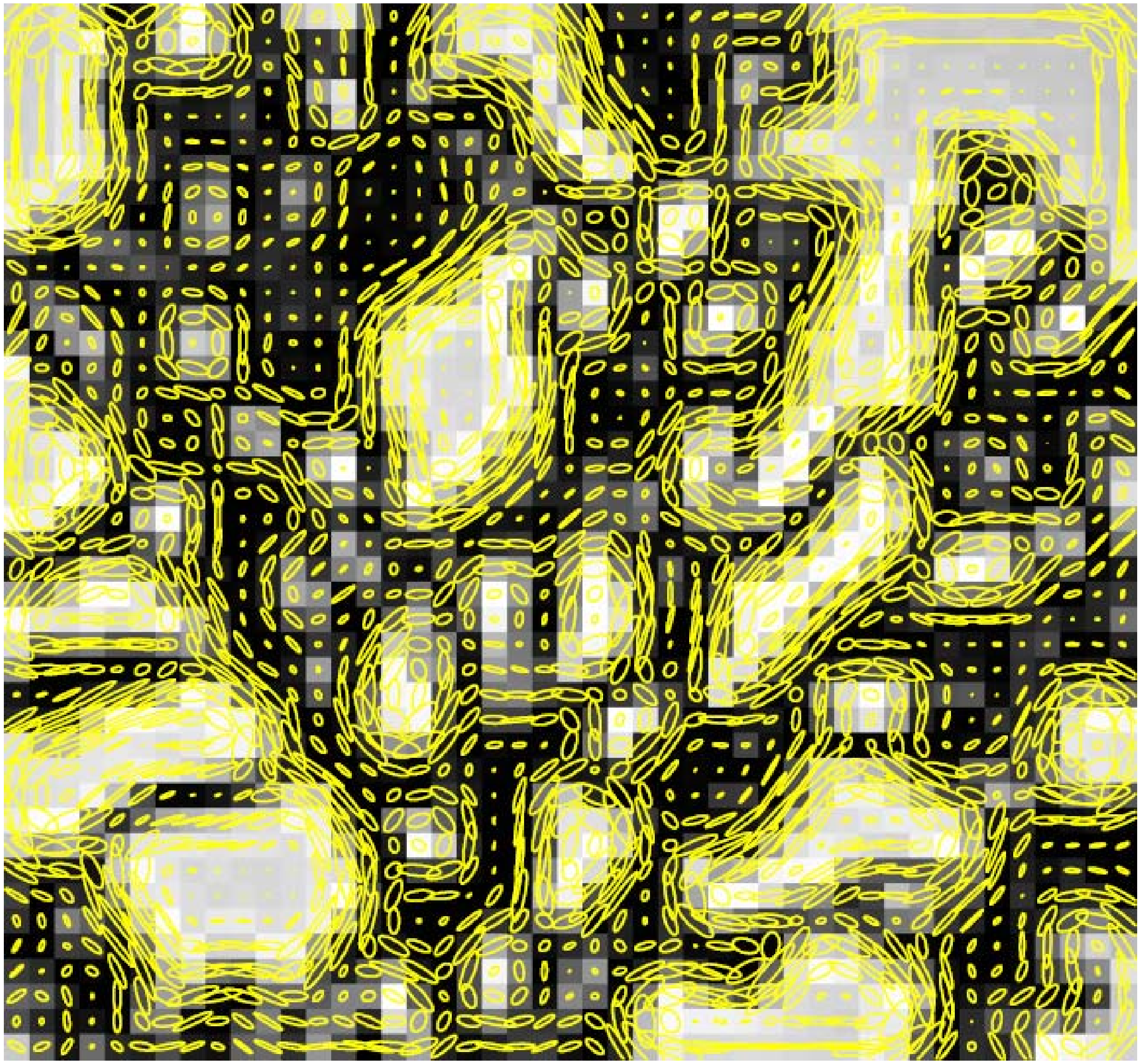
Once you know λ, you find **x** by solving

$$\begin{bmatrix} h_{11} - \lambda & h_{12} \\ h_{21} & h_{22} - \lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

Visualization of second moment matrices

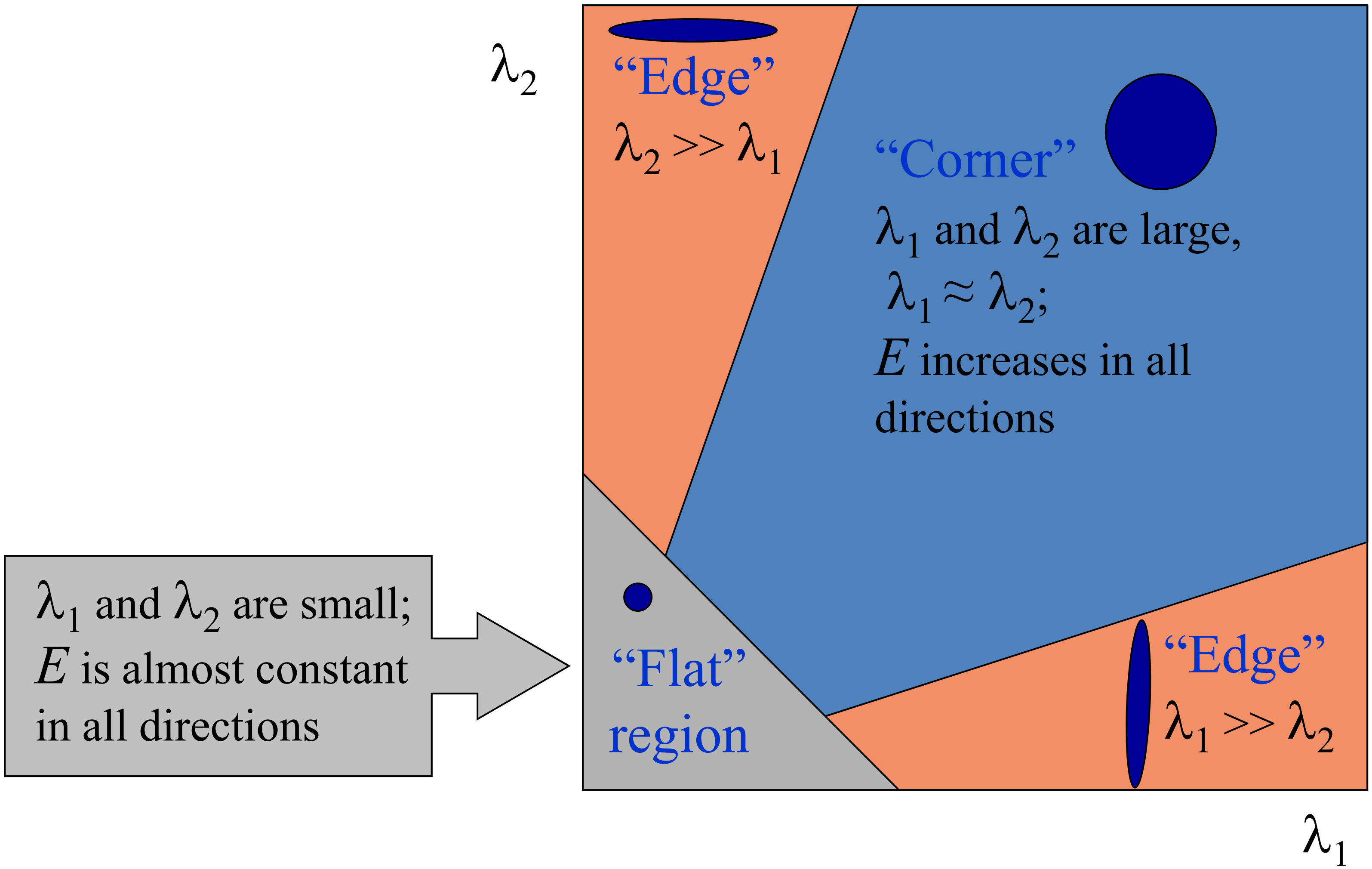


Visualization of second moment matrices



Interpreting the eigenvalues

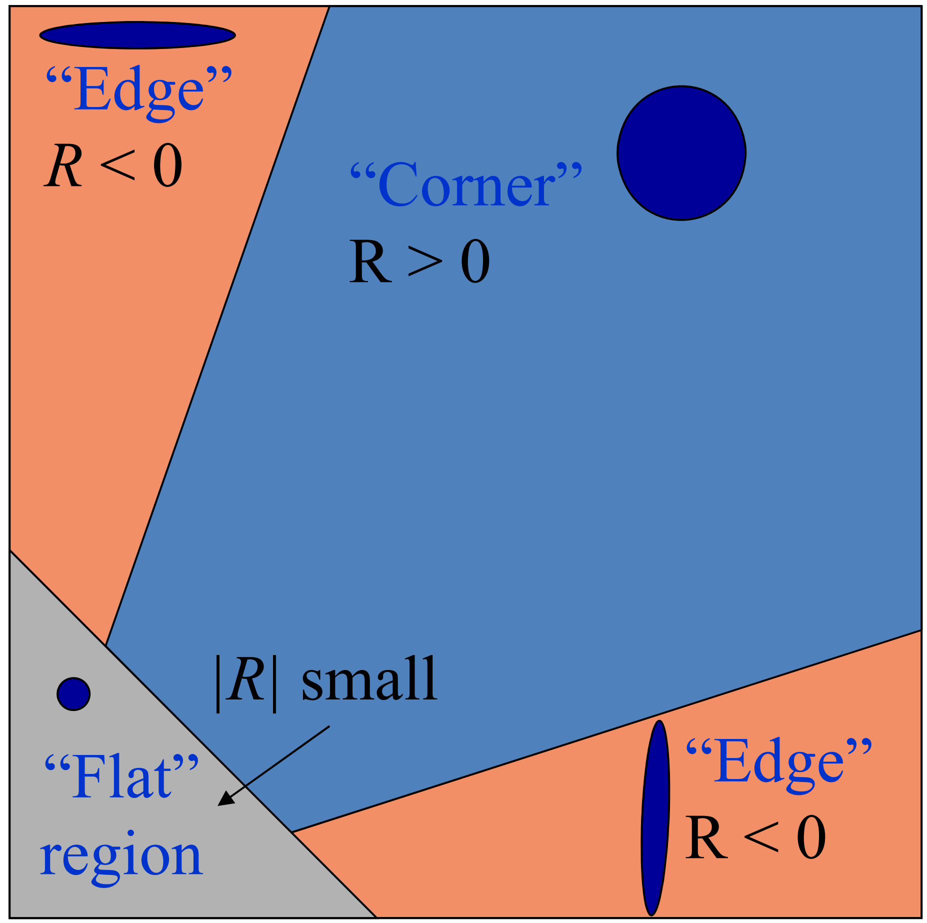
Classification of image points using eigenvalues of M:



Corner response function

$$R = \det(M) - \alpha\, \mathrm{trace}(M)^2 = \lambda_1\lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$

α: constant (0.04 to 0.06)



The Harris corner detector

1. Compute partial derivatives at each pixel
2. Compute second moment matrix *M* in a Gaussian window around each pixel:

$$M = \begin{bmatrix} \sum_{x,y} w(x,y) I_x^2 & \sum_{x,y} w(x,y) I_x I_y \\ \sum_{x,y} w(x,y) I_x I_y & \sum_{x,y} w(x,y) I_y^2 \end{bmatrix}$$

C.Harris and M.Stephens. "A Combined Corner and Edge Detector." *Proceedings of the 4th Alvey Vision Conference*: pages 147—151, 1988.

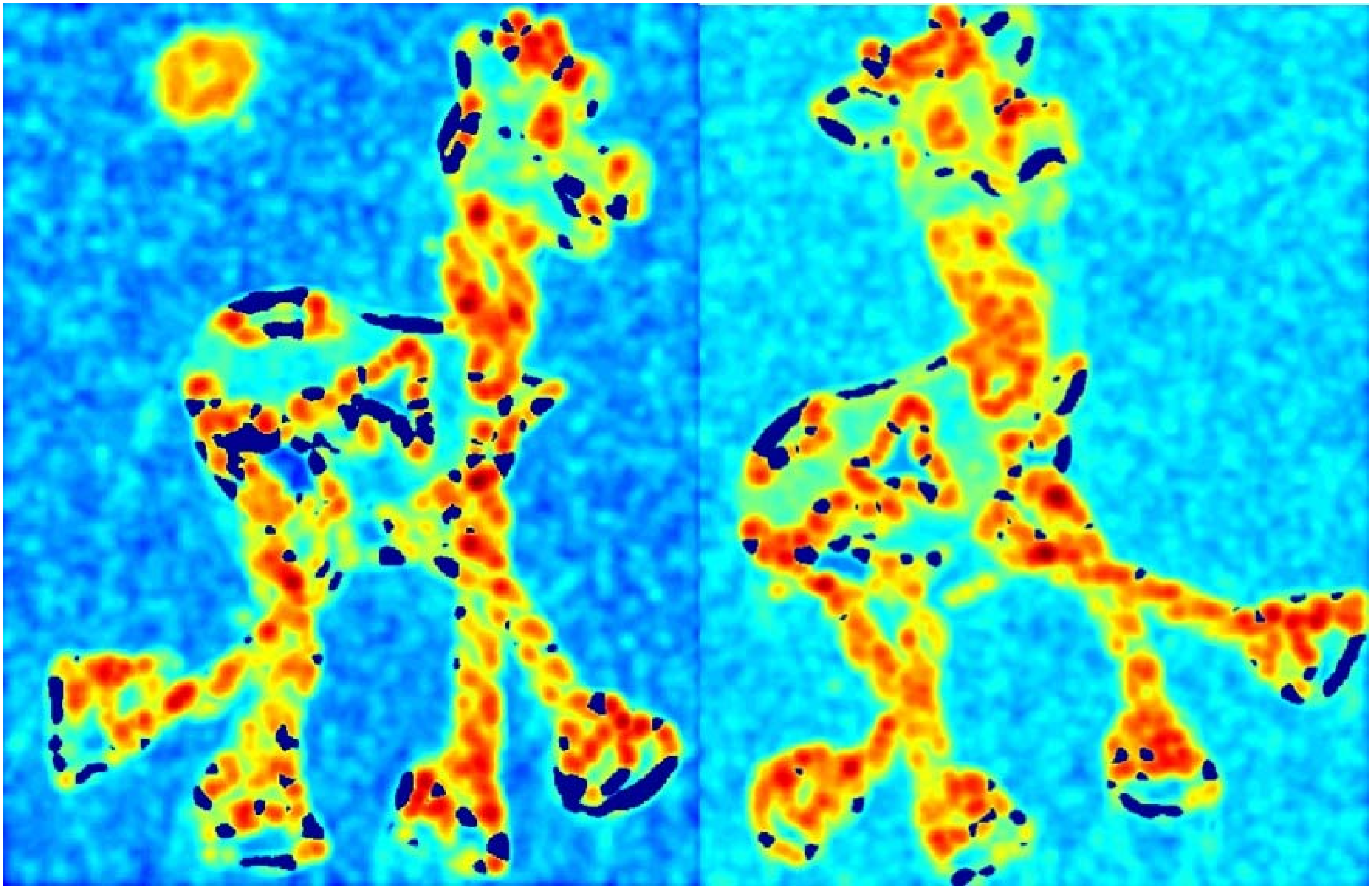
The Harris corner detector

1. Compute partial derivatives at each pixel
2. Compute second moment matrix M in a Gaussian window around each pixel
3. Compute corner response function R

Harris Detector: Steps

Harris Detector: Steps

Compute corner response R

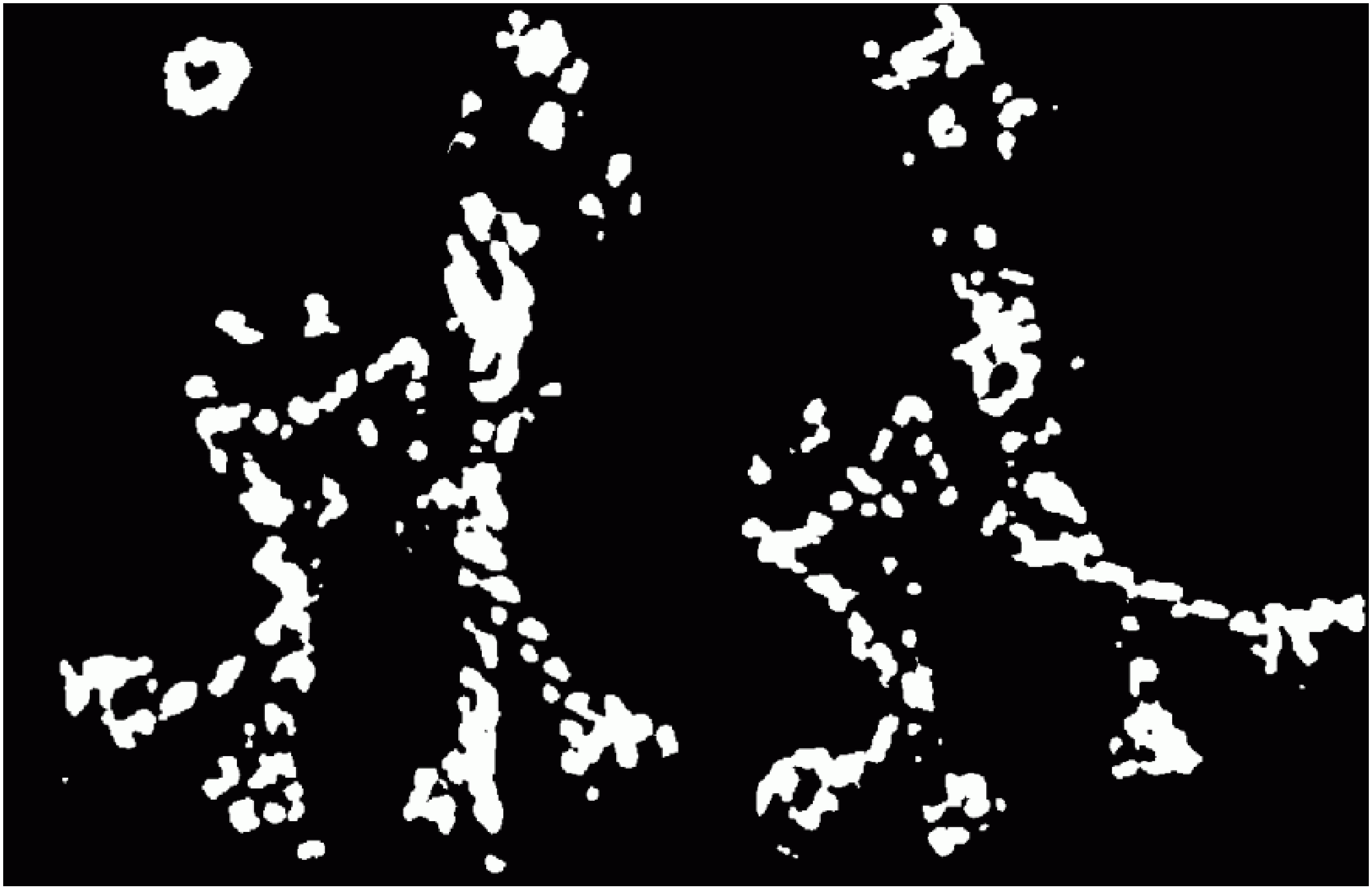


The Harris corner detector

1. Compute partial derivatives at each pixel
2. Compute second moment matrix M in a Gaussian window around each pixel
3. Compute corner response function R
4. Threshold R
5. Find local maxima of response function (non-maximum suppression)

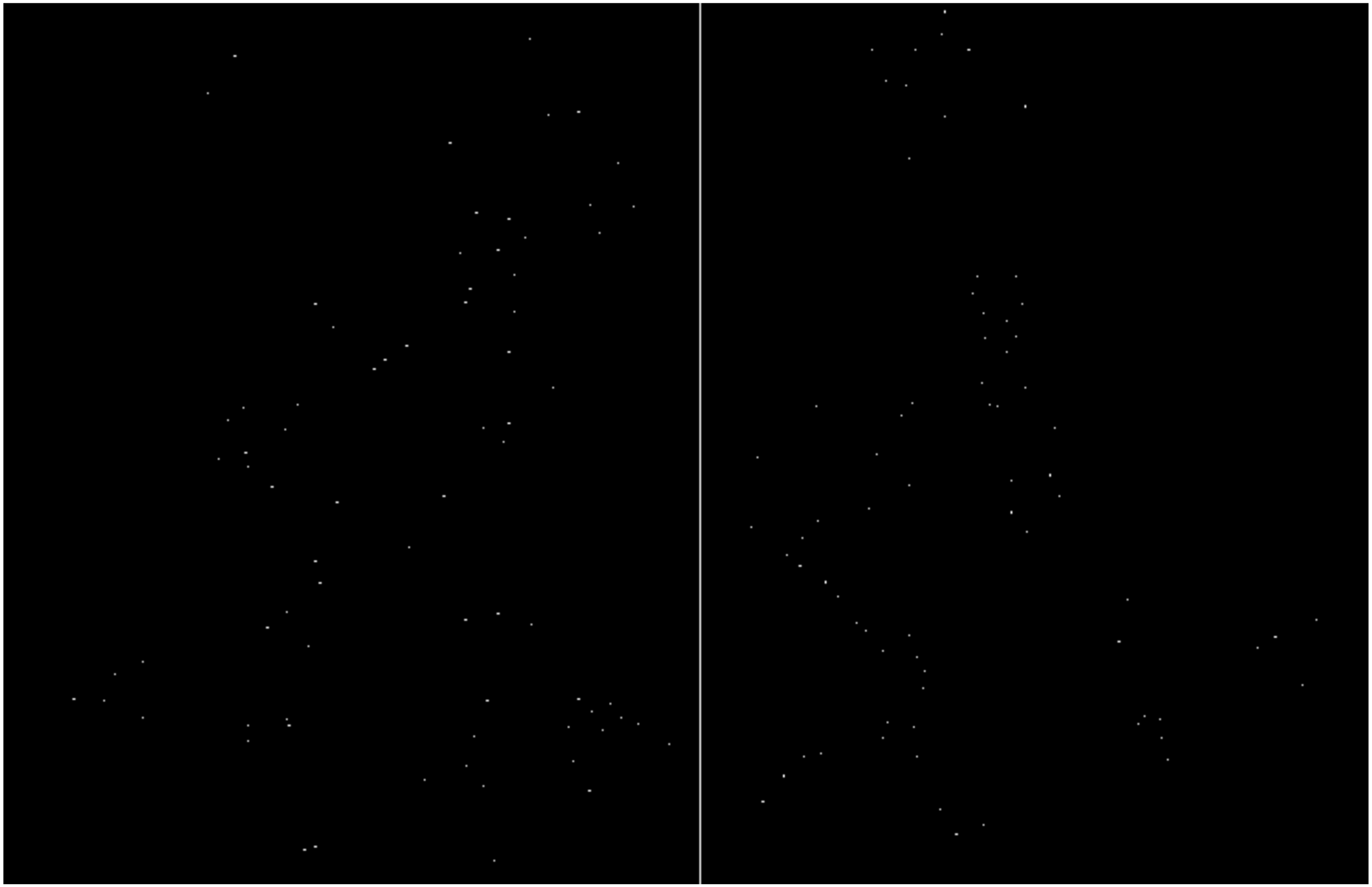
Harris Detector: Steps

Find points with large corner response: $R >$ threshold

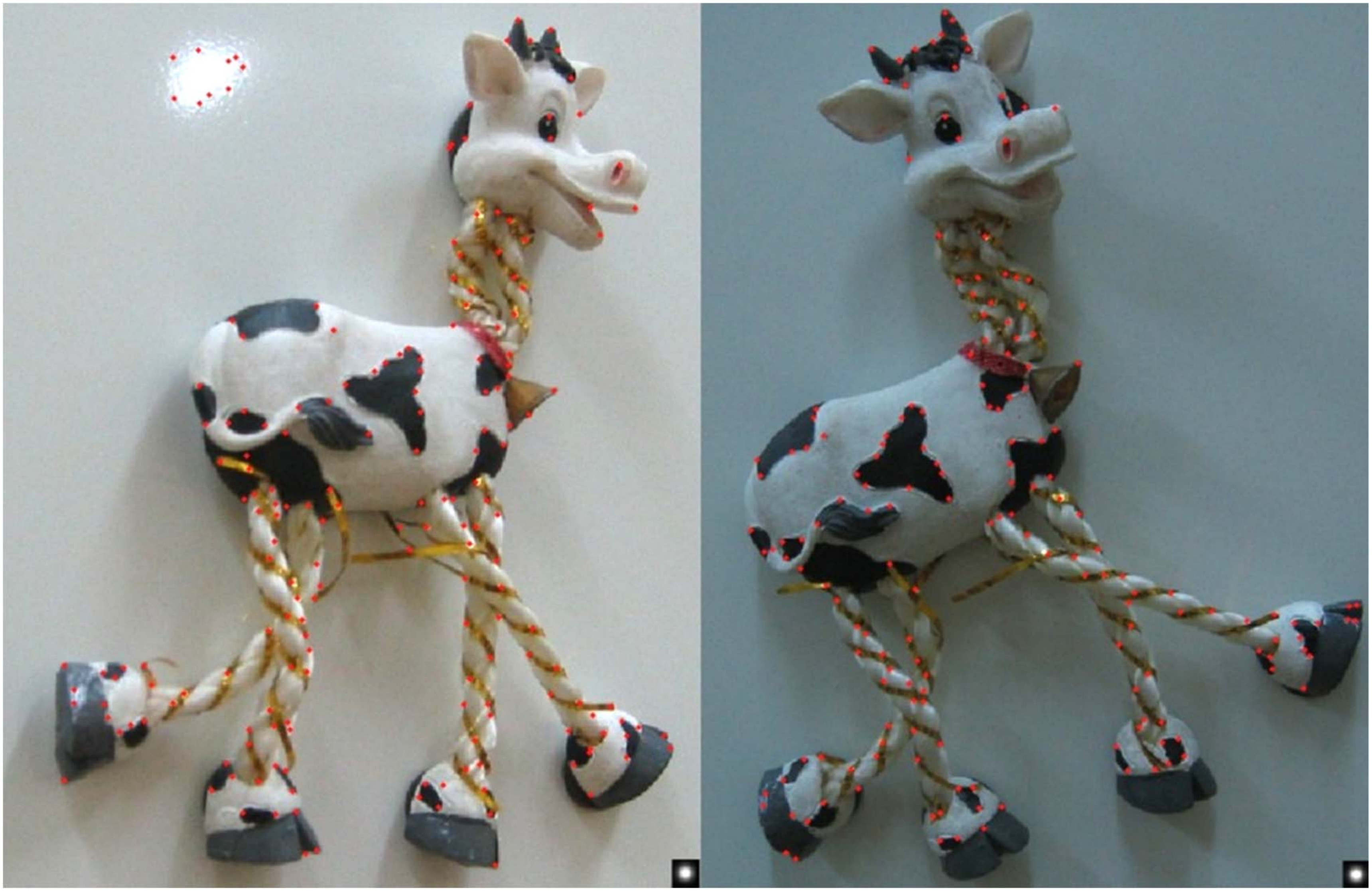


Harris Detector: Steps

Take only the points of local maxima of *R*



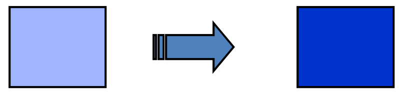
Harris Detector: Steps



Invariance and covariance

- We want corner locations to be *invariant* to photometric transformations and *covariant* to geometric transformations
  - **Invariance:** image is transformed and corner locations do not change
  - **Covariance:** if we have two transformed versions of the same image, features should be detected in corresponding locations

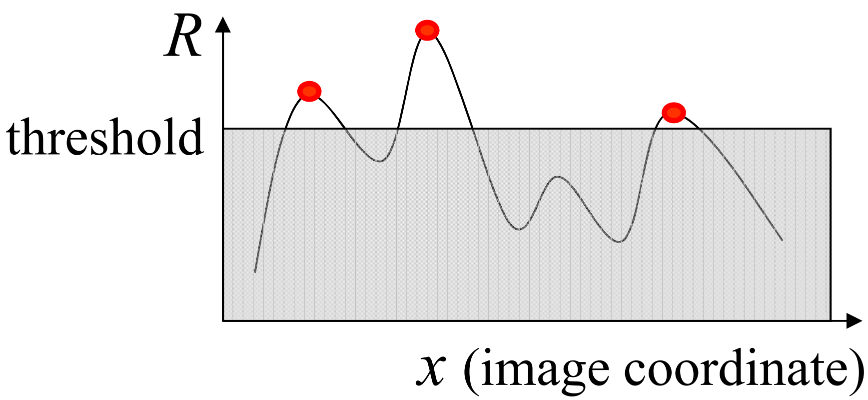
Intensity changes

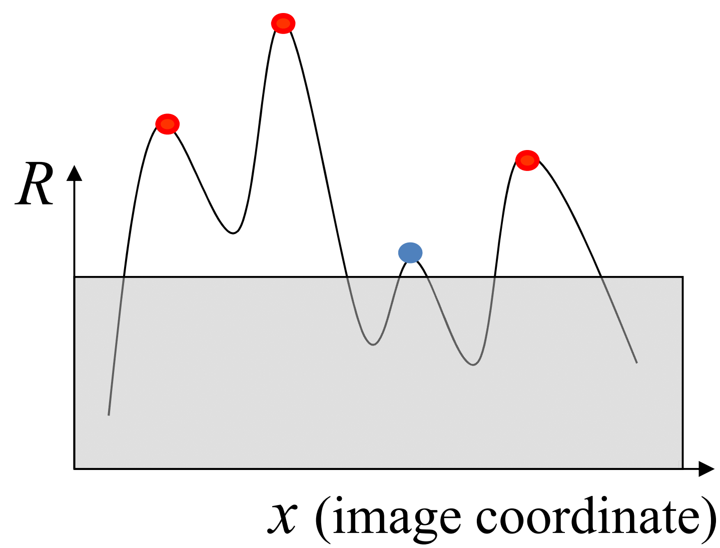


$$I \rightarrow a\,I + b$$

- Only derivatives are used => invariance to intensity shift $I \rightarrow I + b$

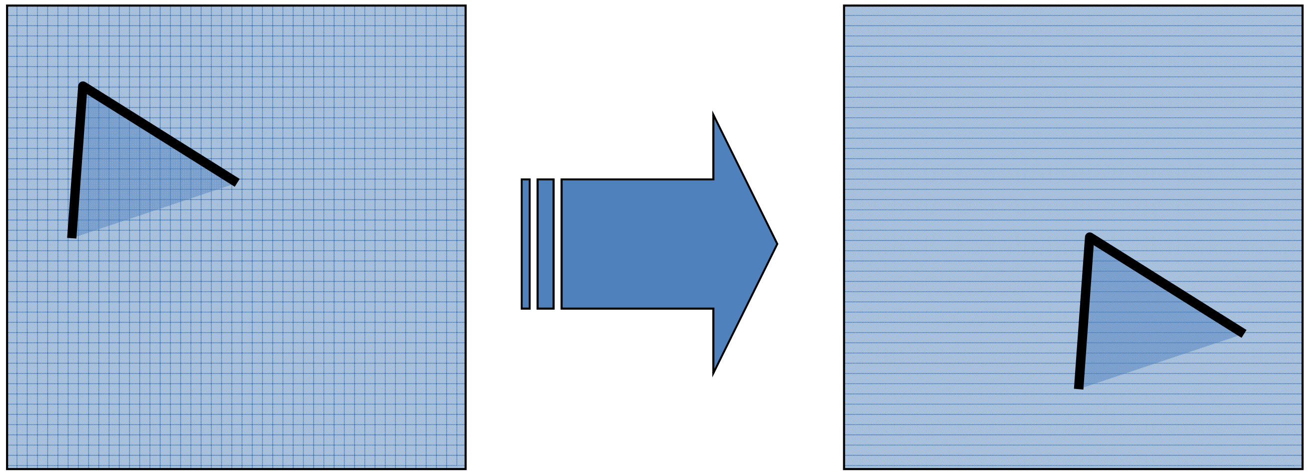
- Intensity scaling: $I \rightarrow a\,I$





Partially invariant to affine intensity change

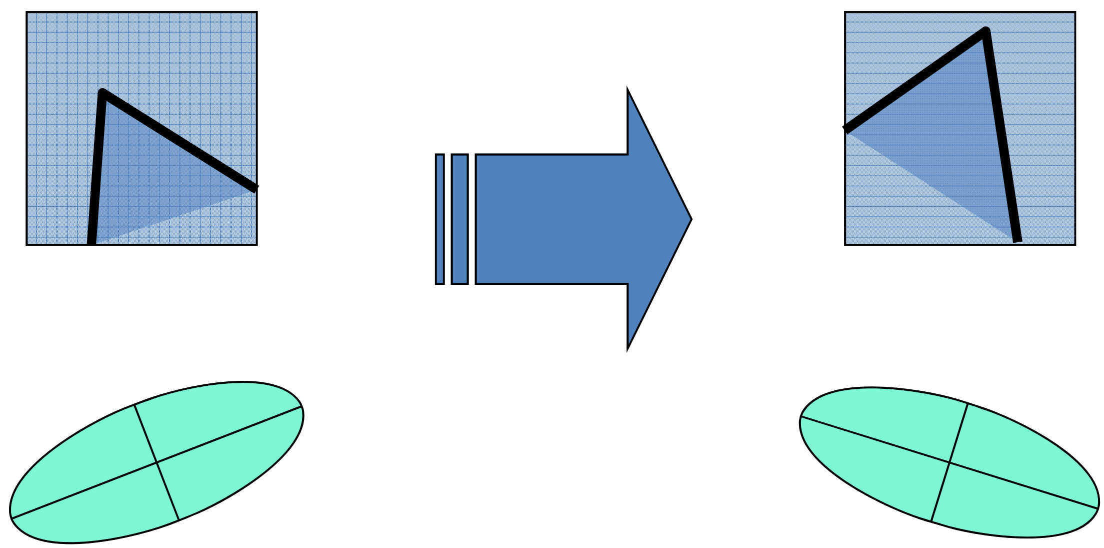
Image translation



- Derivatives and window function are shift-invariant

Corner location is covariant w.r.t. translation

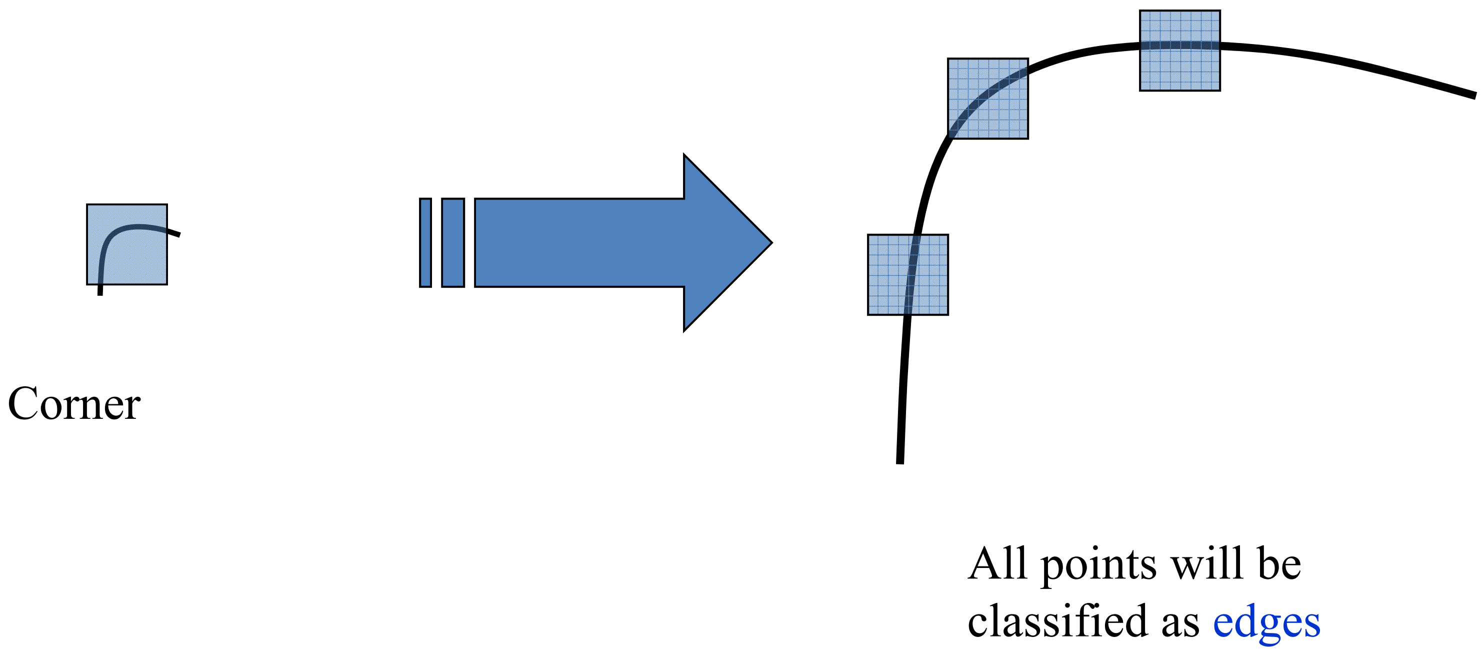
Image rotation



Second moment ellipse rotates but its shape (i.e. eigenvalues) remains the same

Corner location is covariant w.r.t. rotation

Scaling



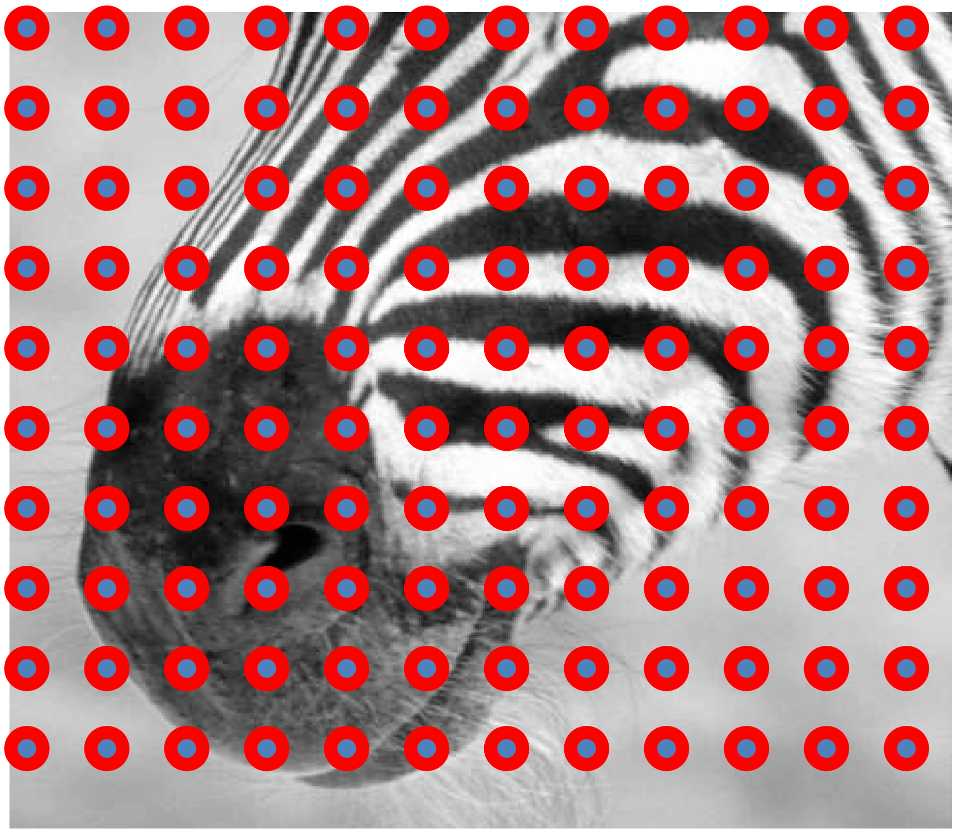
Corner location is not covariant to scaling!

Sampling

Why does a lower resolution image still make sense to us? What do we lose?

Image: http://www.flickr.com/photos/igorms/136916757/

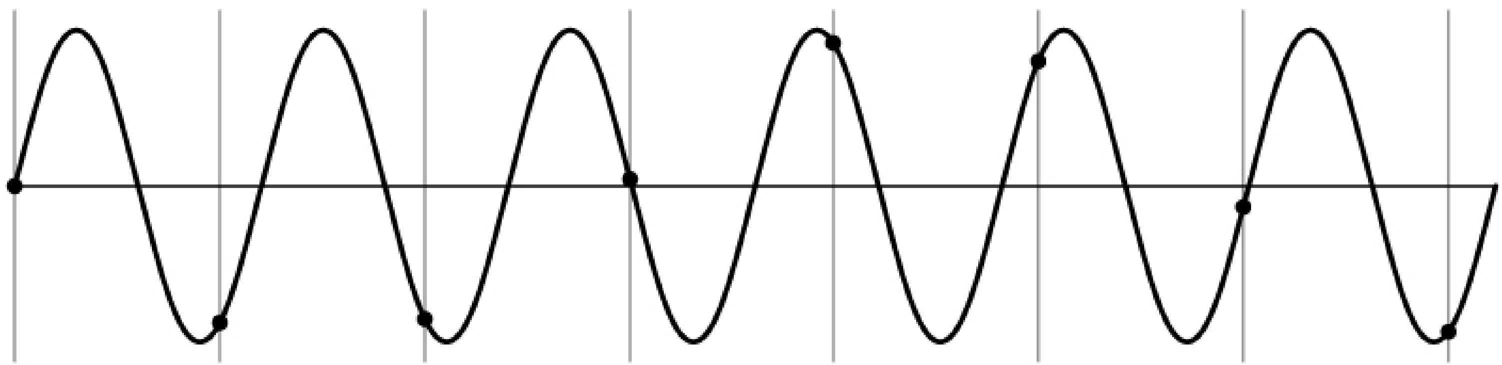
Subsampling by a factor of 2



Throw away every other row and column
to create a 1/2 size image

Aliasing problem

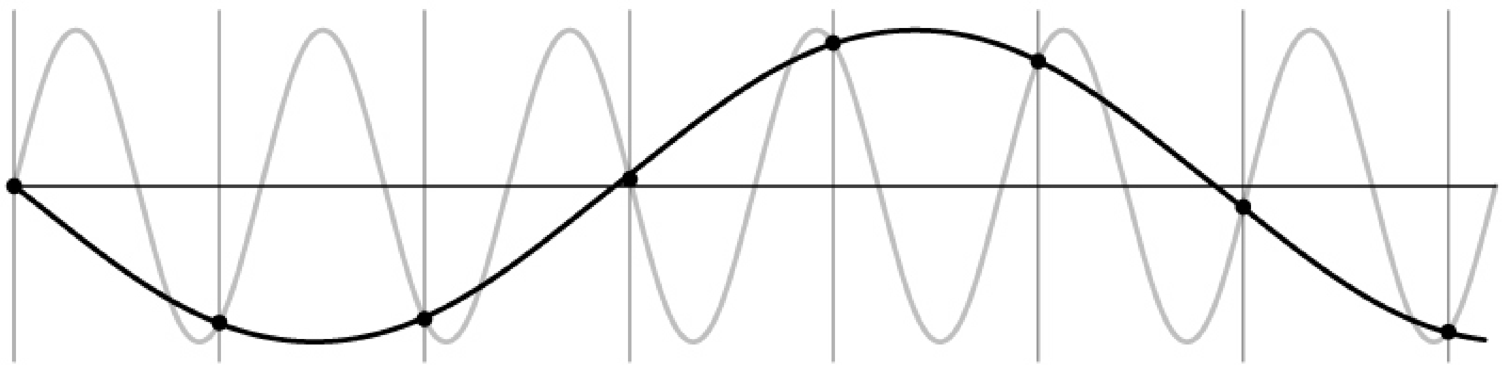
- 1D example (sinewave):



Source: S. Marschner

Aliasing problem

- 1D example (sinewave):



Source: S. Marschner

Aliasing problem

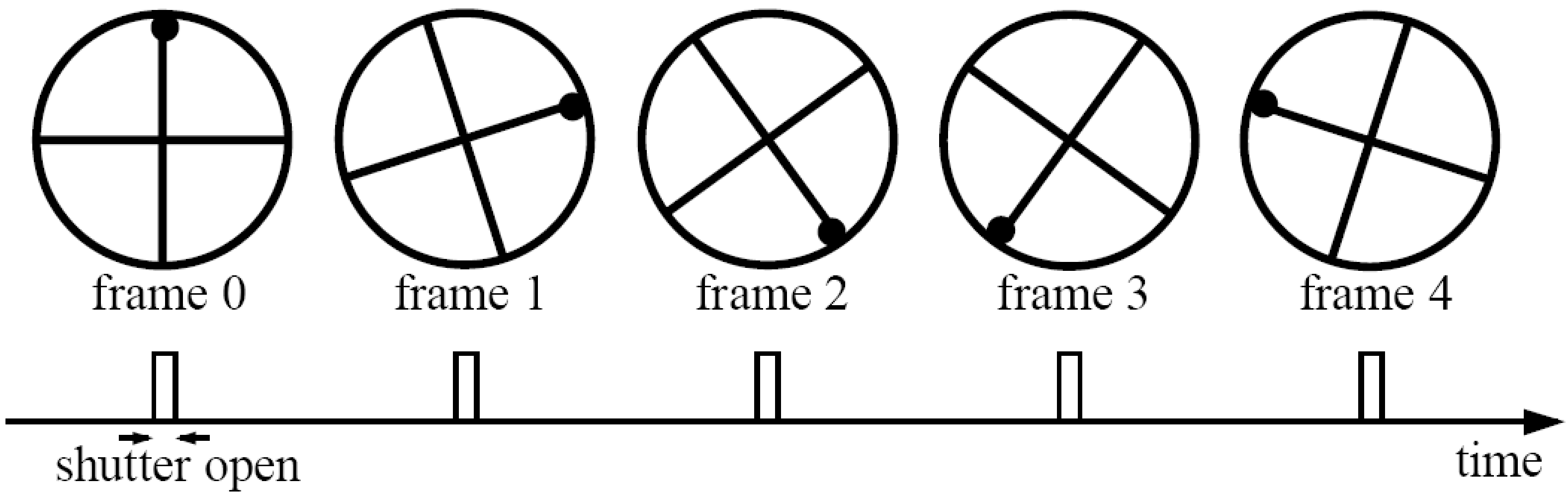
- Sub-sampling may be dangerous….
- Characteristic errors may appear:
  - "Wagon wheels rolling the wrong way in movies"
  - "Checkerboards disintegrate in ray tracing"
  - "Striped shirts look funny on color television"

Source: D. Forsyth

Aliasing in video

Imagine a spoked wheel moving to the right (rotating clockwise). Mark wheel with dot so we can see what's happening.

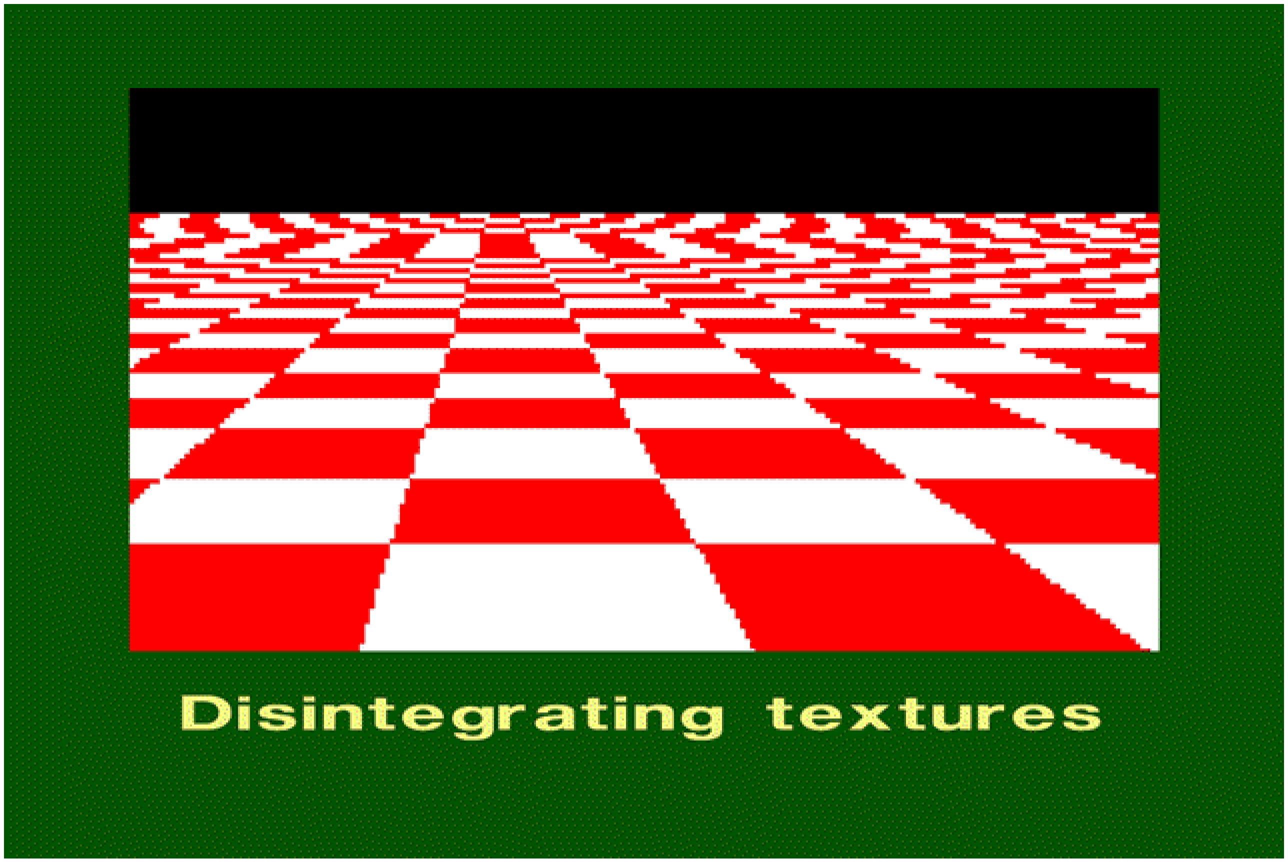
If camera shutter is only open for a fraction of a frame time (frame time = 1/30 sec. for video, 1/24 sec. for film):



Without dot, wheel appears to be rotating slowly backwards! (counterclockwise)

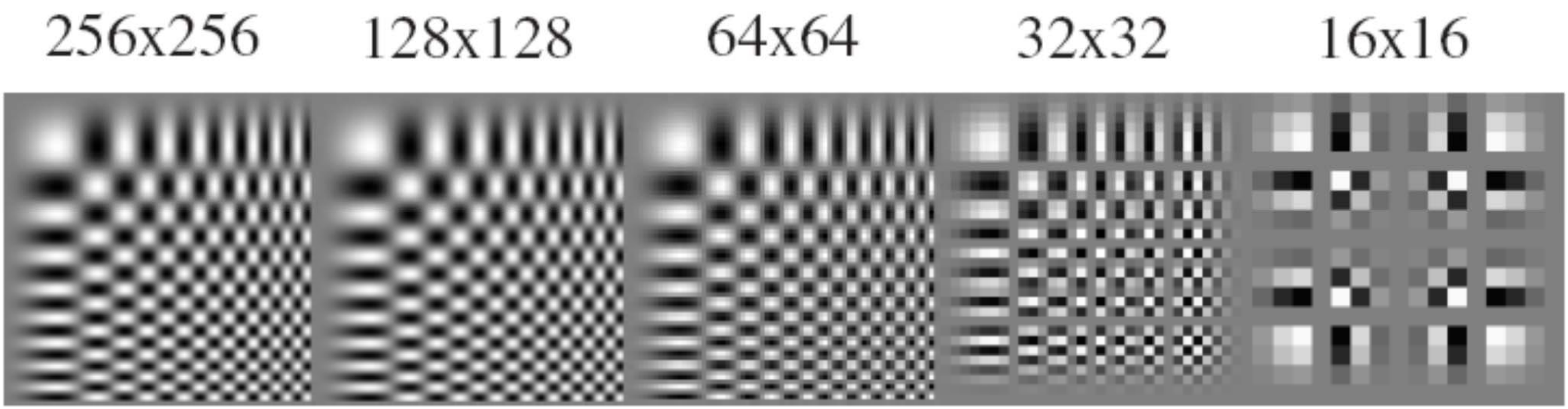
Slide by Steve Seitz

Aliasing in graphics



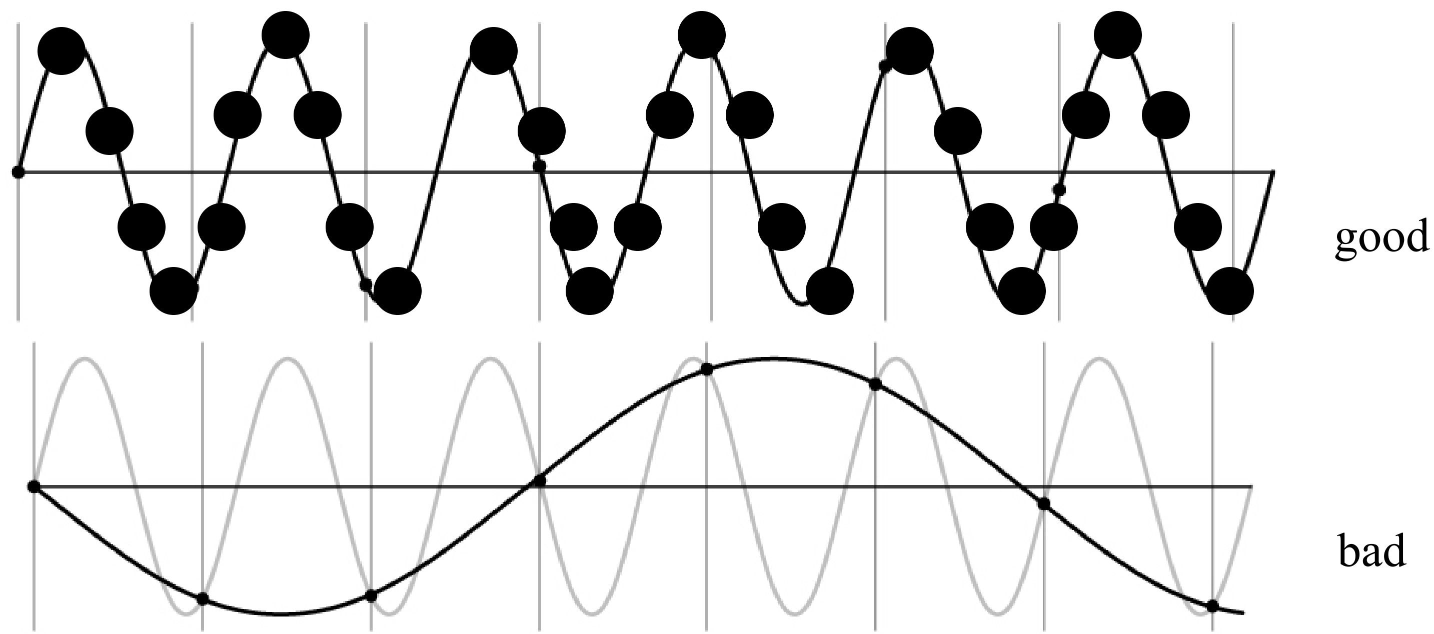
Source: A. Efros

Sampling and aliasing



Nyquist-Shannon Sampling Theorem

- When sampling a signal at discrete intervals, the sampling frequency must be $\geq 2 \times f_{max}$
- f_{max} = max frequency of the input signal
- This will allows to reconstruct the original perfectly from the sampled version



Anti-aliasing

Solutions:

- Sample more often
- Get rid of all frequencies that are greater than half the new sampling frequency
  - Will lose information
  - But it’s better than aliasing
  - Apply a smoothing filter

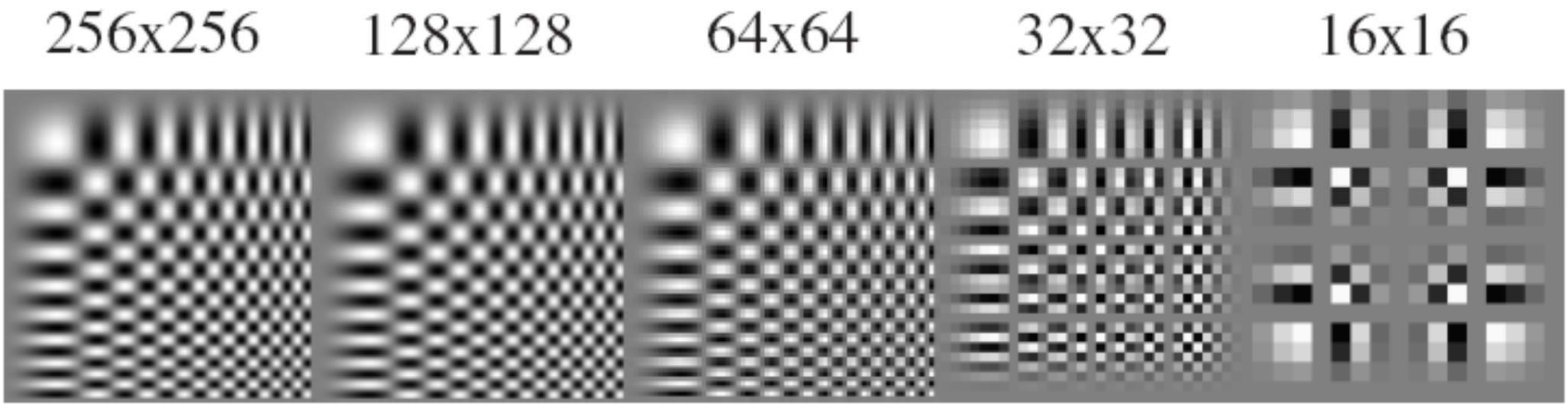
Algorithm for downsampling by factor of 2

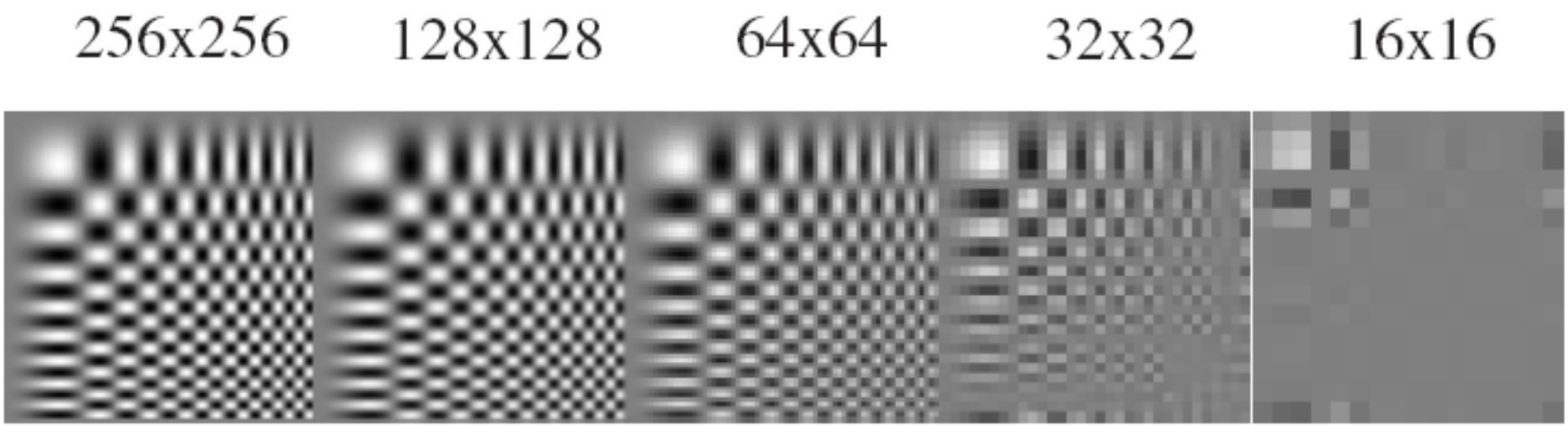
1. Start with image(h, w)
2. Apply low-pass filter

   ```
   im_blur = imfilter(image, fspecial('gaussian', 7, 1))
   ```
3. Sample every other pixel

   ```
   im_small = im_blur(1:2:end, 1:2:end);
   ```

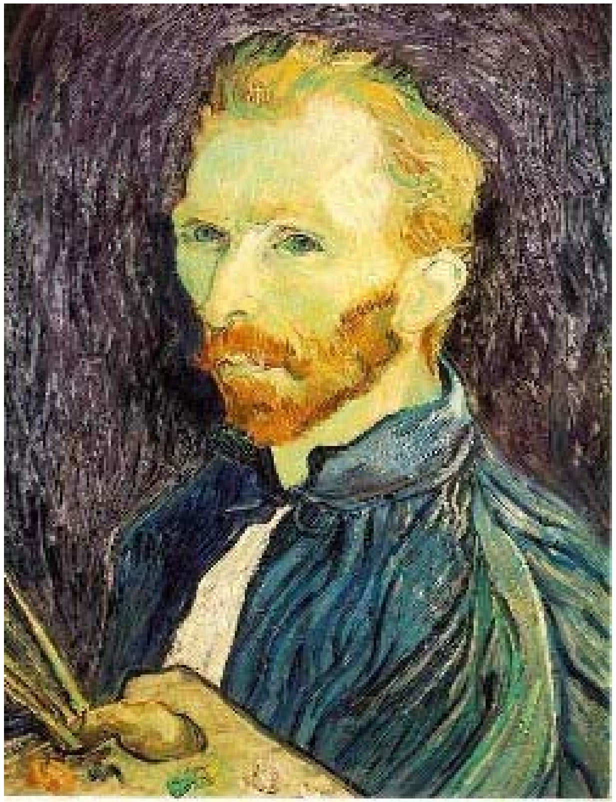
Anti-aliasing



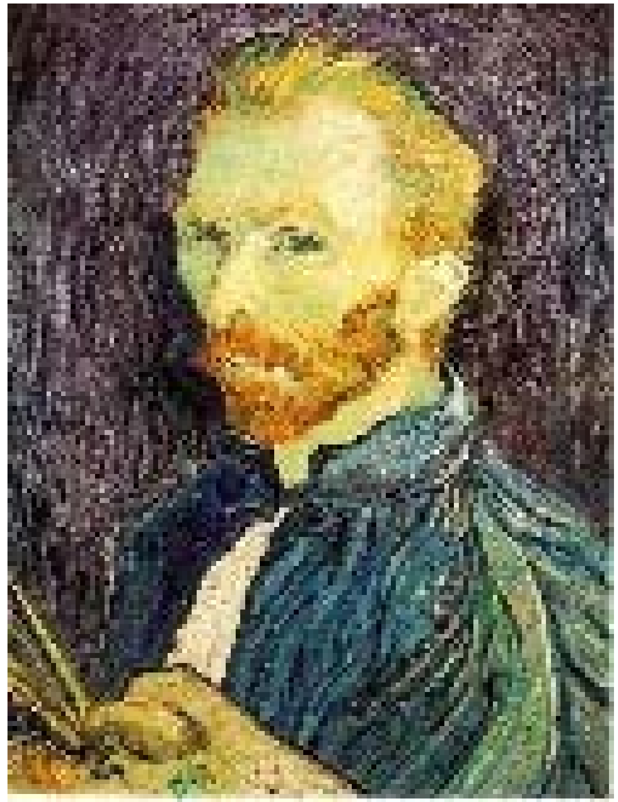


Forsyth and Ponce 2002

Subsampling without pre-filtering



1/2

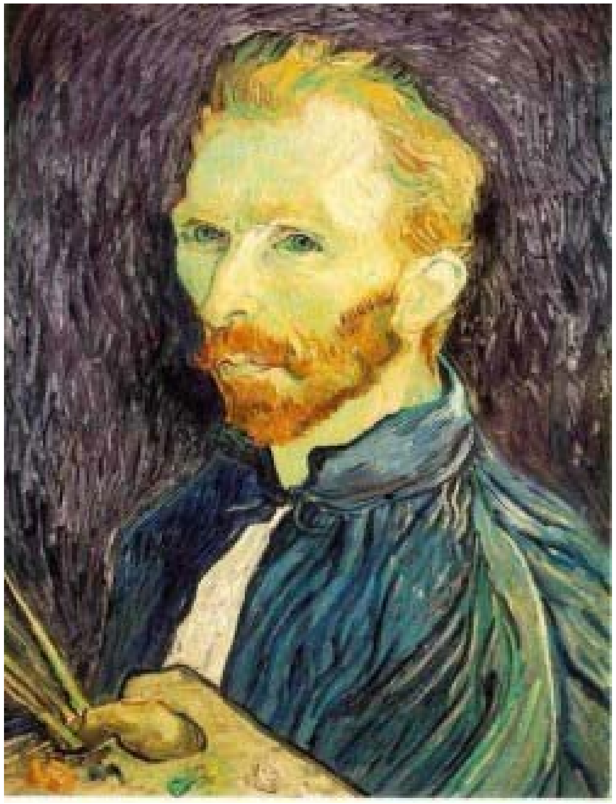


1/4 (2x zoom)

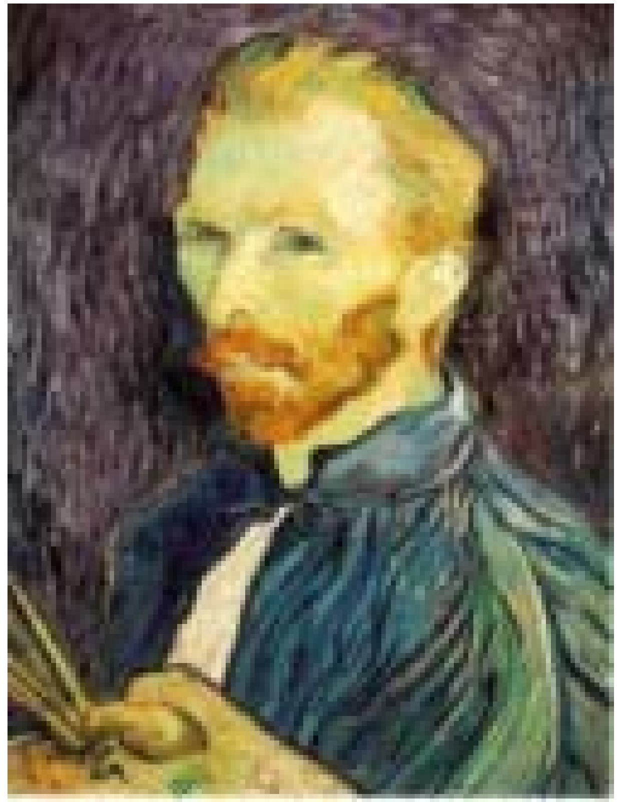
1/8 (4x zoom)

Slide by Steve Seitz

Subsampling with Gaussian pre-filtering



Gaussian 1/2



G 1/4

G 1/8

Slide by Steve Seitz

Slide Credits

- This set of sides contains contributions kindly made available by the following authors
  - Svetlana Lazebnik
  - Alexei Efros
  - Li Fei-Fei
  - Kristen Grauman
  - Martial Hebert
  - Derek Hoiem
  - David Lowe
  - Steve Marschner
  - David Forsyth
  - Steve Seitz
  - Richard Szeliski