

# CS 558: Computer Vision

## 10<sup>th</sup> Set of Notes

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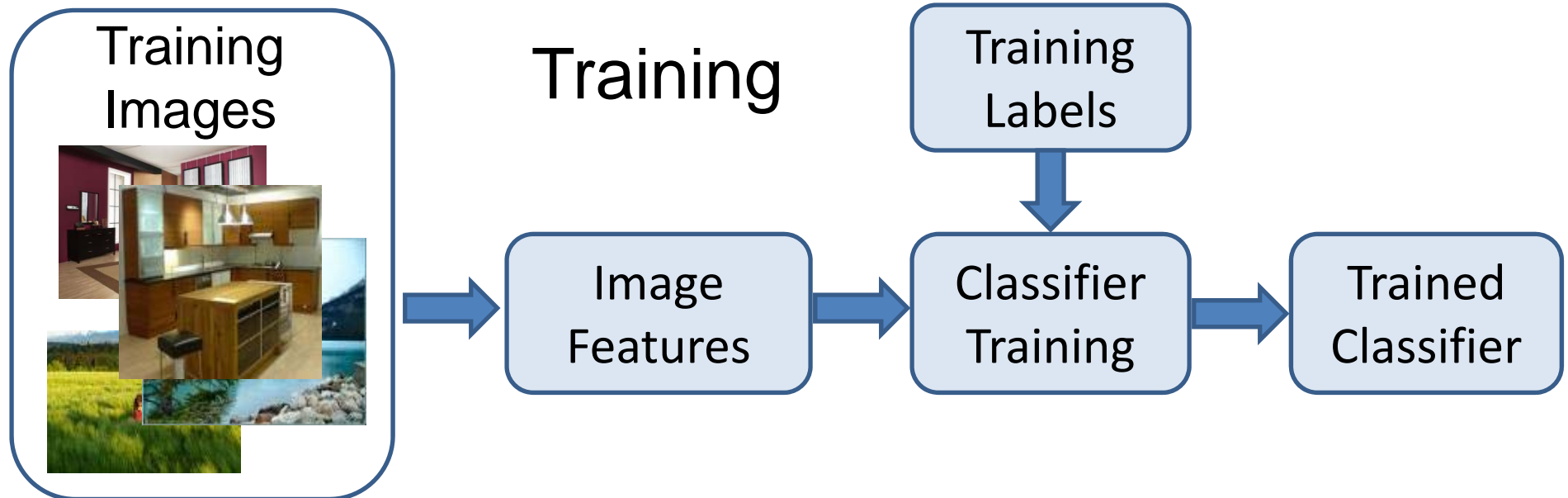
# Overview

- Image Features and Categorization
  - Histograms
  - Bags of features/visual words
  - Vocabulary trees
  - Spatial layout and context (preview)
  - Based on slides by K. Grauman, D. Hoiem and S. Lazebnik

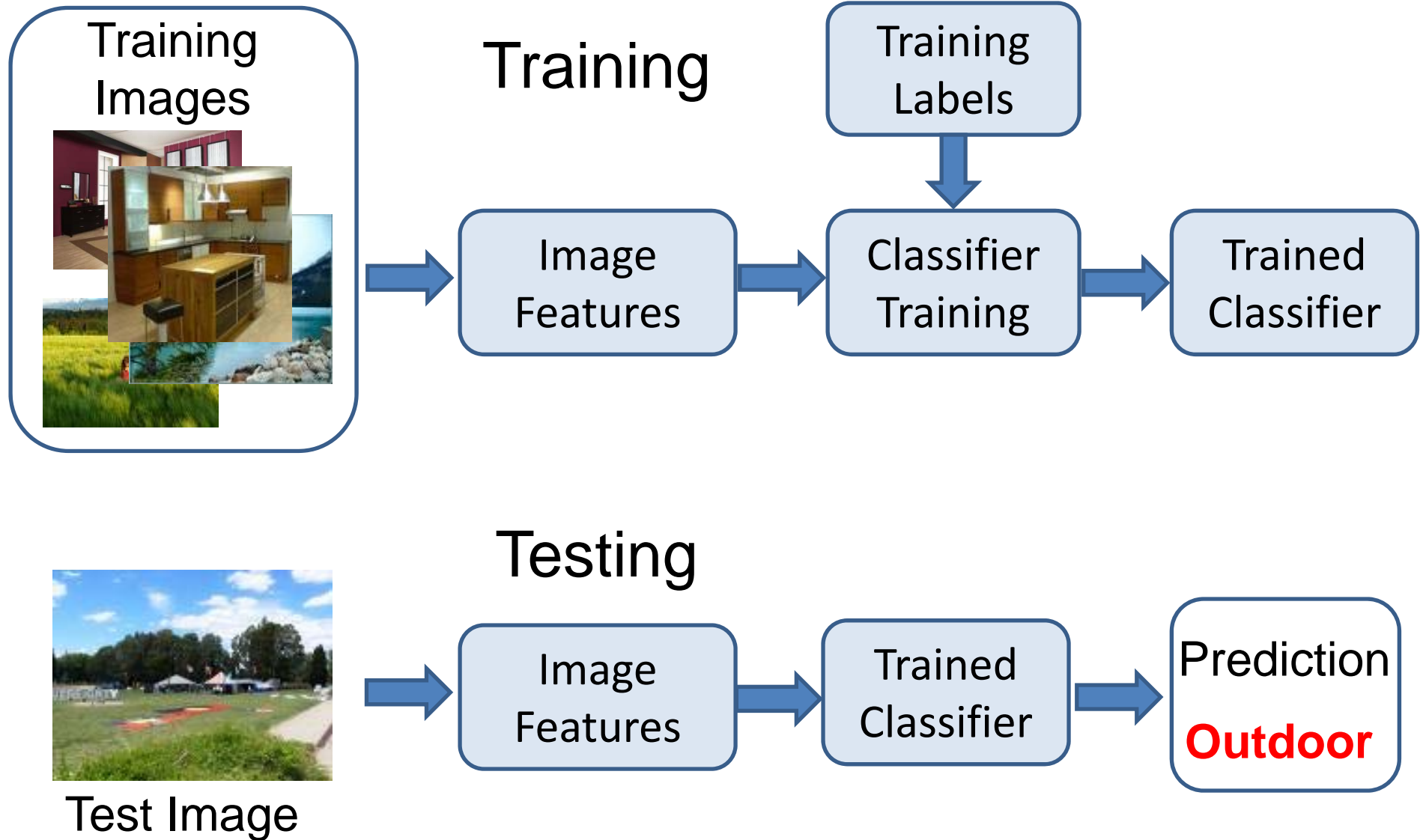
# Image Features and Categorization



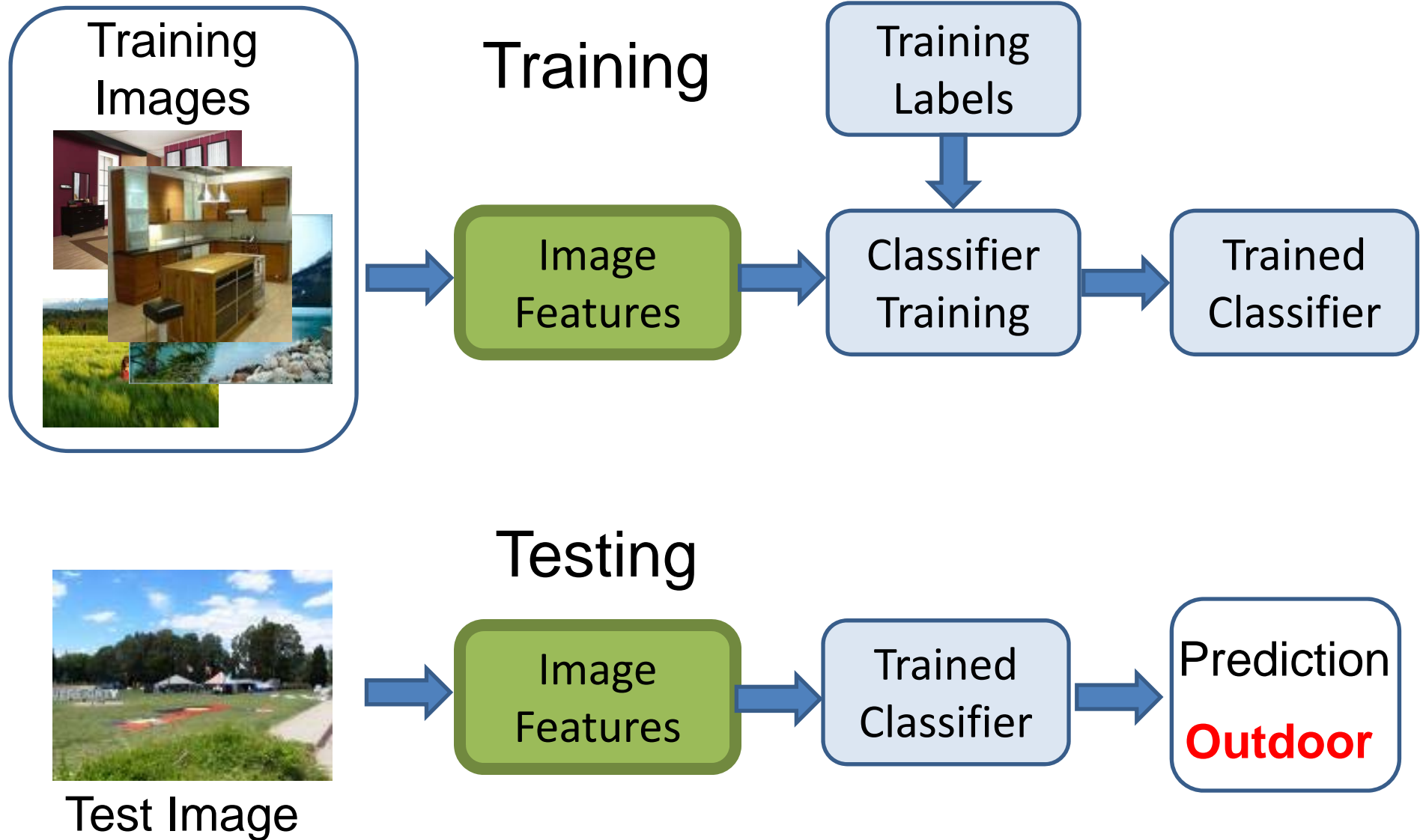
# Training phase



# Testing phase



# Testing phase



# Q: What are good features for...

- recognizing a beach?



# Q: What are good features for...

- recognizing fabrics?





# Q: What are good features for...

- recognizing a mug?



# What are the right features?

Depends on what we want to know!

- Object: shape
  - Local shape info, shading, shadows, texture
- Scene : geometric layout
  - linear perspective, gradients, line segments
- Material properties: albedo, feel, hardness
  - Color, texture
- Action: motion
  - Optical flow, tracked points

# General Principles of Representation

- Coverage
  - Ensure that all relevant information is captured
- Conciseness
  - Minimize number of features without sacrificing coverage
- Directness
  - Ideal features are independently useful for prediction

# Image Representations

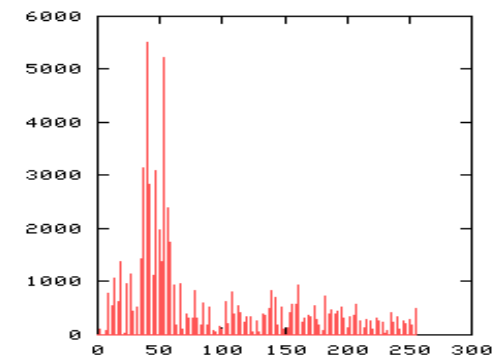
- Templates
  - Intensity, gradients, etc.



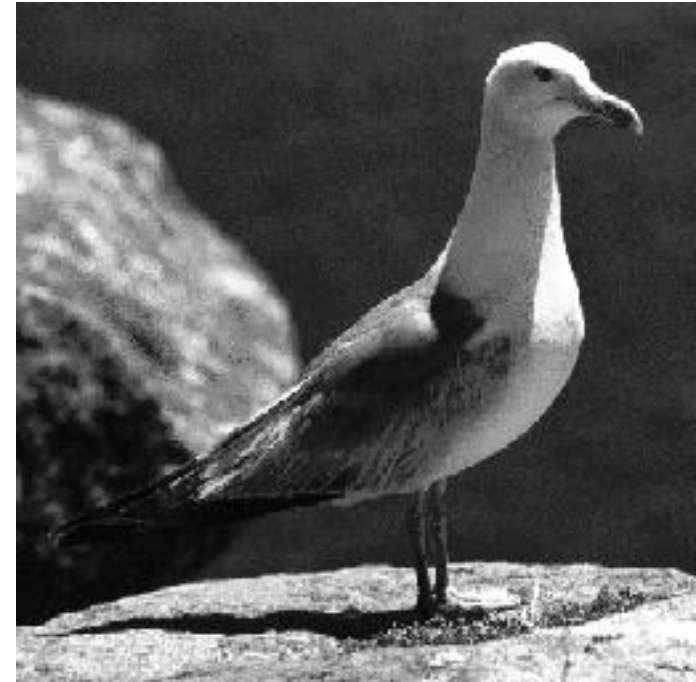
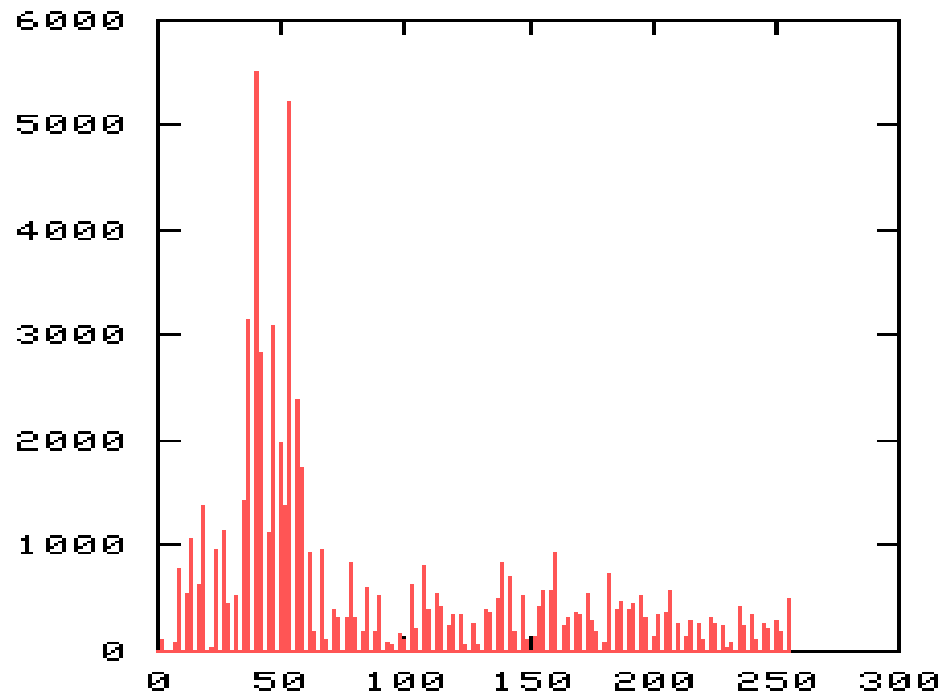
Image  
Intensity

Gradient  
template

- Histograms
  - Color, texture, SIFT descriptors, etc.
- Average of features



# Image representations: histograms

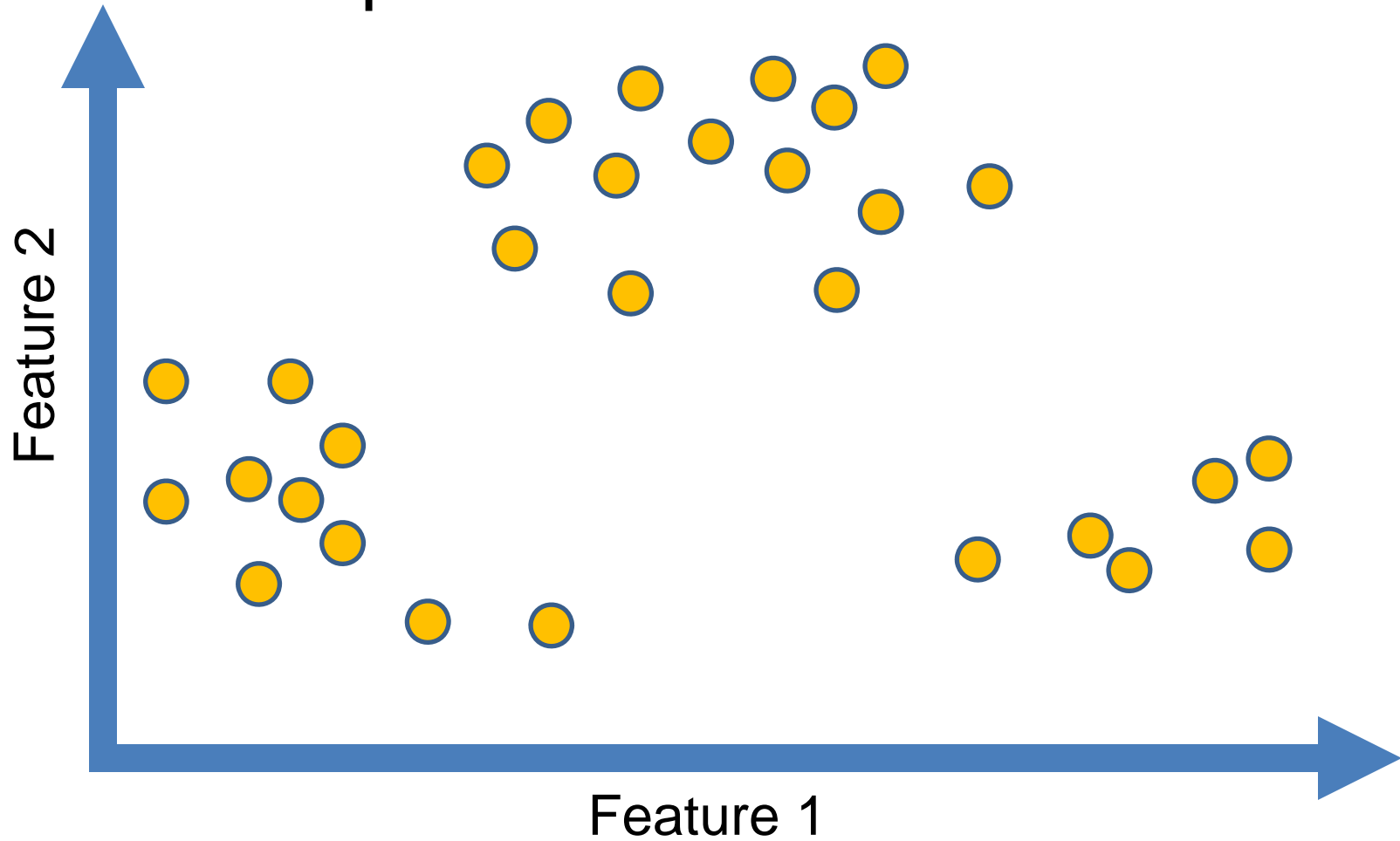


## Global histogram

- Represent distribution of features
  - Color, texture, depth, ...

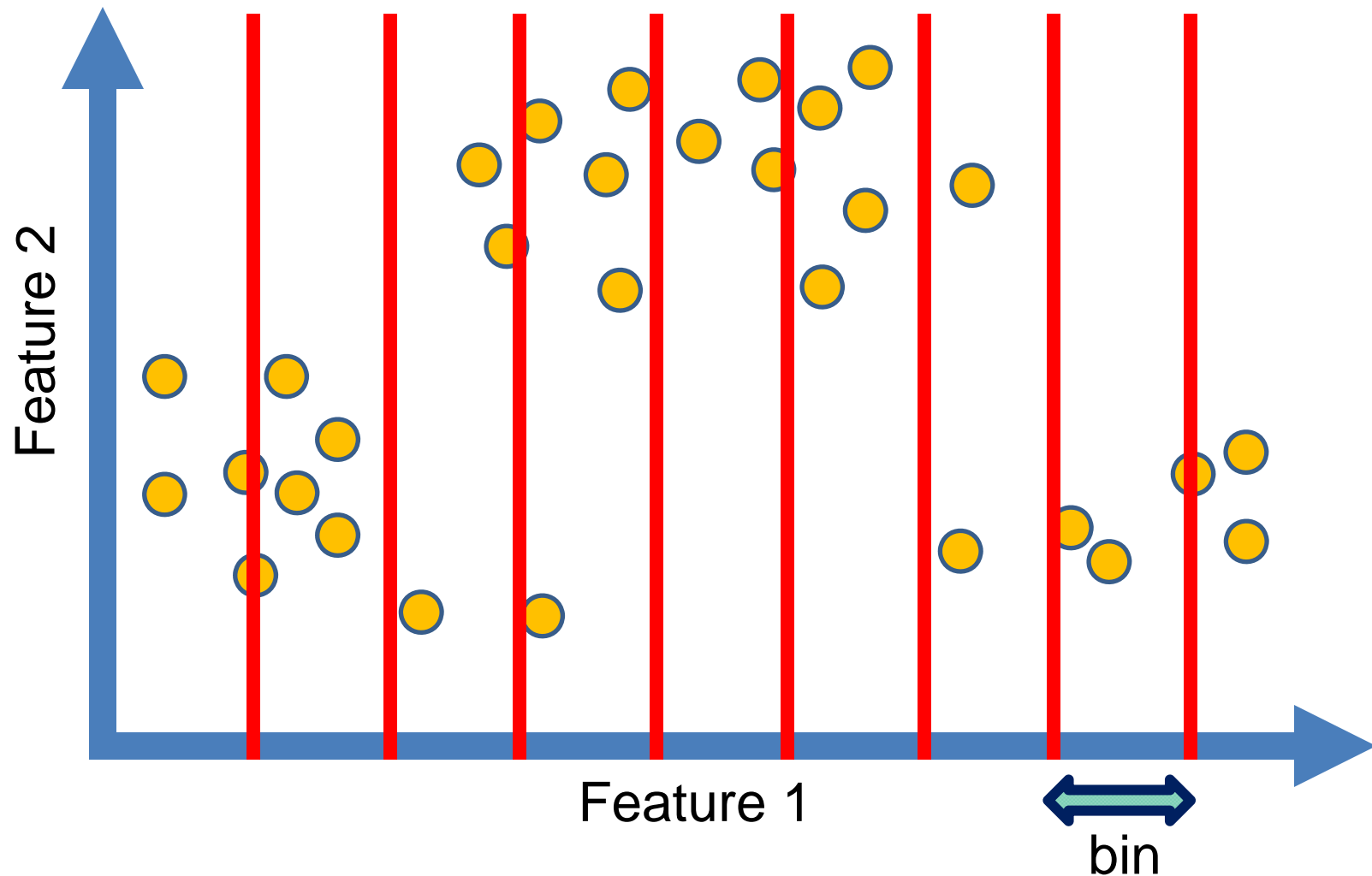
# Image representations: histograms

- Data samples in 2D



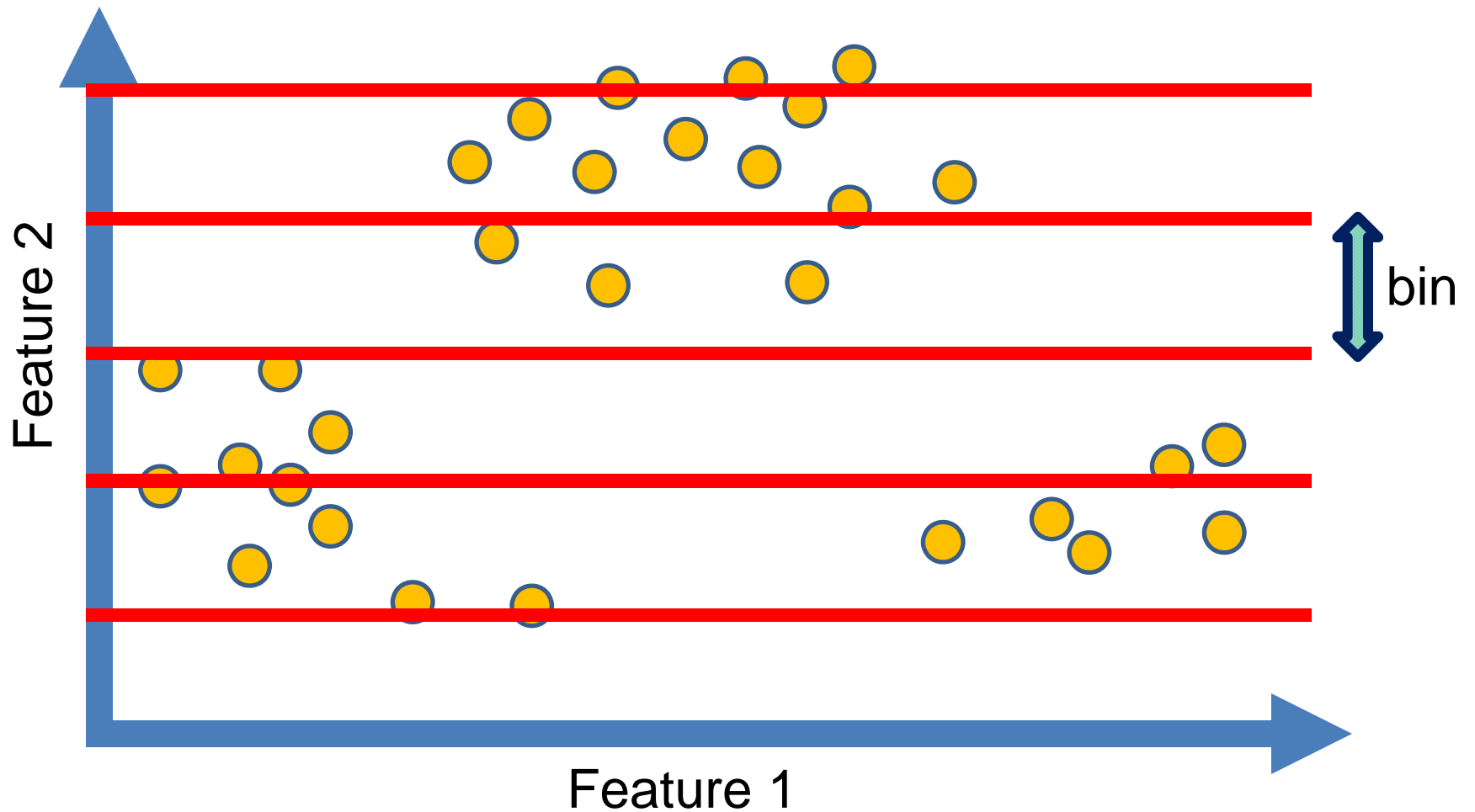
# Image representations: histograms

- Probability or count of data in each bin
- Marginal histogram on feature 1



# Image representations: histograms

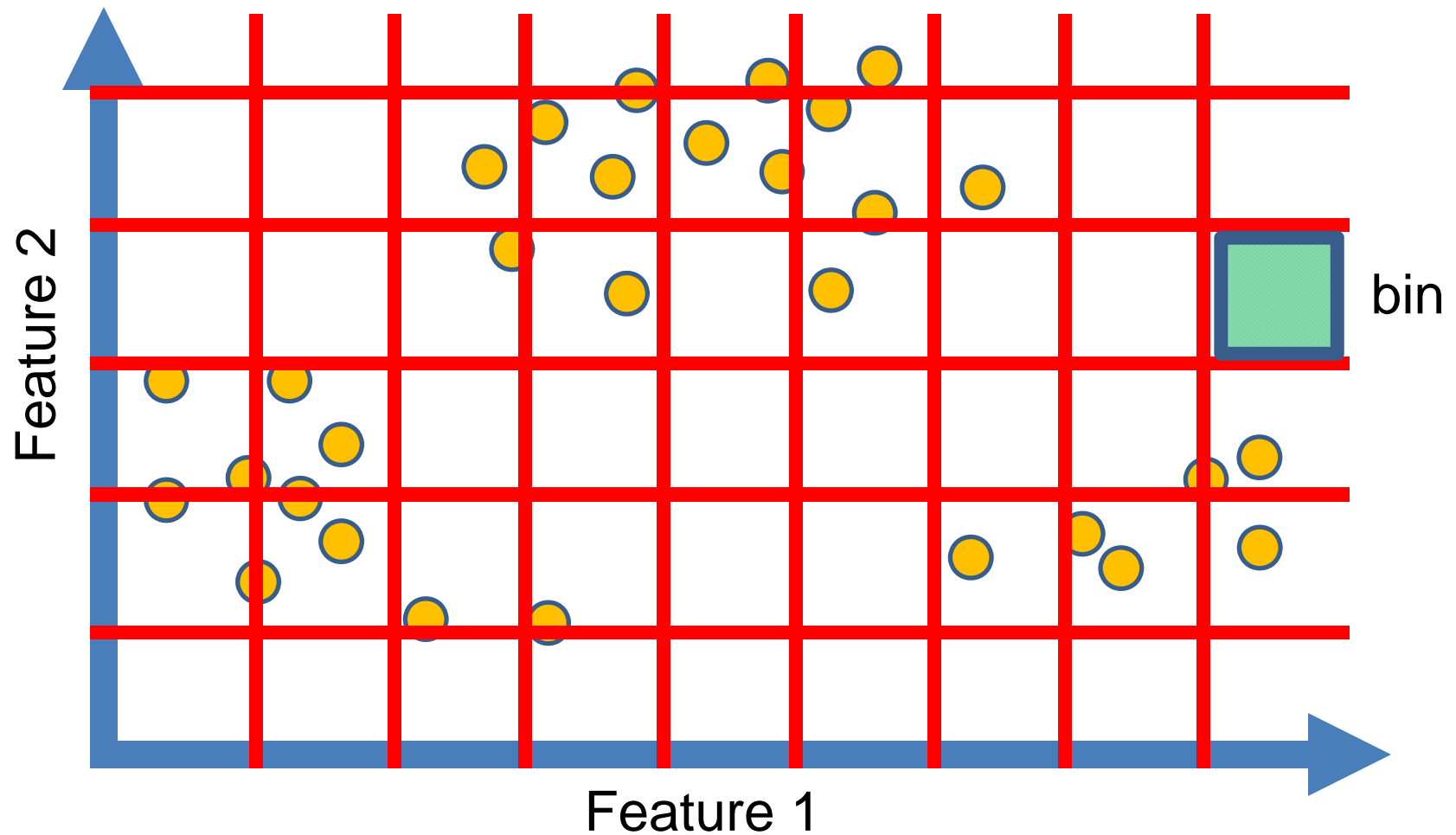
- Marginal histogram on feature 2



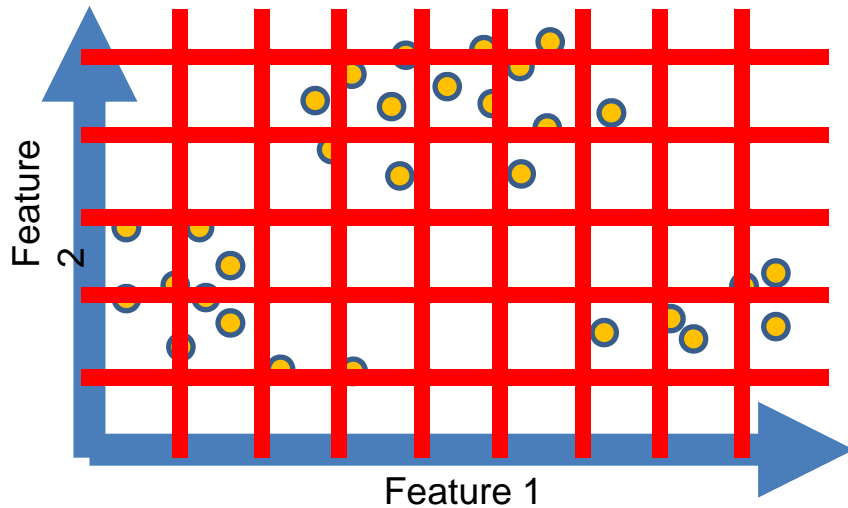


# Image representations: histograms

- Joint histogram

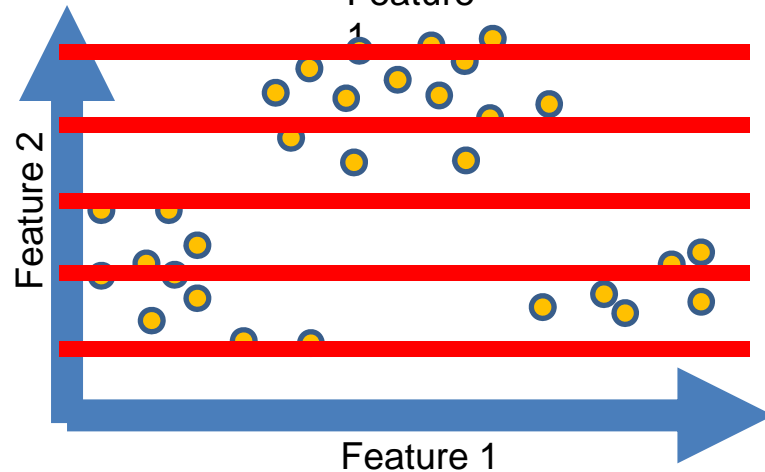
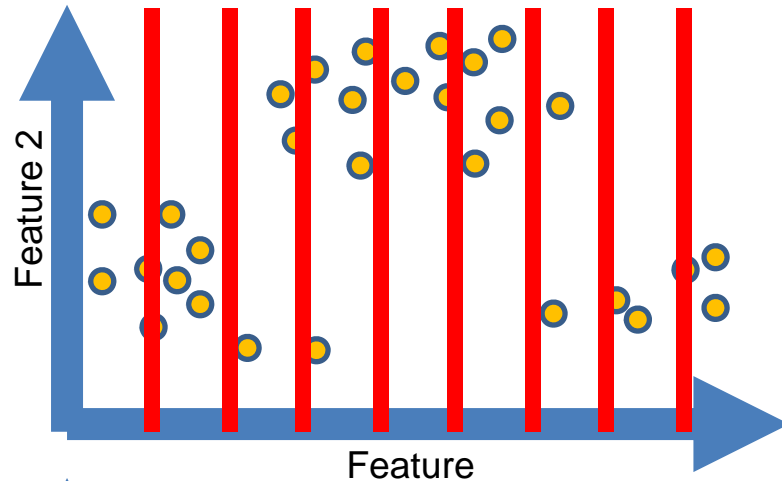


# Modeling multi-dimensional data



## Joint histogram

- Requires lots of data
- Loss of resolution to avoid empty bins



## Marginal histogram

- Requires independent features
- More data/bin than joint histogram

# Computing histogram distance

- Histogram intersection

$$\text{histint}(h_i, h_j) = 1 - \sum_{m=1}^K \min(h_i(m), h_j(m))$$

- Chi-squared Histogram matching distance

$$\chi^2(h_i, h_j) = \frac{1}{2} \sum_{m=1}^K \frac{[h_i(m) - h_j(m)]^2}{h_i(m) + h_j(m)}$$

- Earth mover's distance  
(Cross-bin similarity measure)
  - minimal cost paid to transform one distribution into the other

# Histograms: implementation issues

- Quantization
  - Grids: fast but applicable only with few dimensions
  - Clustering: slower but can quantize data in higher dimensions (see next slides)



Few Bins

Need less data

Coarser representation

Many Bins

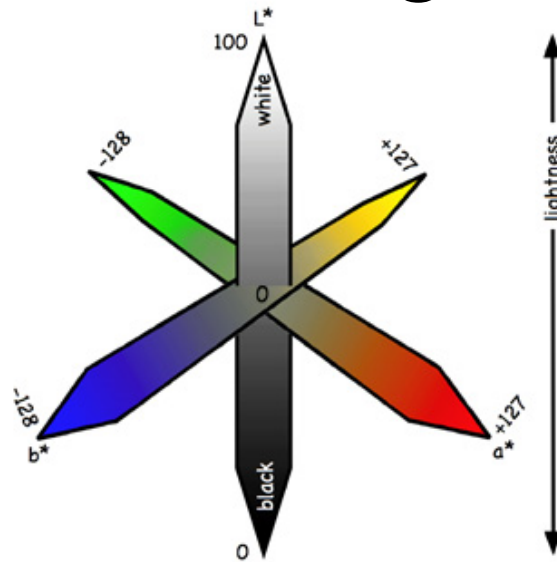
Need more data

Finer representation

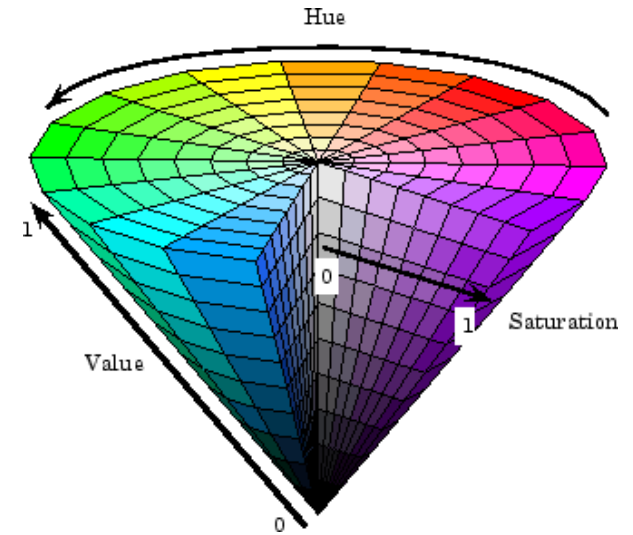
- Matching
  - Histogram intersection or Euclidean distance may be faster
  - Chi-squared distance often works better
  - Earth mover's distance is good when nearby bins represent similar values

# What kind of things do we compute histograms of?

- Color

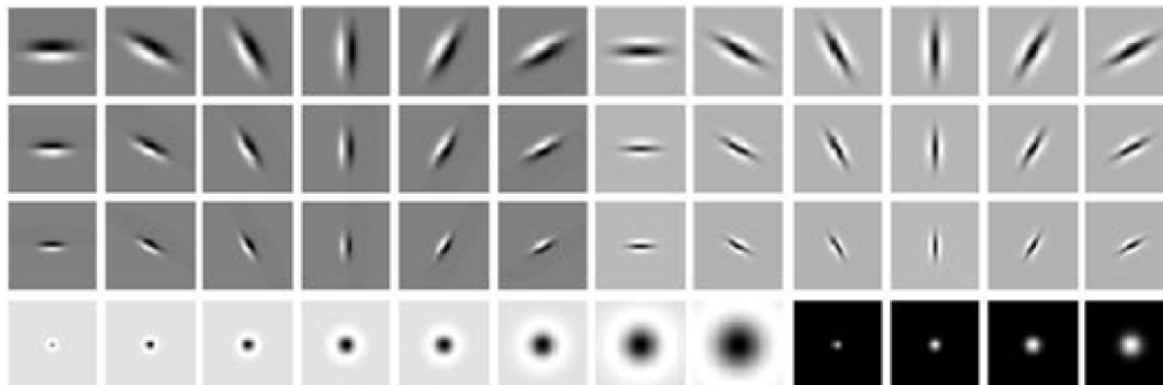


L\*a\*b\* color space



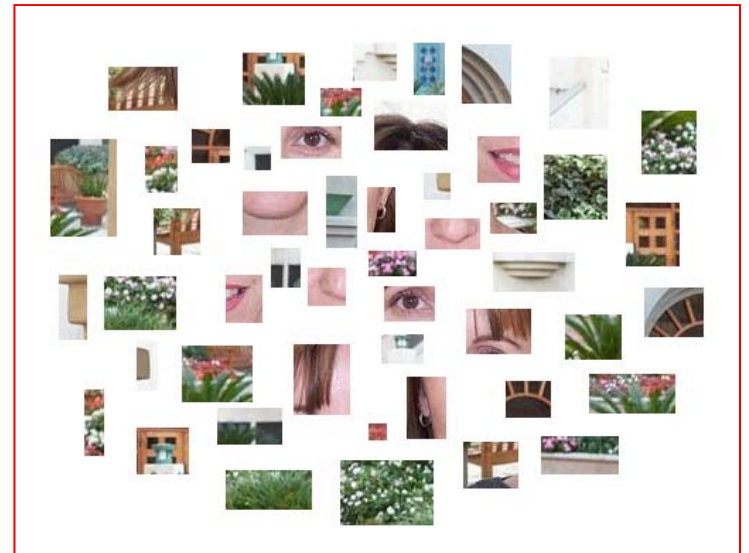
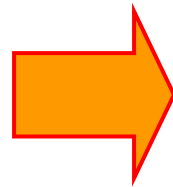
HSV color space

- Texture (filter banks or descriptors)



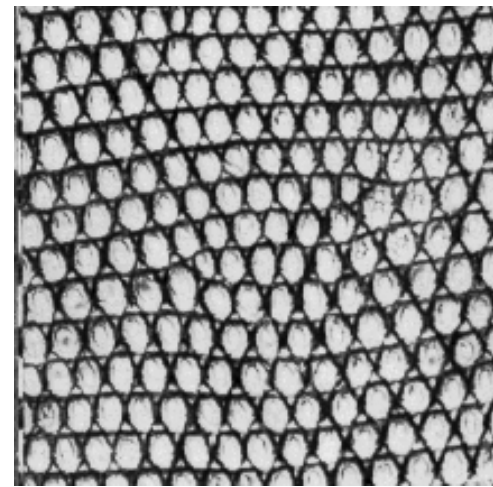
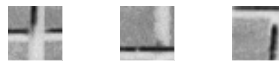
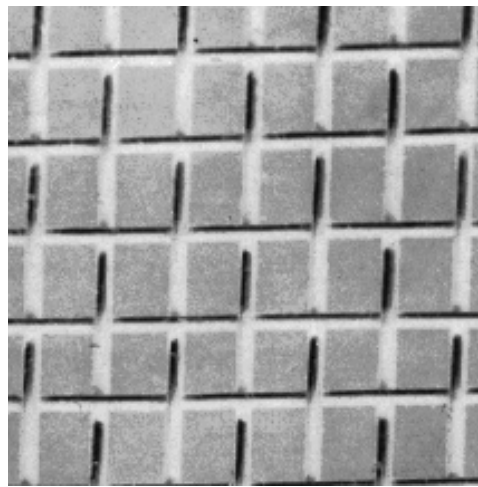
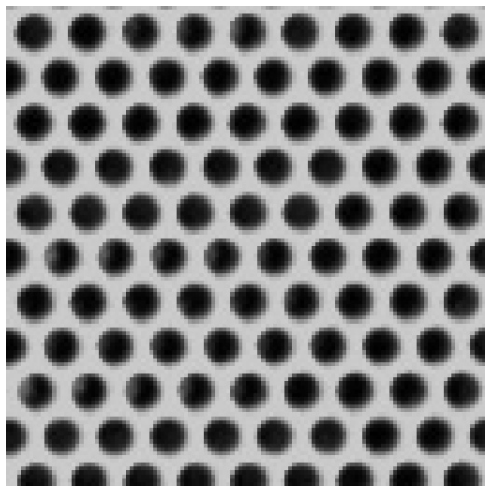
# Bags of Features/Visual Words

# Bags of features



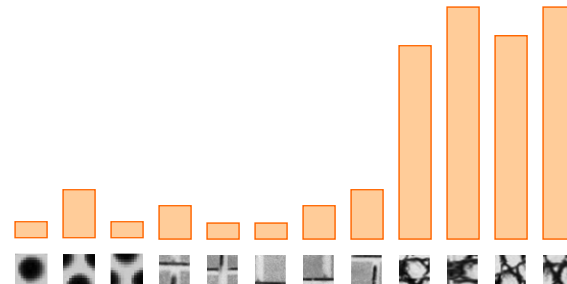
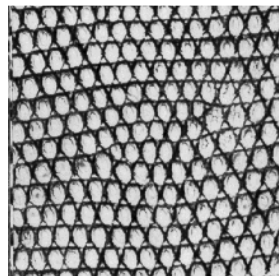
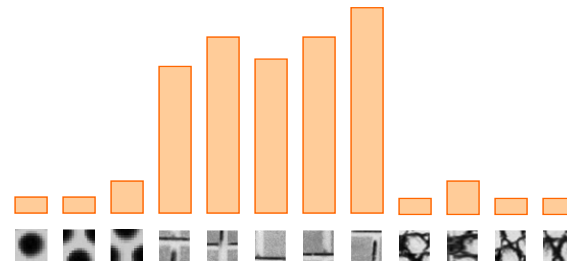
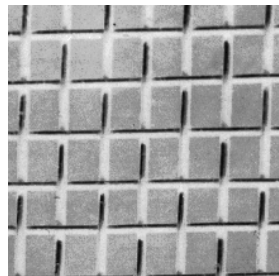
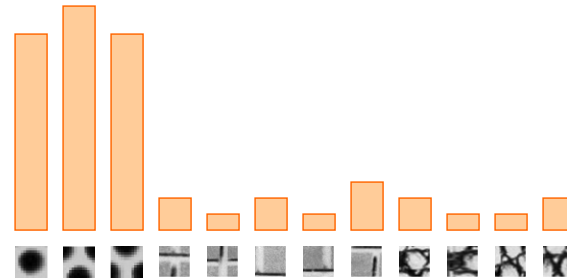
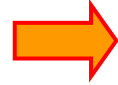
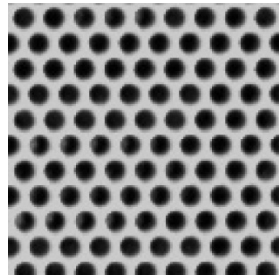
# Origin 1: Texture recognition

- Texture is characterized by the repetition of basic elements or *textons*
- For stochastic textures, it is the identity of the textons, not their spatial arrangement, that matters





# Origin 1: Texture recognition



# Origin 2: Bag-of-words models

- Orderless document representation: frequencies of words from a dictionary Salton & McGill (1983)

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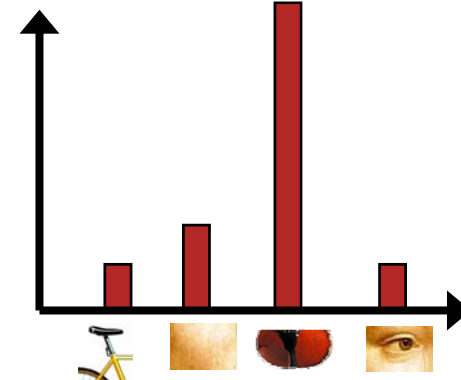
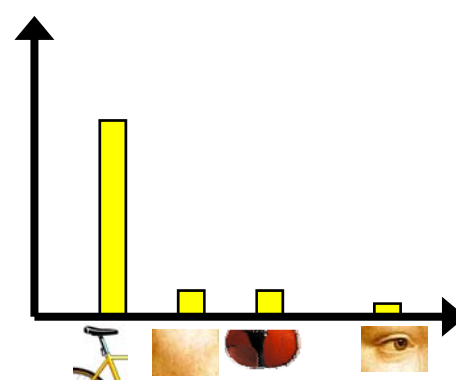
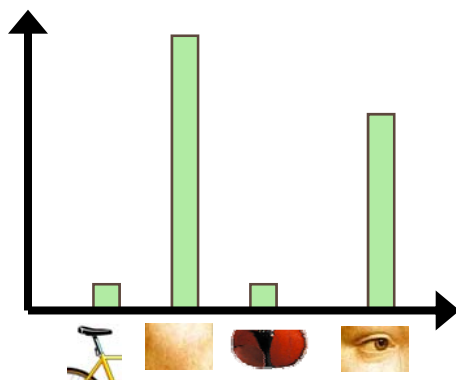
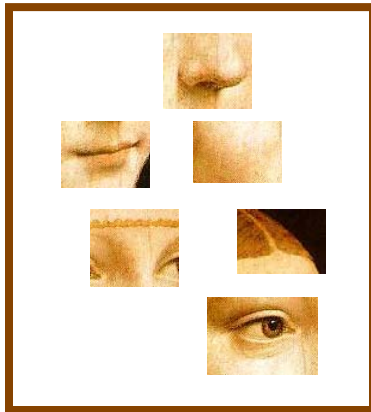
# Origin 2: Bag-of-words models

- Orderless document representation: frequencies of words from a dictionary Salton & McGill (1983)



# Bag-of-features steps

1. Extract local features
2. Learn “visual vocabulary”
3. Quantize local features using visual vocabulary
4. Represent images by frequencies of “visual words”

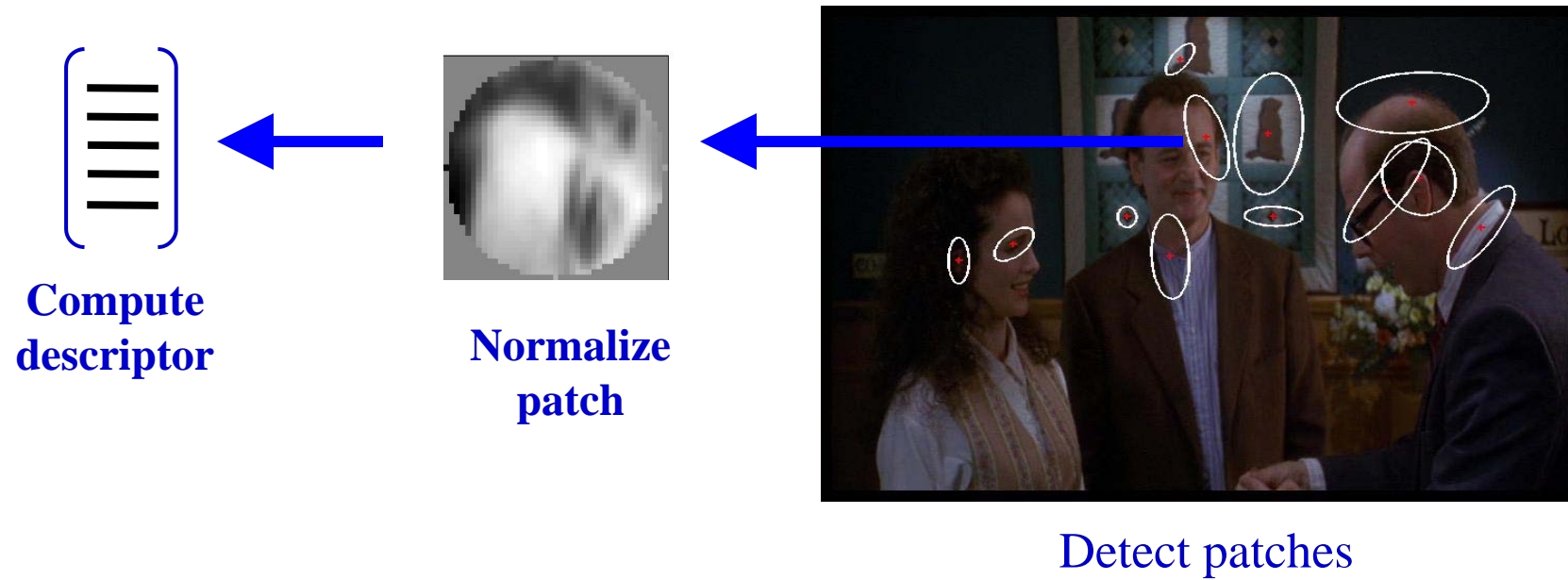


# Local feature extraction

- Regular grid or interest regions

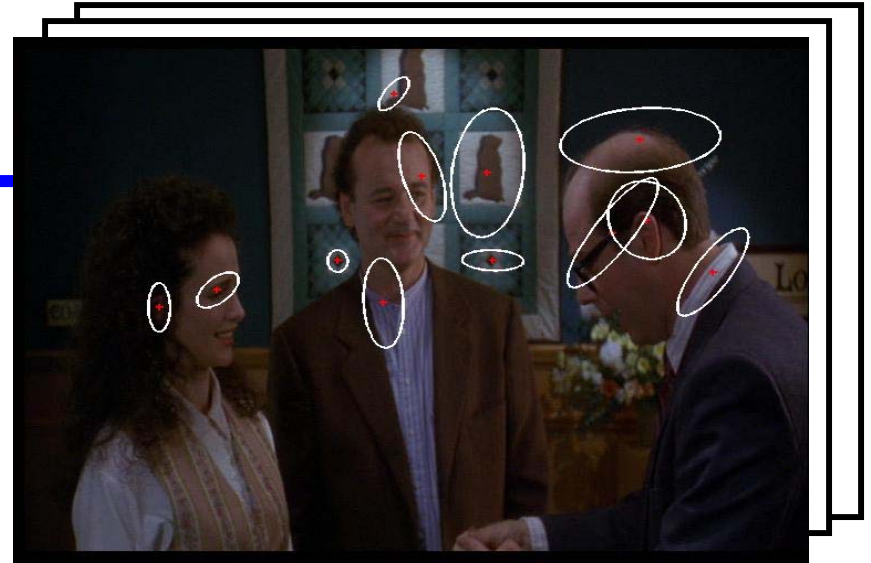
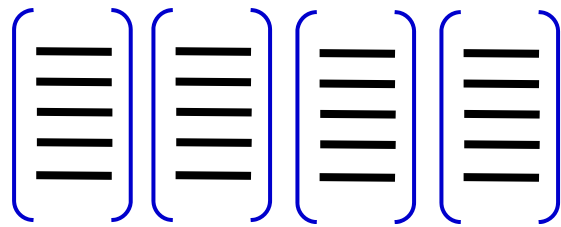


# Local feature extraction

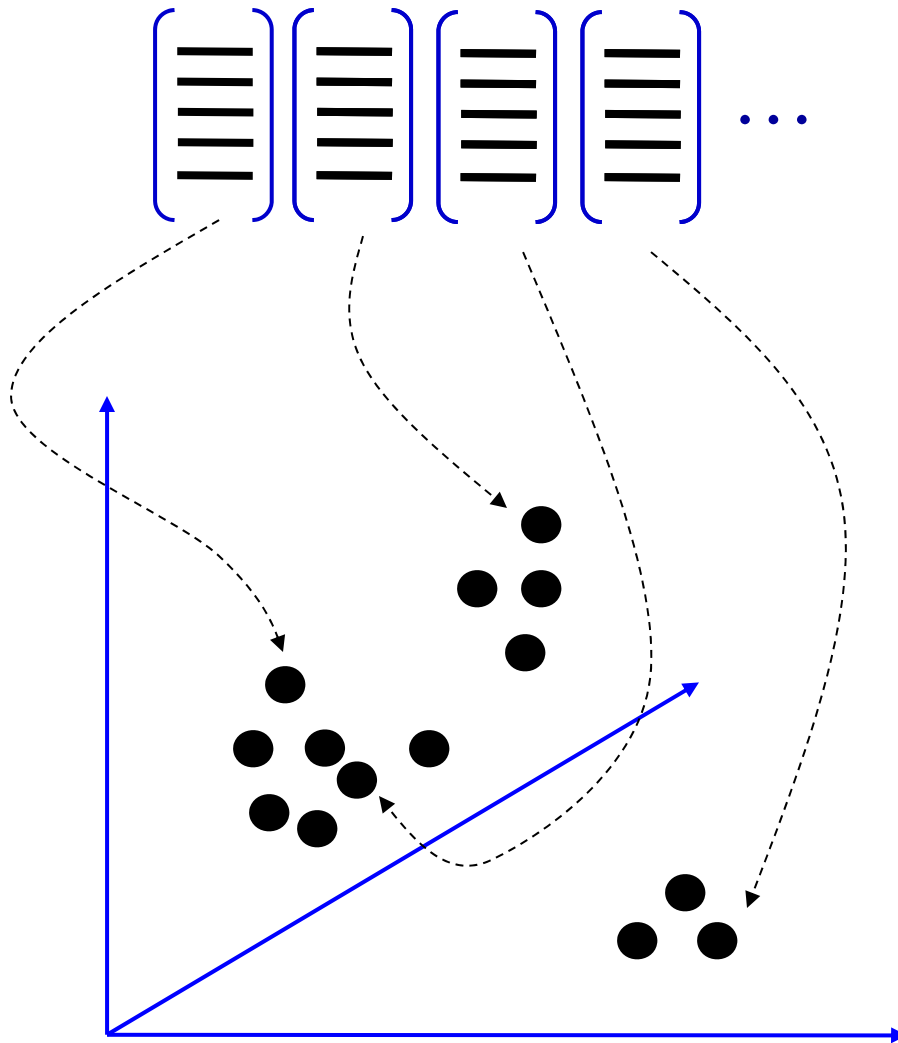




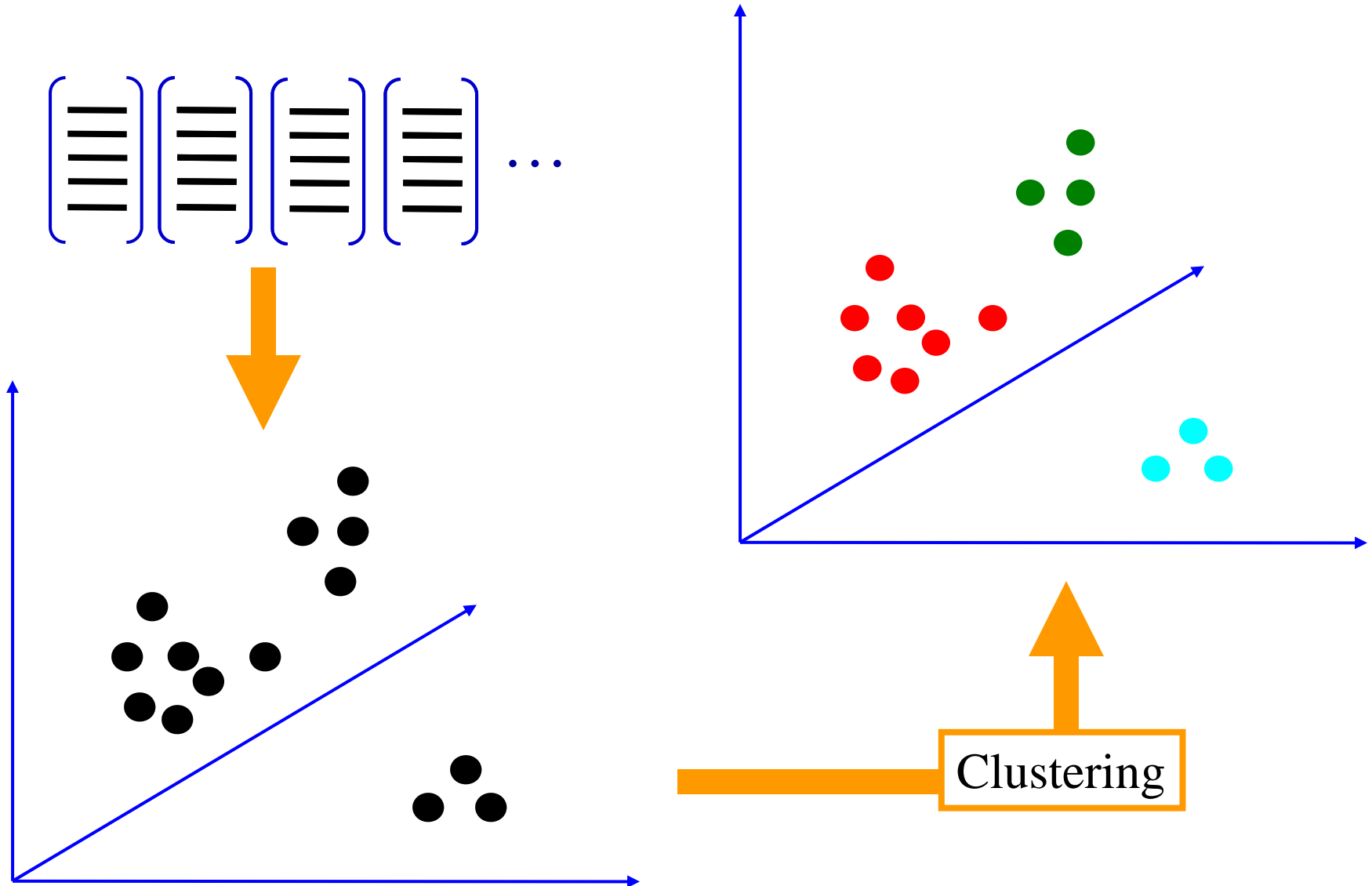
# Local feature extraction



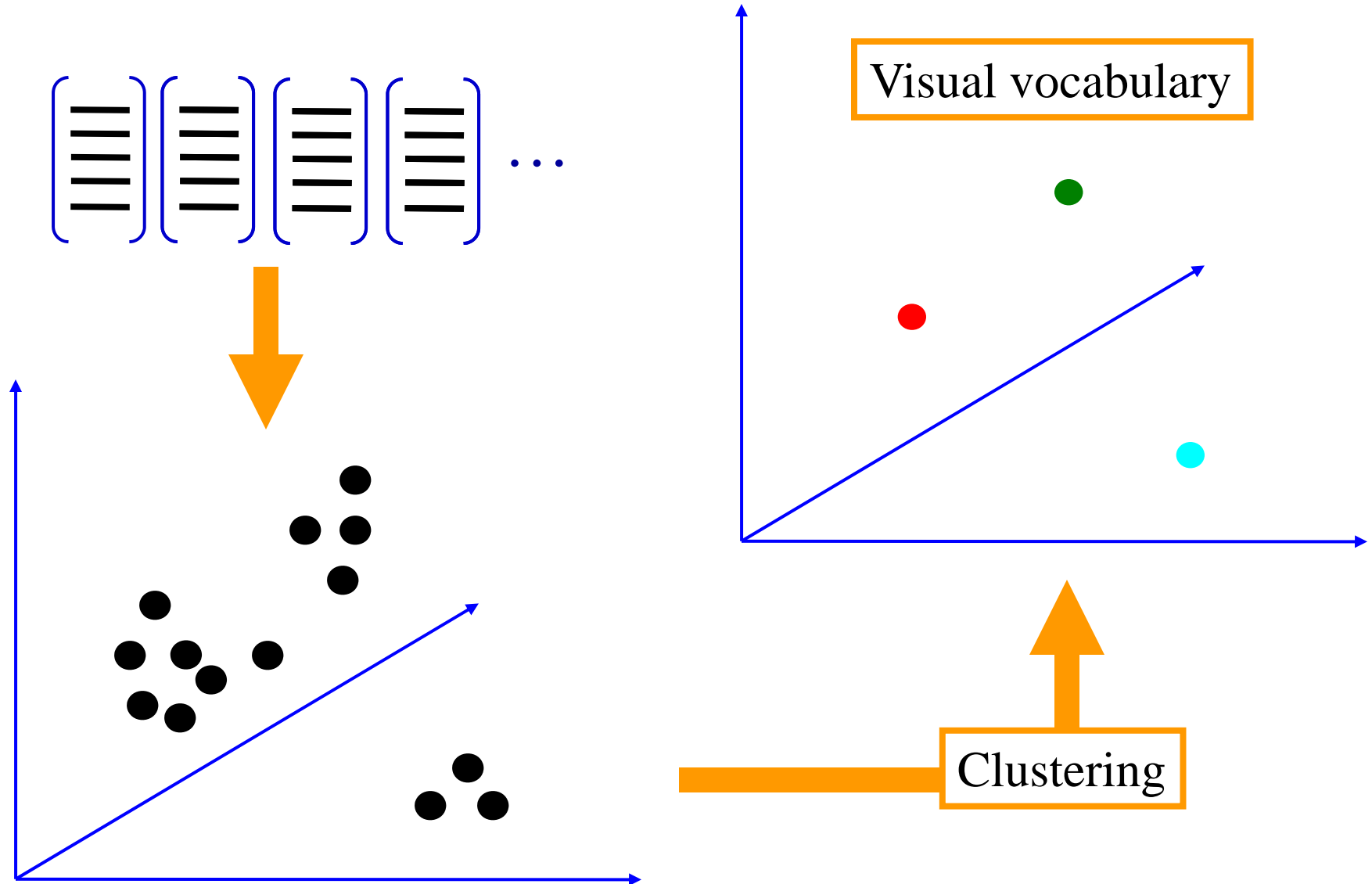
# Learning the visual vocabulary



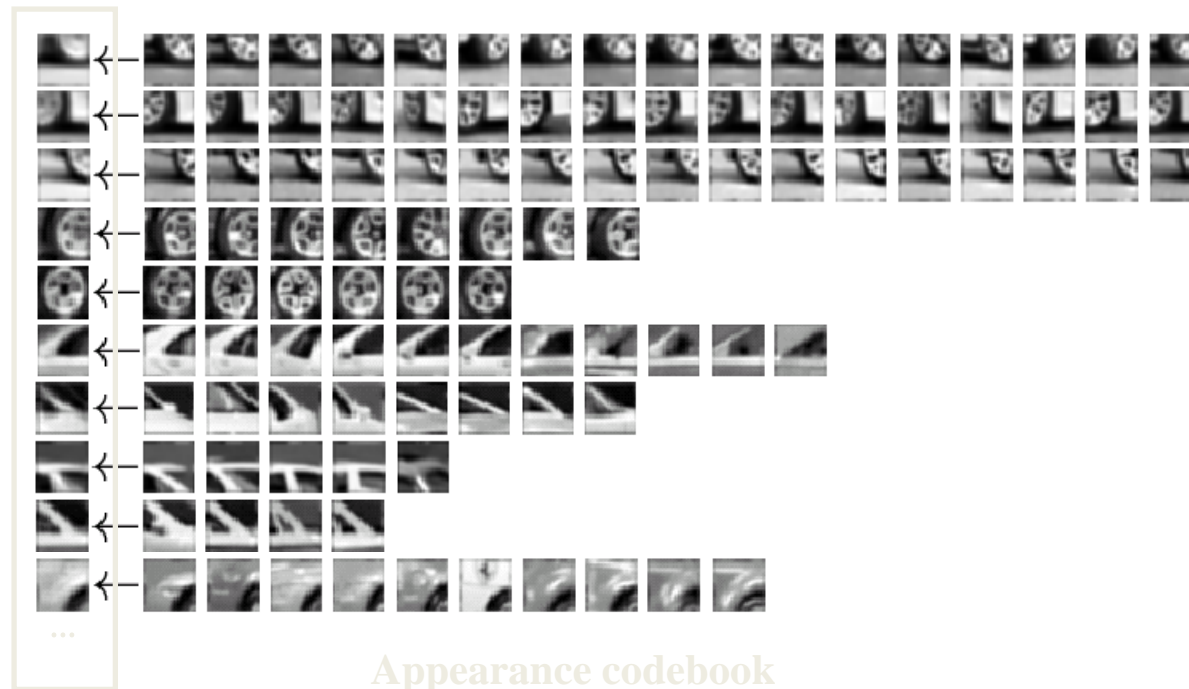
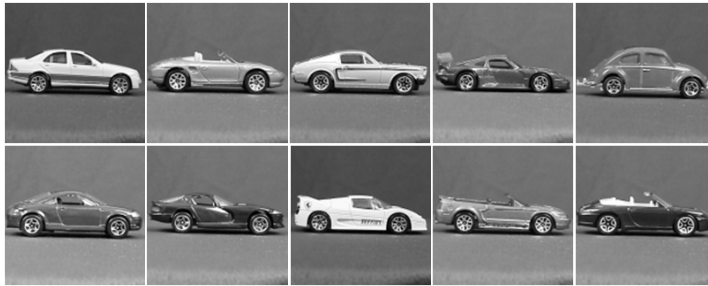
# Learning the visual vocabulary



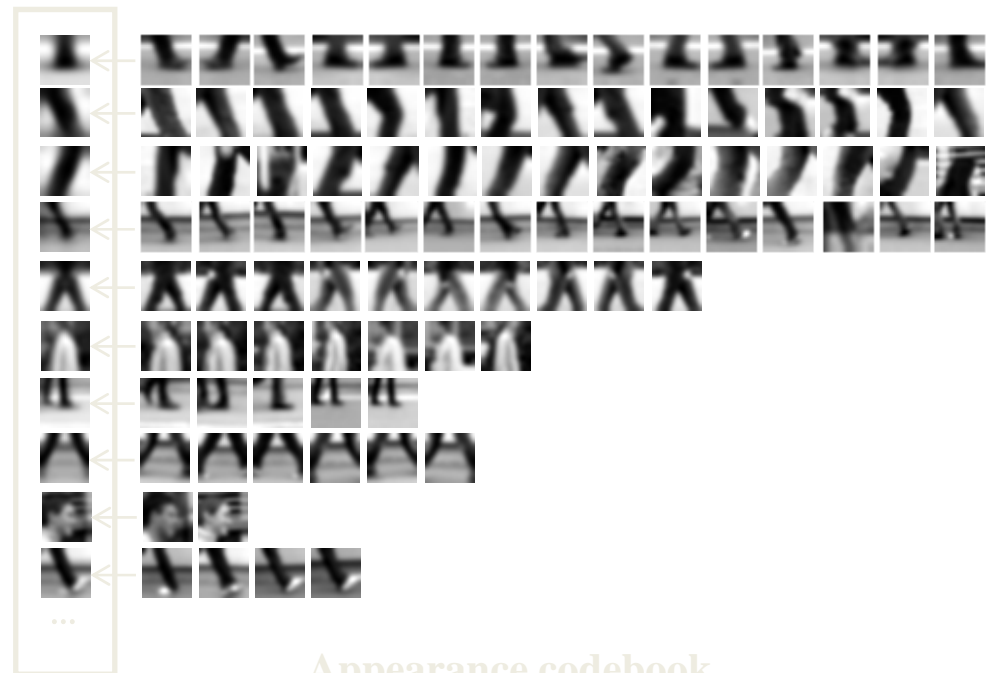
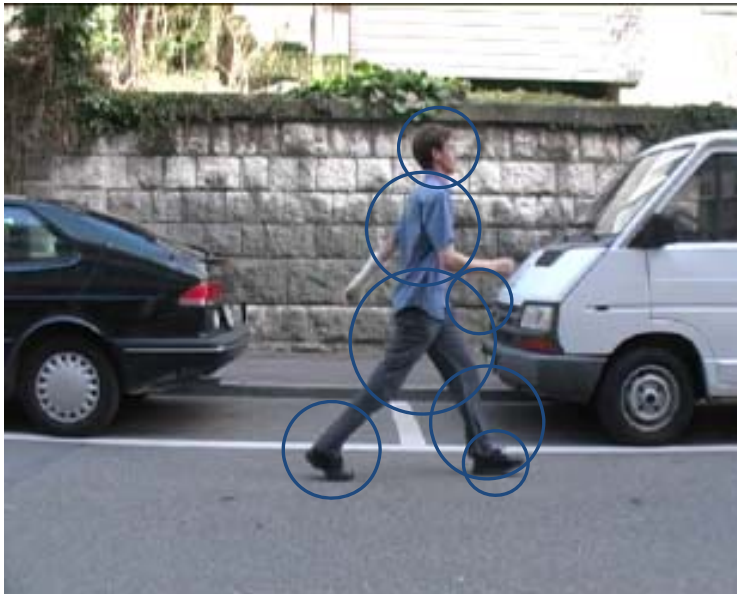
# Learning the visual vocabulary



# Example codebook

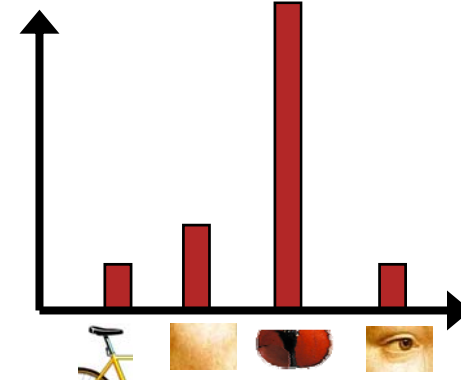
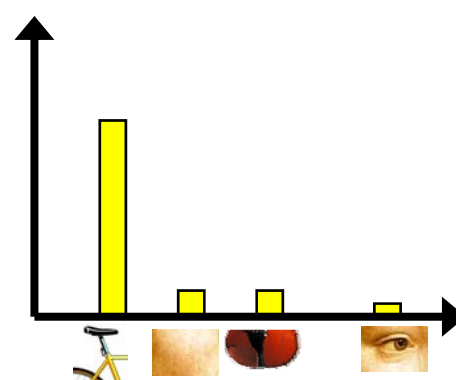
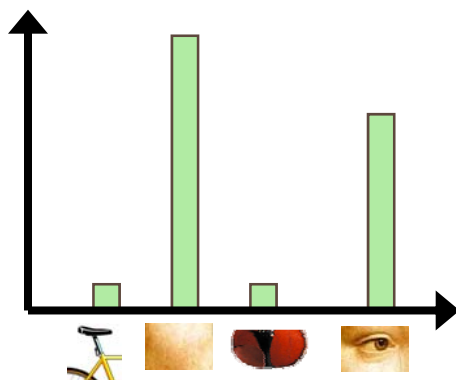
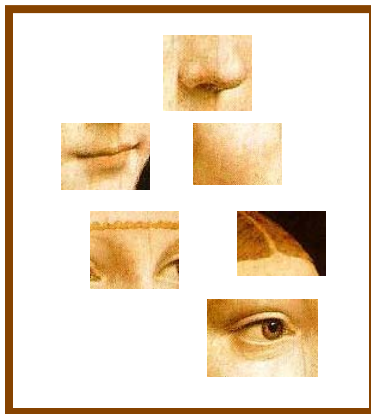


# Another codebook



# Bag-of-features steps

1. Extract local features
2. Learn “visual vocabulary”
3. Quantize local features using visual vocabulary
4. Represent images by frequencies of “visual words”



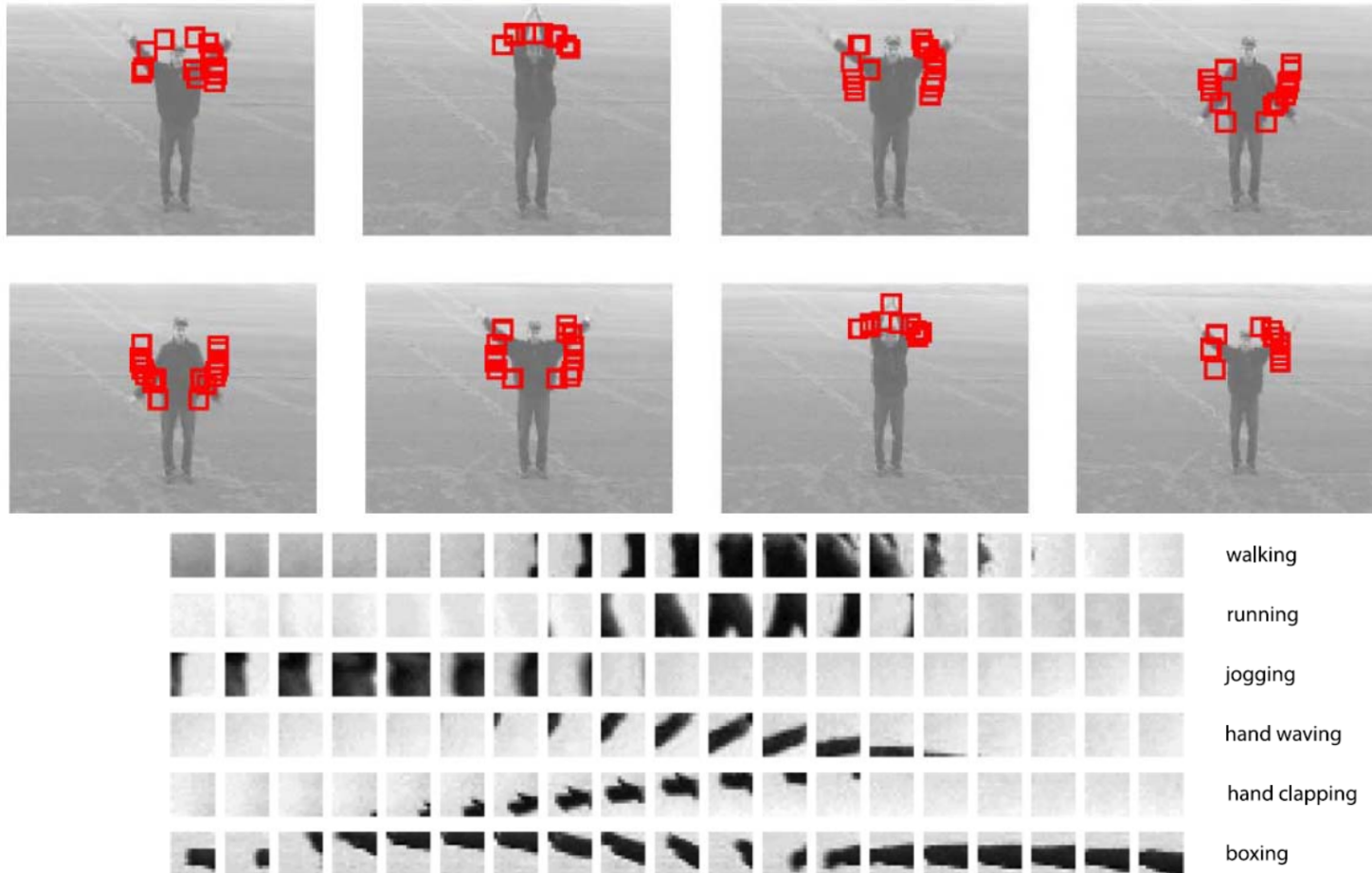
# Visual vocabularies: Details

- How to choose vocabulary size?
  - Too small: visual words not representative of all patches
  - Too large: quantization artifacts, overfitting
  - Right size is application-dependent
- Improving efficiency of quantization
  - Vocabulary trees (Nister and Stewenius, 2006)
- Improving vocabulary quality
  - Discriminative/supervised training of codebooks
  - Sparse coding, non-exclusive assignment to codewords
- More discriminative bag-of-words representations
  - Fisher Vectors (Perronnin et al., 2007), VLAD (Jegou et al., 2010)
- Incorporating spatial information



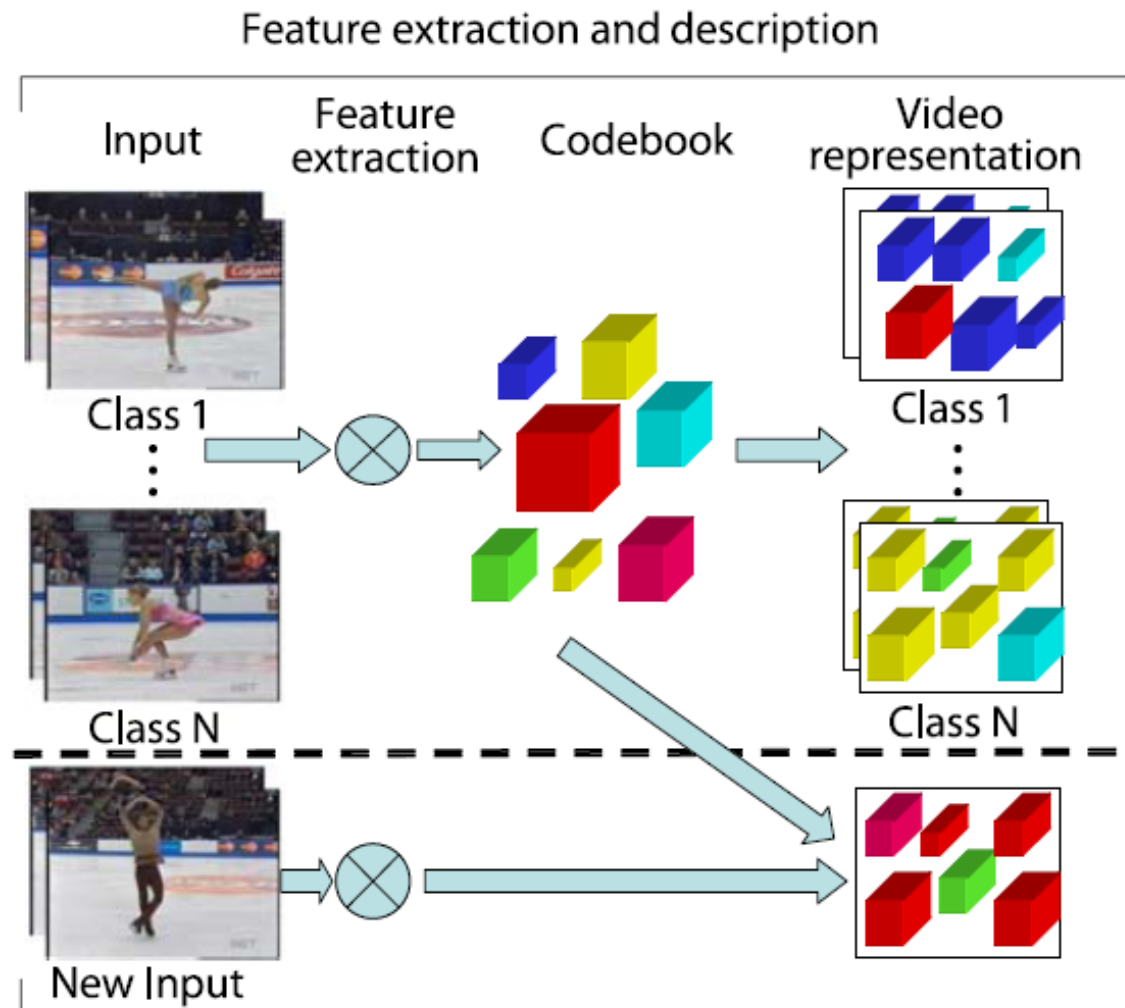
# Bags of features for action recognition

Space-time interest points



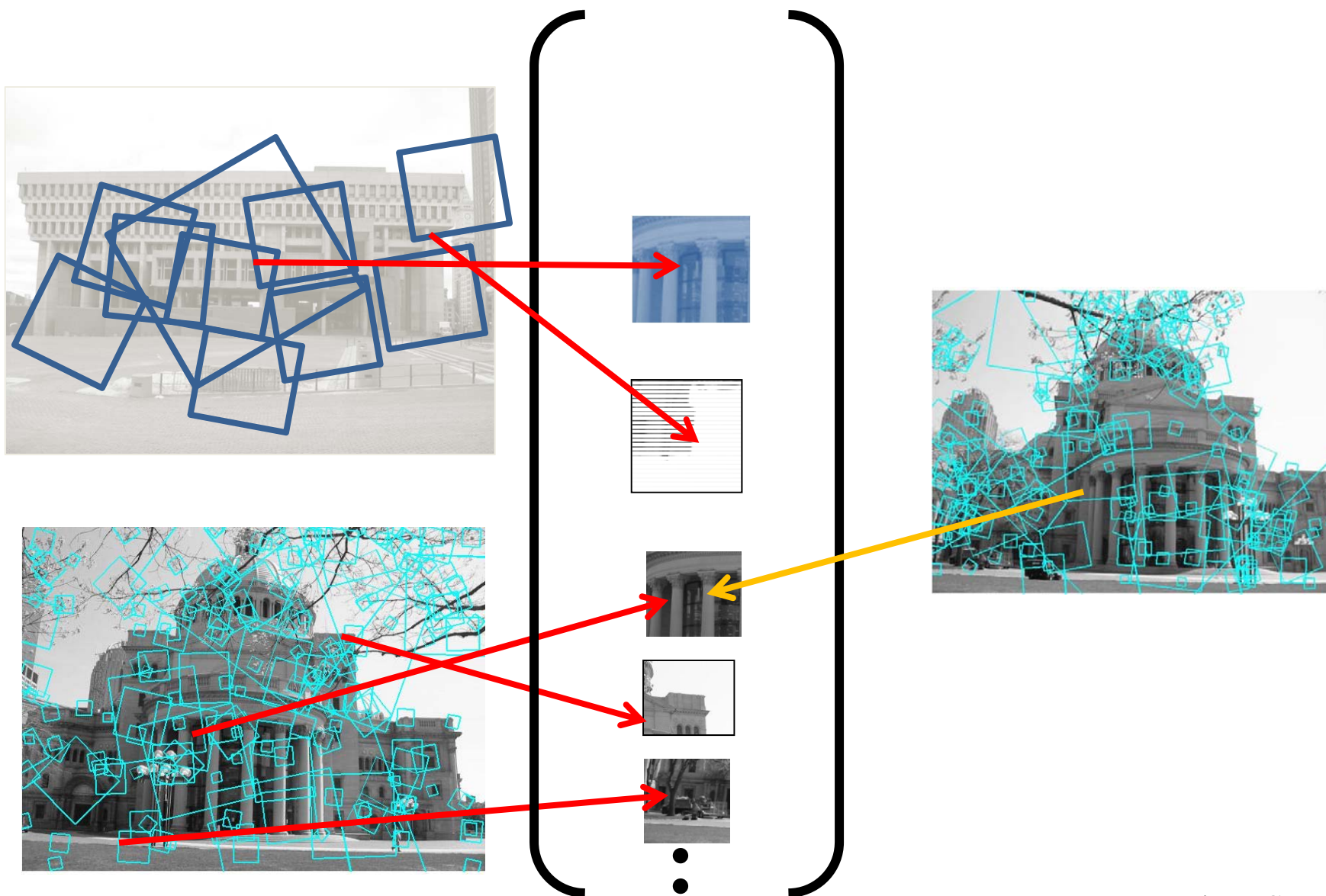
Juan Carlos Niebles, Hongcheng Wang and Li Fei-Fei, [Unsupervised Learning of Human Action Categories Using Spatial-Temporal Words](#), IJCV 2008.

# Bags of features for action recognition



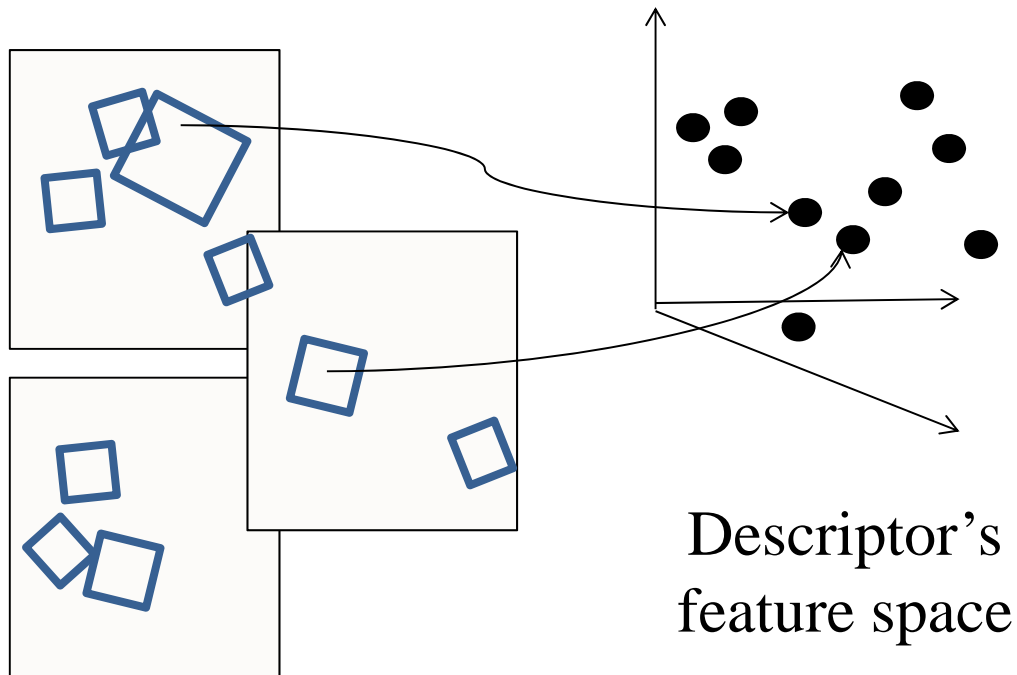
Juan Carlos Niebles, Hongcheng Wang and Li Fei-Fei, [Unsupervised Learning of Human Action Categories Using Spatial-Temporal Words](#), IJCV 2008.

# Indexing local features



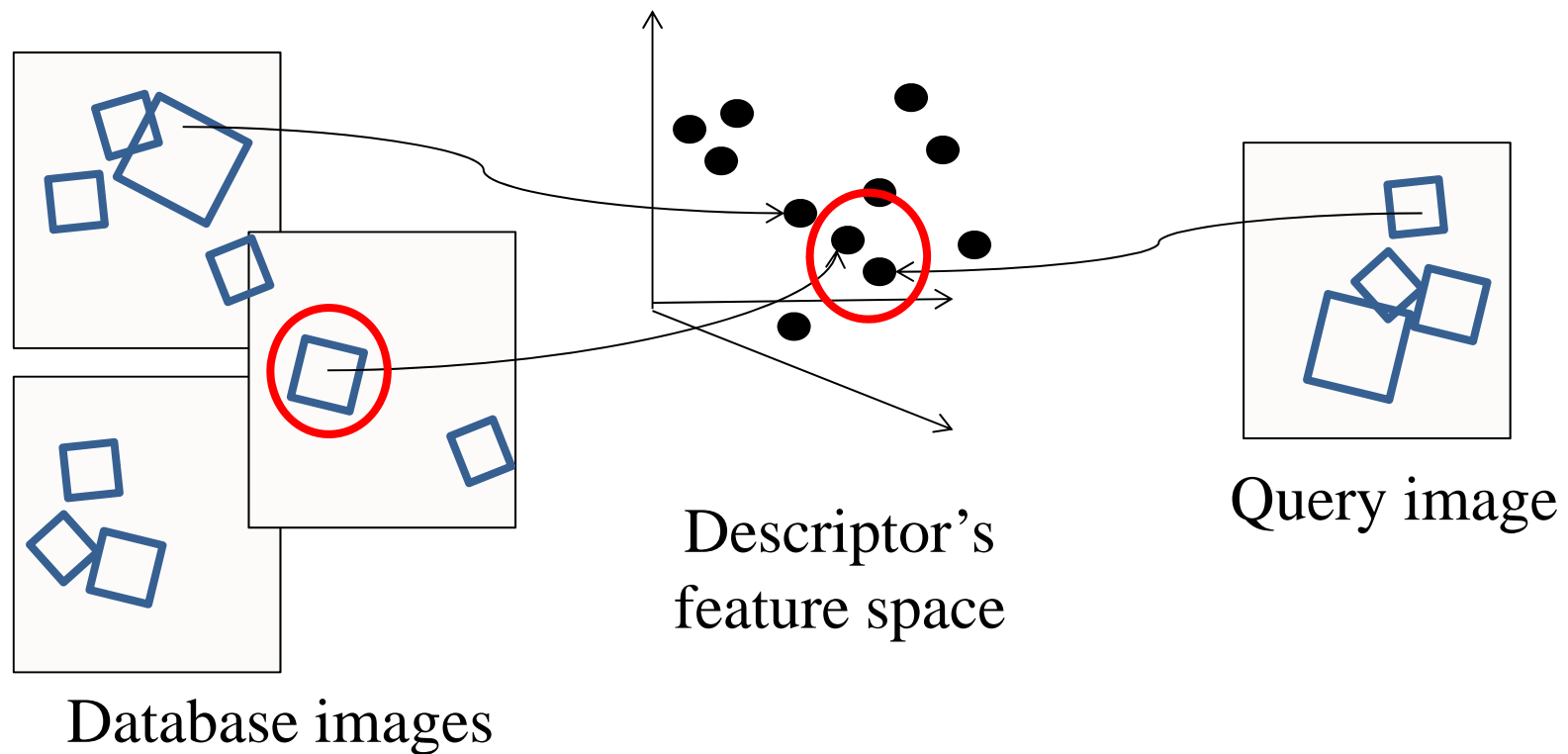
# Indexing local features

- Each patch / region has a descriptor, which is a point in some high-dimensional feature space (e.g., SIFT)



# Indexing local features

- When we see close points in feature space, we have similar descriptors, which indicates similar local content.



# Indexing local features

- With potentially thousands of features per image, and hundreds to millions of images to search, how to efficiently find those that are relevant to a new image?

# Indexing local features: inverted file index

Index		
"Along I-75," From Detroit to Florida; <i>inside back cover</i>	Butterfly Center, McGuire; 134	Driving Lanes; 85
"Drive I-95," From Boston to Florida; <i>inside back cover</i>	CAA (see AAA)	Duval County; 163
1929 Spanish Trail Roadway; 101-102,104	CCC, The; 111,113,115,135,142	Eau Gallie; 175
511 Traffic Information; 83	Ca. d'Zan; 147	Edison, Thomas; 152
A1A (Barrier Isl) - I-95 Access; 86	Caloosahatchee River; 152	Eglin AFB; 116-118
AAA (and CAA); 83	Name; 150	Eight Reale; 176
AAA National Office; 88	Canaveral Natnl Seashore; 173	Ellenton; 144-145
Abbreviations,	Cannon Creek Airpark; 130	Emanuel Point Wreck; 120
Colored 25 mile Maps; cover	Canopy Road; 106,169	Emergency Callboxes; 83
Exit Services; 196	Cape Canaveral; 174	Epiphytes; 142,148,157,159
Travelogue; 85	Castillo San Marcos; 169	Escambia Bay; 119
Africa; 177	Cave Diving; 131	Bridge (I-10); 119
Agricultural Inspection Stns; 126	Cayo Costa, Name; 150	County; 120
Ah-Tah-Thi-Ki Museum; 160	Celebration; 93	Estero; 153
Air Conditioning, First; 112	Charlotte County; 149	Everglade,90,95,139-140,154-160
Alabama; 124	Charlotte Harbor; 150	Draining of; 156,181
Alachua; 132	Chautauqua; 116	Wildlife MA; 160
County; 131	ChIPLEY; 114	Wonder Gardens; 154
Alafia River; 143	Name; 115	Falling Waters SP; 115
Alapaha, Name; 126	Choctawatchee, Name; 115	Fantasy of Flight; 95
Alfred B Maclay Gardens; 106	Circus Museum, Ringling; 147	Fayer Dykes SP; 171
Alligator Alley; 154-155	Citrus; 88,97,130,136,140,180	Fires, Forest; 166
Alligator Farm, St Augustine; 169	CityPlace, W Palm Beach; 180	Fires, Prescribed ; 148
Alligator Hole (definition); 157	City Maps,	Fisherman's Village; 151
Alligator, Buddy; 155	Ft Lauderdale Expwys; 194-195	Flagler County; 171
Alligators; 100,135,138,147,156	Jacksonville; 163	Flagler, Henry; 97,165,167,171
Anastasia Island; 170	Kissimmee Expwys; 192-193	Florida Aquarium; 186
Anhaica; 109-109,146	Miami Expressways; 194-195	Florida,
Apalachicola River; 112	Orlando Expressways; 192-193	12,000 years ago; 167
Appleton Mus of Art; 136	Pensacola; 26	Cavern SP; 114
Aquifer; 102	Tallahassee; 191	Map of all Expressways; 2-3
Arabian Nights; 94	Tampa-St. Petersburg; 63	Mus of Natural History; 134
Art Museum, Ringling; 147	St. Augustine; 191	National Cemetery ; 141
Aruba Beach Cafe; 183	Civil War; 100,108,127,138,141	Part of Africa; 177
Aucilla River Project; 106	Clearwater Marine Aquarium; 187	Platform; 167
Babcock-Web WMA; 151	Collier County; 154	Sheriff's Boys Camp; 126
Bahia Mar Marina; 184	Collier, Barron; 152	Sports Hall of Fame; 130
Baker County; 99	Colonial Spanish Quarters; 168	Sun 'n Fun Museum; 97
Barefoot Mailmen; 182	Columbia County; 101,128	Supreme Court; 107
Barge Canal; 137	Coquina Building Material; 165	Florida's Turnpike (FTP), 178,189
Bee Line Expy; 80	Corkscrew Swamp, Name; 154	25 mile Strip Maps; 66
Belz Outlet Mall; 89	Cowboys; 95	Administration; 189
Bernard Castro; 136	Crab Trap II; 144	Coin System; 190
Big "I"; 165	Cracker, Florida; 88,95,132	Exit Services; 189
Big Cypress; 155,158	Crosstown Expy; 11,35,98,143	HEFT; 76,161,190
Big Foot Monster; 105	Cuban Bread; 184	History; 189
Billie Swamp Safari; 160	Dade Battlefield; 140	Names; 189
Blackwater River SP; 117	Dade, Maj. Francis; 139-140,161	Service Plazas; 190
Blue Angels	Dania Beach Hurricane; 184	Spur SR91; 76
	Daniel Boone, Florida Walk; 117	Ticket System; 190
	Daytona Beach; 172-173	Toll Plazas; 190
	De Land; 87	Ford, Henry; 152

- For text documents, an efficient way to find all *pages* on which a *word* occurs is to use an index...
- We want to find all *images* in which a *feature* occurs.
- To use this idea, we'll need to map our features to "visual words".

# Text retrieval vs. image search

- What makes the problems similar, different?



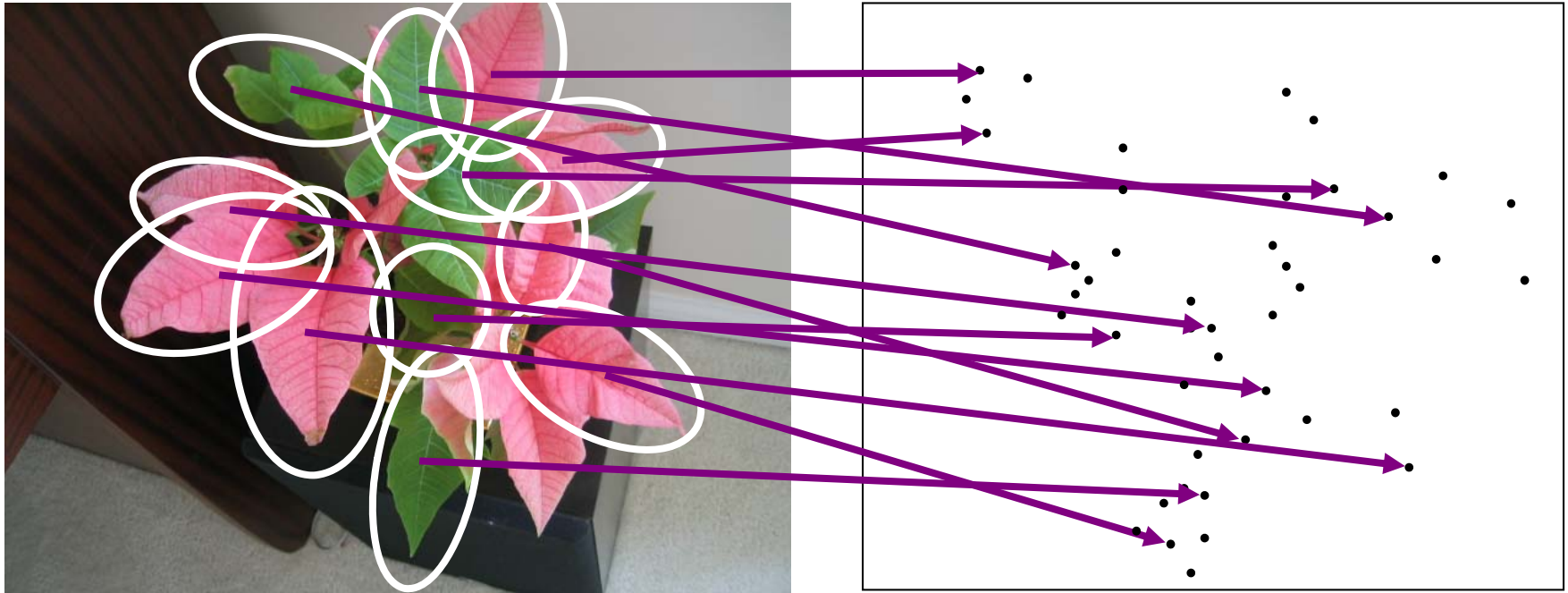
# Visual words: main idea

- Extract some local features from a number of images ...

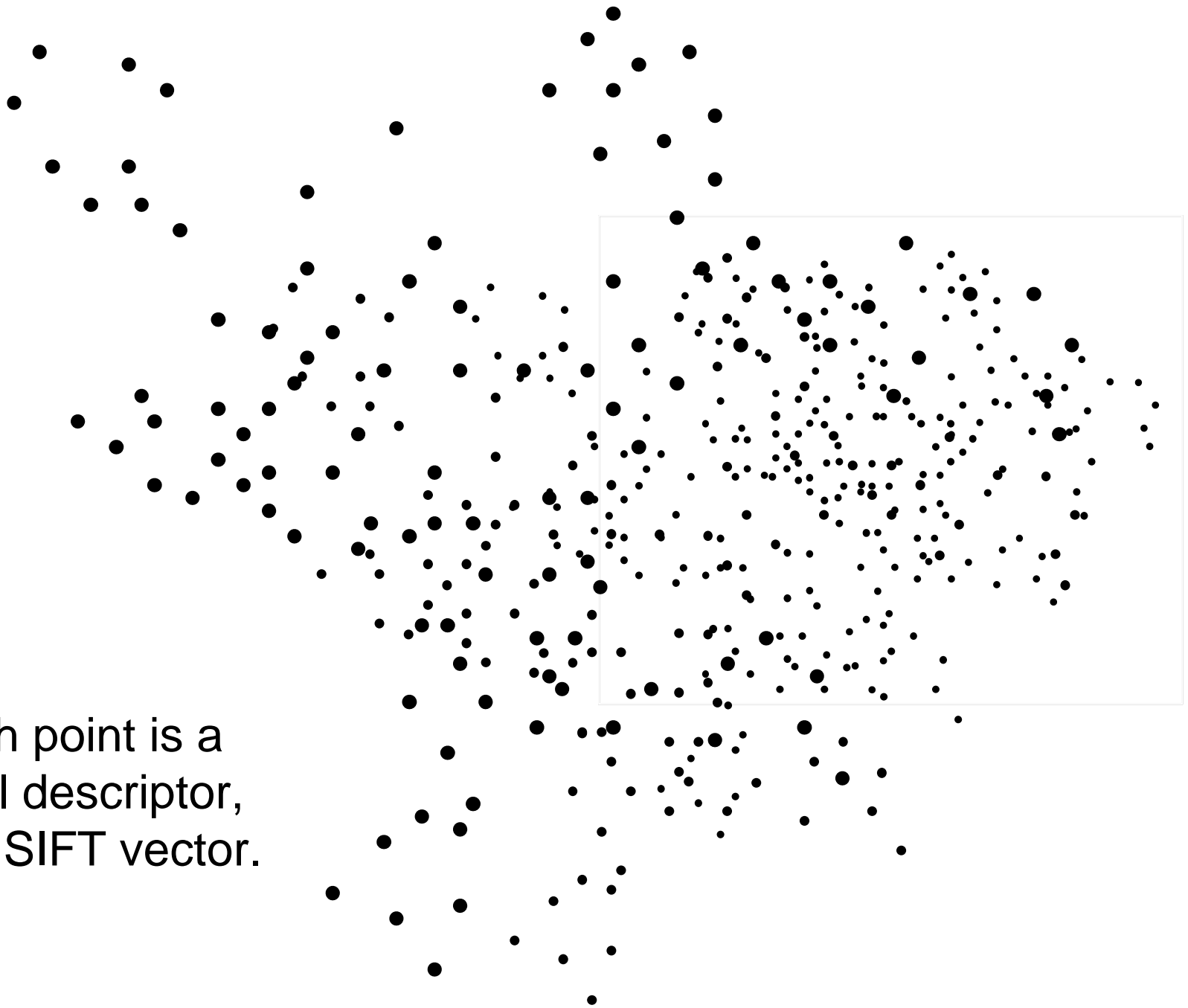


e.g., SIFT descriptor space: each point is 128-dimensional

# Visual words: main idea

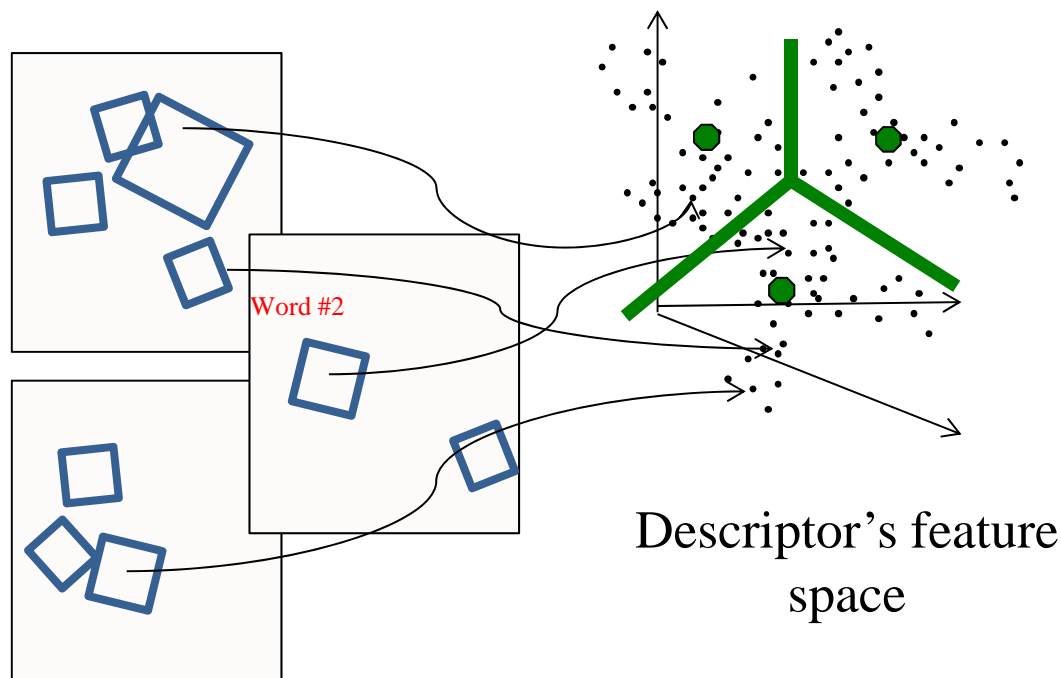


Each point is a  
local descriptor,  
e.g. SIFT vector.



# Visual words

- Map high-dimensional descriptors to tokens/words by quantizing the feature space



- Quantize via clustering, let cluster centers be the prototype “words”
- Determine which word to assign to each new image region by finding the closest cluster center.

# Visual words

- Example: each group of patches belongs to the same visual word

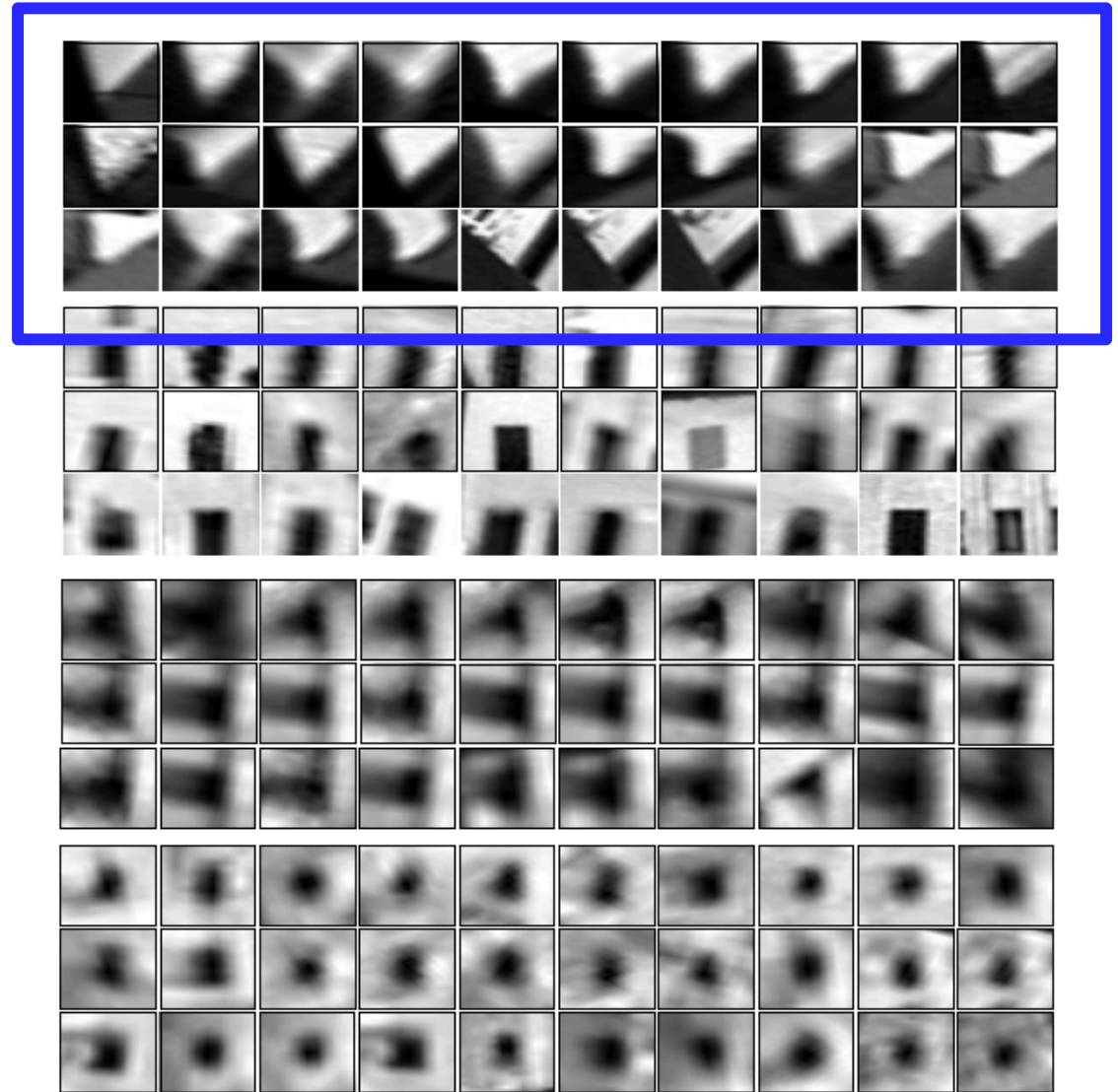
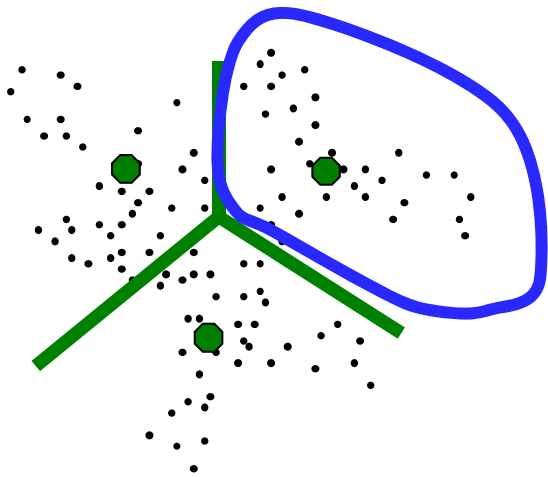
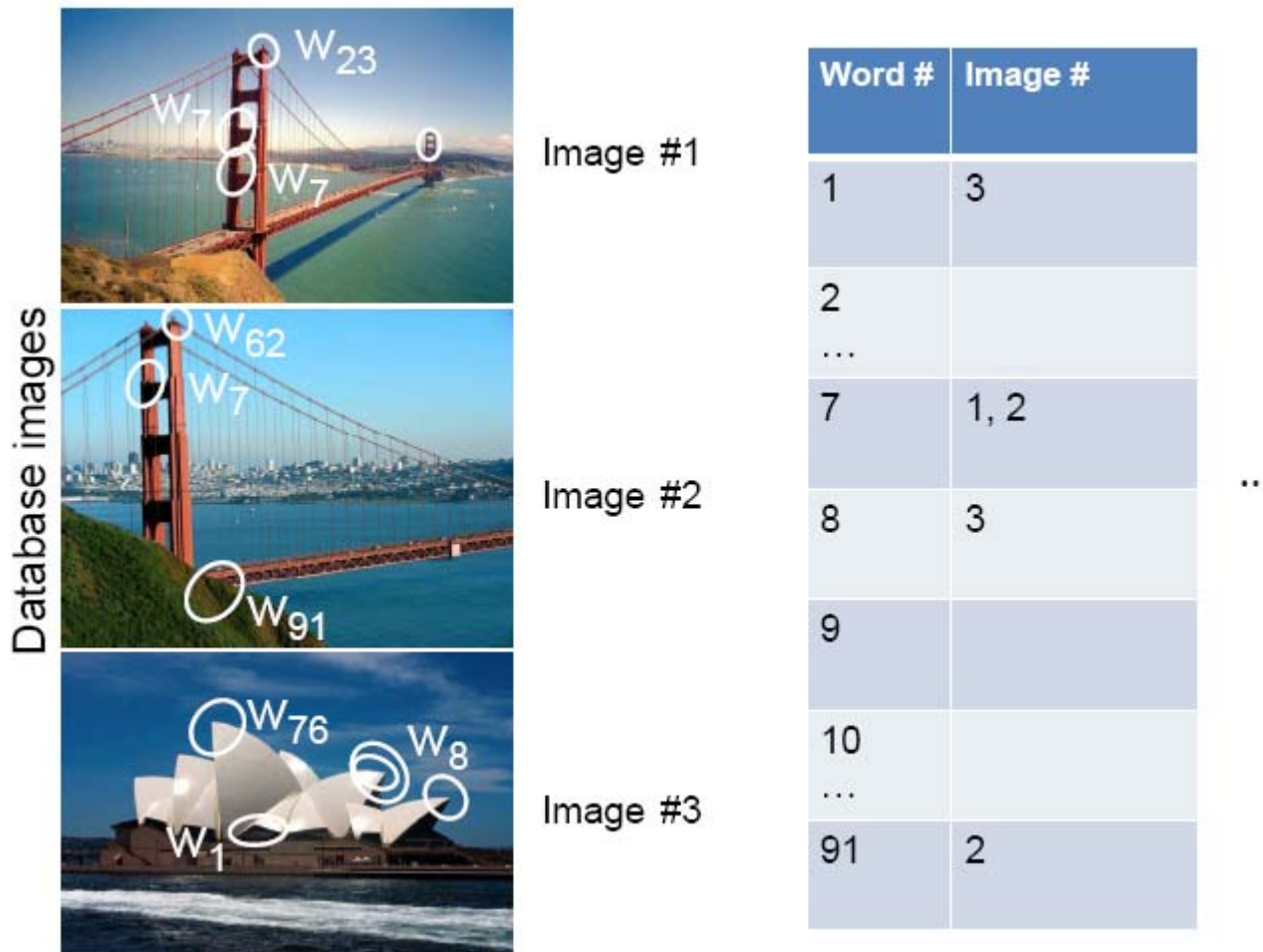


Figure from Sivic & Zisserman, ICCV 2003

# Inverted file index



- Database images are loaded into the index mapping words to image numbers

# Inverted file index

*When will this give us a significant gain in efficiency?*



New query image

Word #	Image #
1	3
2	
...	
7	1, 2
8	3
9	
10	
...	
91	2

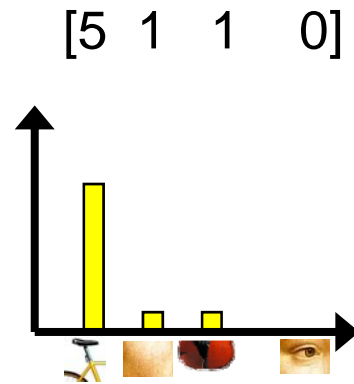
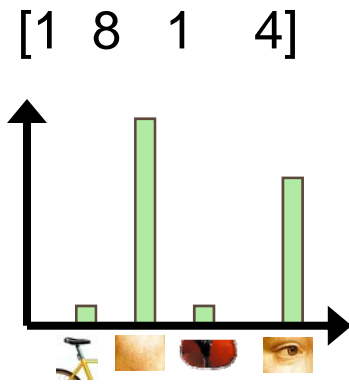
- New query image is mapped to indices of database images that share a word.

- If a local image region is a visual word, how can we summarize an image (the document)?



# Comparing bags of words

- Rank frames by normalized scalar product between their (possibly weighted) occurrence counts---*nearest neighbor* search for similar images.



$\vec{d}_j$



$\vec{q}$

$$\text{sim}(d_j, q) = \frac{\langle d_j, q \rangle}{\|d_j\| \|q\|}$$

$$= \frac{\sum_{i=1}^V d_j(i) * q(i)}{\sqrt{\sum_{i=1}^V d_j(i)^2} * \sqrt{\sum_{i=1}^V q(i)^2}}$$

for vocabulary of  $V$  words

# *tf-idf* weighting

- Term frequency - inverse document frequency
- Describe frame by frequency of each word within it, downweight words that appear often in the database
- (Standard weighting for text retrieval)

Number of occurrences  
of word  $i$  in document  $d$

$$t_i = \frac{n_{id}}{n_d} \log \frac{N}{n_i}$$

Total number of  
documents in database

Number of words in  
document  $d$

Number of documents  
word  $i$  occurs in, in whole  
database

# Query Expansion

Query: *golf green*

Results:

- How can the grass on the *greens* at a *golf* course be so perfect?
- For example, a skilled *golfer* expects to reach the *green* on a par-four hole in ...
- Manufactures and sells synthetic *golf* putting *greens* and mats.

Irrelevant result can cause a `topic drift`:

- Volkswagen *Golf*, 1999, *Green*, 2000cc, petrol, manual, , hatchback, 94000miles, 2.0 GTi, 2 Registered Keepers, HPI Checked, Air-Conditioning, Front and Rear Parking Sensors, ABS, Alarm, Alloy

# Query Expansion

Results



Query image

Spatial verification



New results



New query



Chum, Philbin, Sivic, Isard, Zisserman: Total Recall..., ICCV 2007

Slide credit: Ondrej Chum

# Bags of words for content-based image retrieval

Visually defined query

“Find this clock”



“Find this place”



“Groundhog Day” [Rammis, 1993]



# Example



## retrieved shots



# Video Google System

1. Collect all words within query region
2. Inverted file index to find relevant frames
3. Compare word counts
4. Spatial verification

Sivic & Zisserman, ICCV 2003



Query region



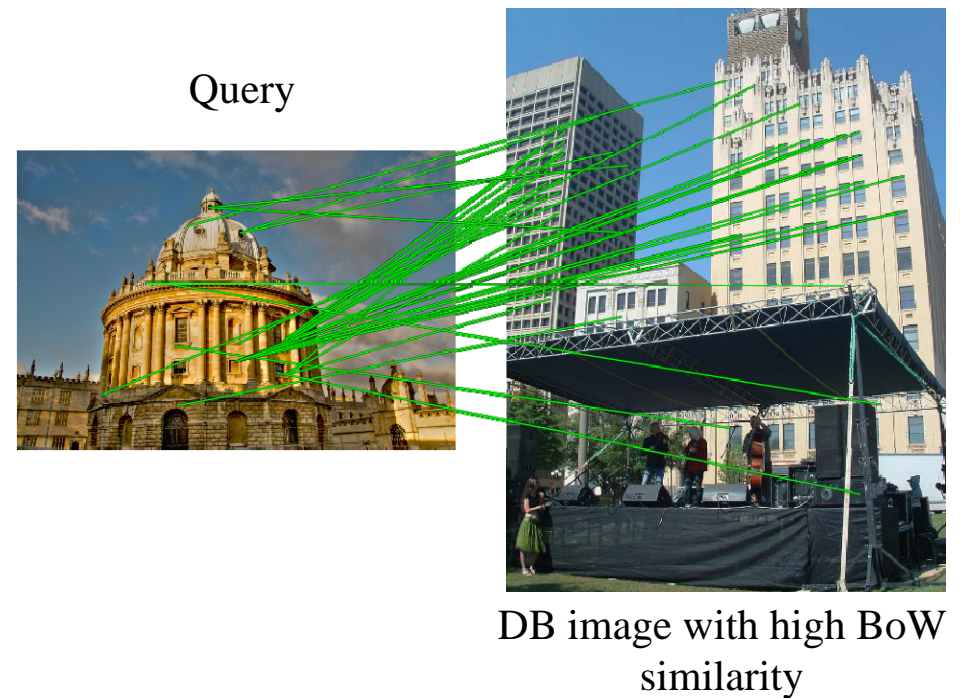
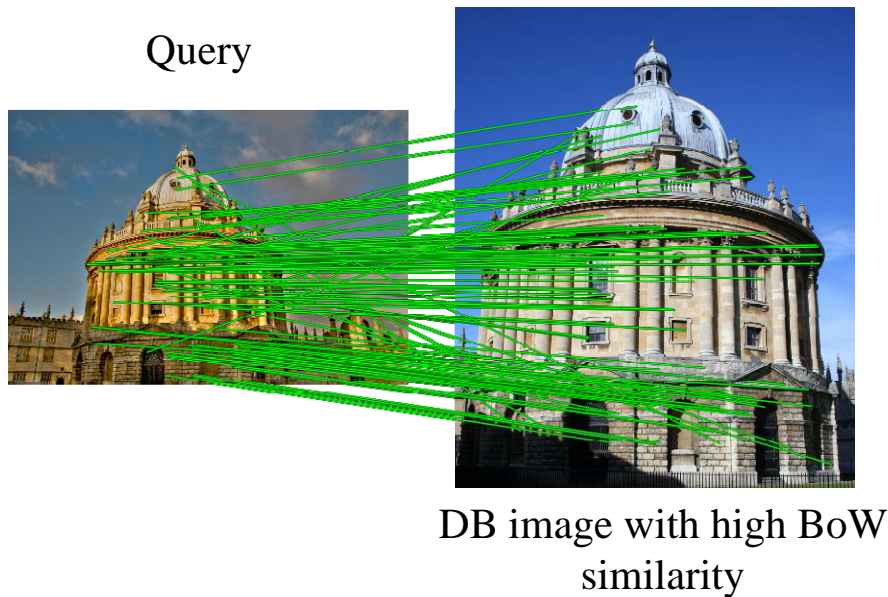
Retrieved frames

Is having the same set of visual words enough to identify the object or scene?

How to verify spatial agreement?



# Spatial Verification



Both image pairs have many visual words in common.

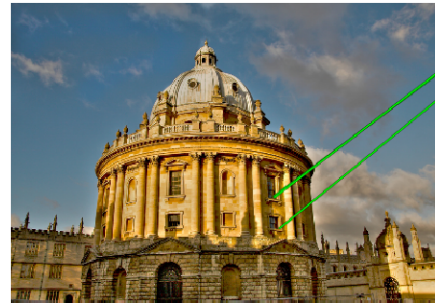
# Spatial Verification

Query



DB image with high BoW similarity

Query



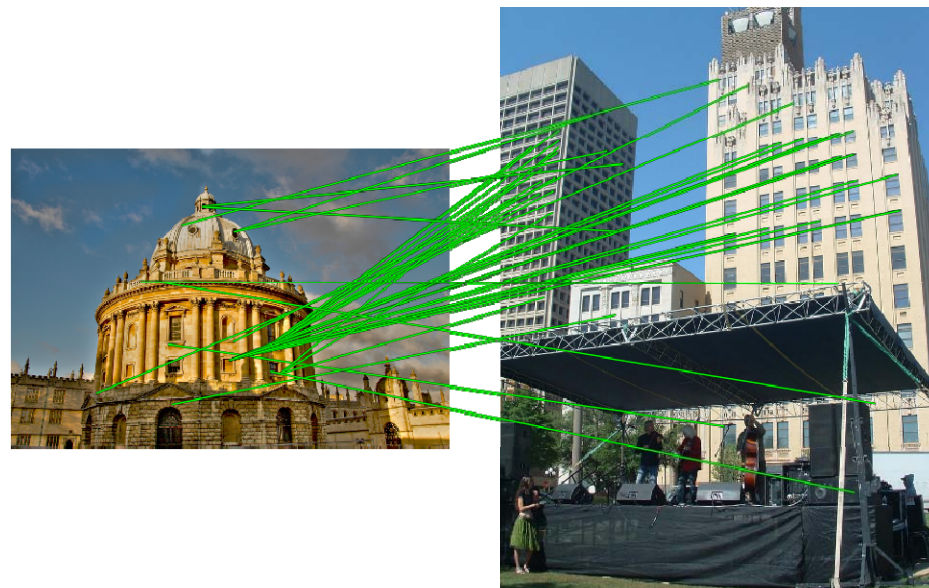
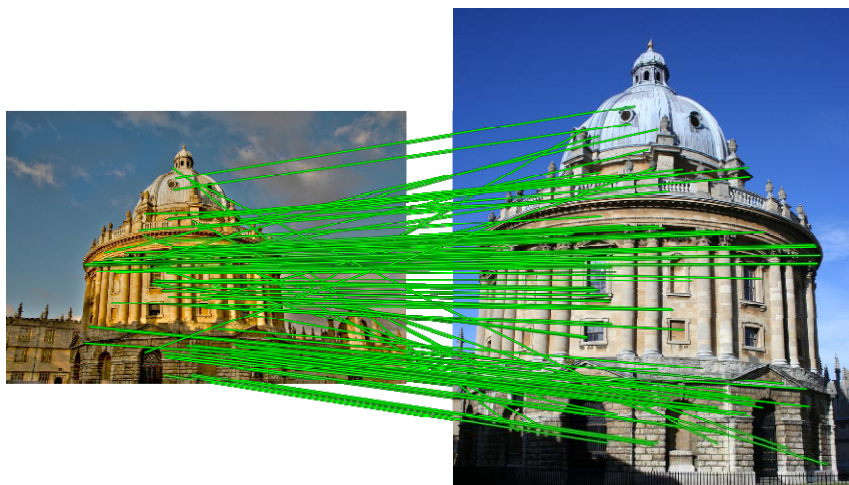
DB image with high BoW similarity

Only some of the matches are mutually consistent

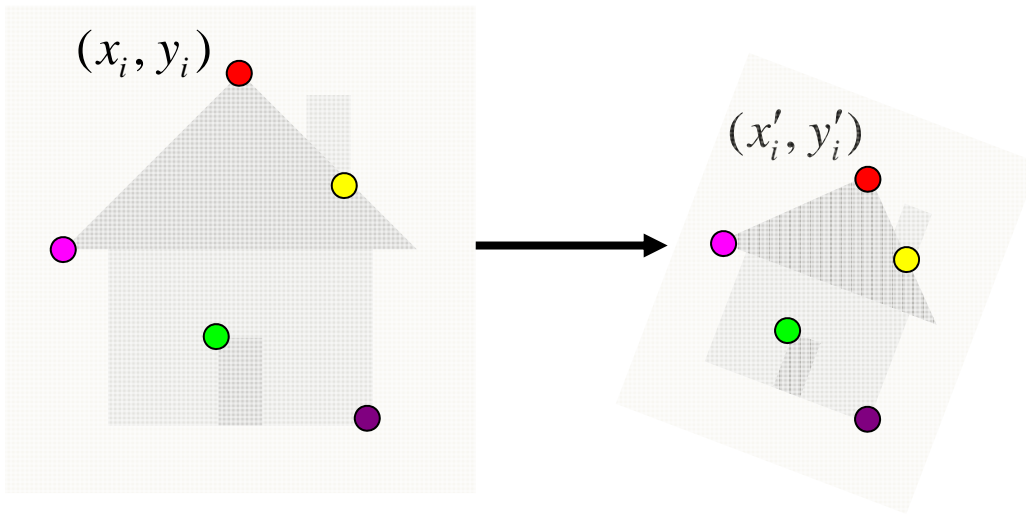
# Spatial Verification: two basic strategies

- RANSAC
  - Typically sort by BoW similarity as initial filter
  - Verify by checking support (inliers) for possible transformations
    - e.g., “success” if a transformation with  $> N$  inlier correspondences can be found
- Generalized Hough Transform
  - Let each matched feature cast a vote on location, scale, orientation of the model object
  - Verify parameters with enough votes

# RANSAC verification



# Recall: Fitting an affine transformation

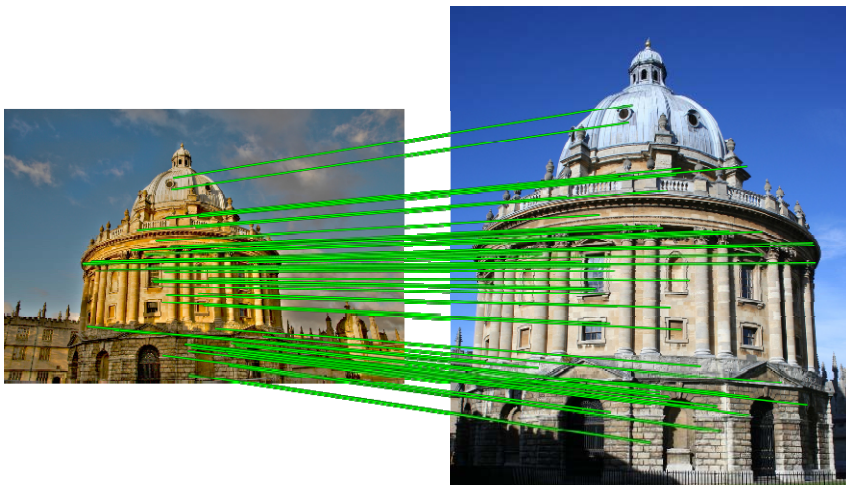
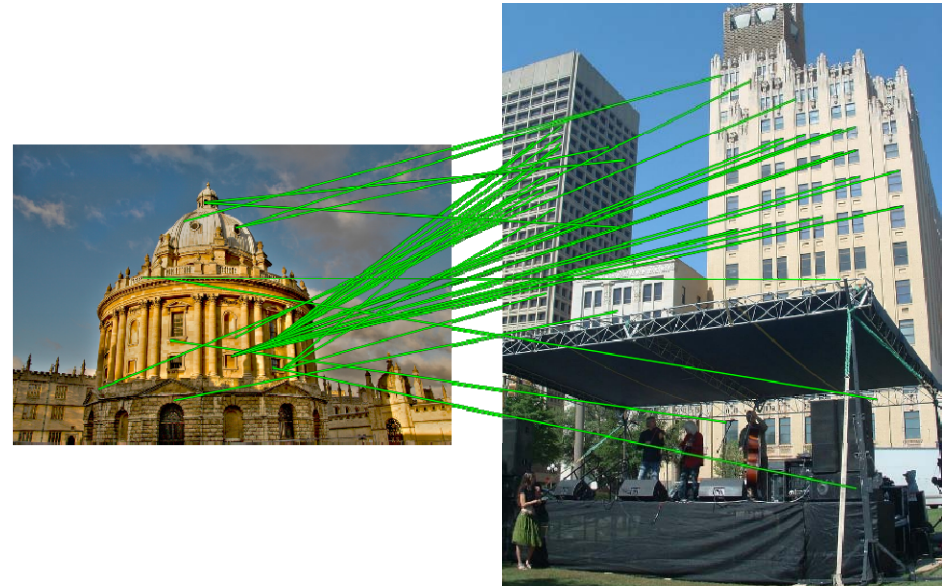
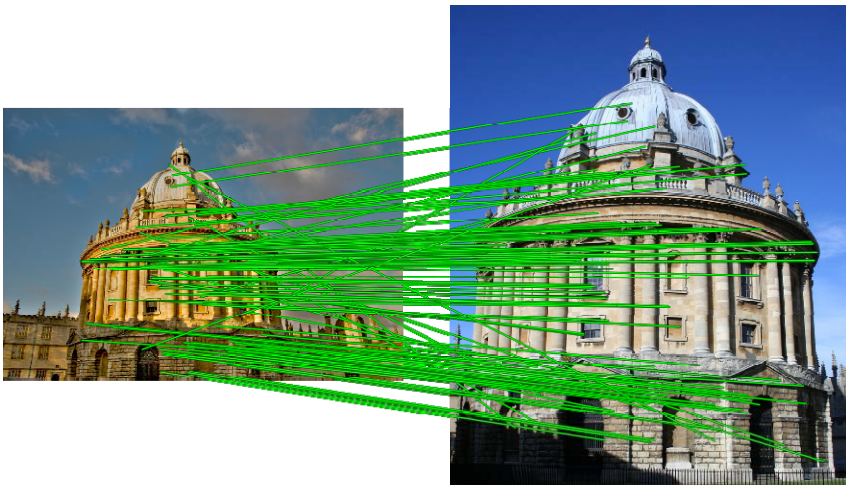


Approximates viewpoint changes for roughly planar objects and roughly orthographic cameras.

$$\begin{bmatrix} x'_i \\ y'_i \end{bmatrix} = \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$$

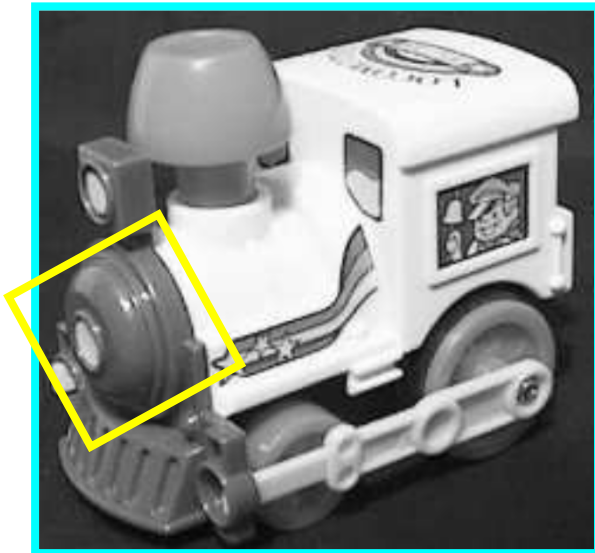
$$\begin{bmatrix} \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ x_i & y_i & 0 & 0 & 1 & 0 & m_1 \\ 0 & 0 & x_i & y_i & 0 & 1 & m_2 \\ \dots & \dots & \dots & \dots & \dots & \dots & m_3 \\ & & & & & & m_4 \\ & & & & & & t_1 \\ & & & & & & t_2 \end{bmatrix} = \begin{bmatrix} \dots \\ x'_i \\ y'_i \\ \dots \end{bmatrix}$$

# RANSAC verification



# Voting: Generalized Hough Transform

- If we use scale, rotation, and translation invariant local features, then each feature match gives an alignment hypothesis (for scale, translation, and orientation of model in image).



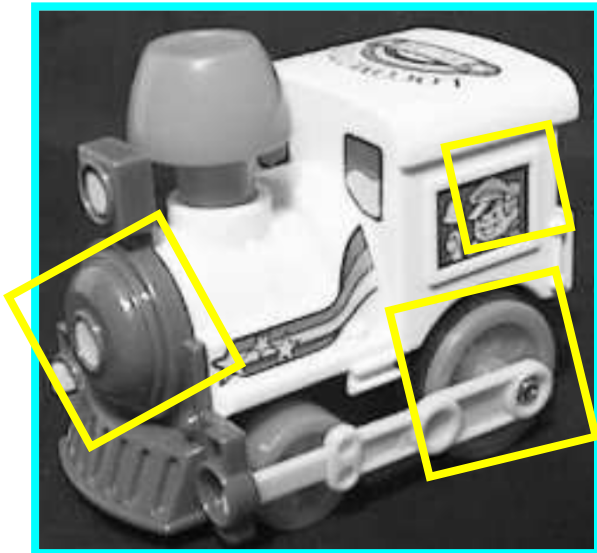
Model



Novel image

# Voting: Generalized Hough Transform

- A hypothesis generated by a single match may be unreliable,
- So let each match vote for a hypothesis in Hough space



Model



Novel image



# Generalized Hough Transform details

- **Training phase:** For each model feature, record 2D location, scale, and orientation of model (relative to normalized feature frame)
- **Test phase:** Let each match between a test SIFT feature and a model feature vote in a 4D Hough space
  - Use broad bin sizes of 30 degrees for orientation, a factor of 2 for scale, and 0.25 times image size for location
  - Vote for two closest bins in each dimension
- Find all bins with at least three votes and perform geometric verification
  - Estimate least squares *affine* transformation
  - Search for additional features that agree with the alignment

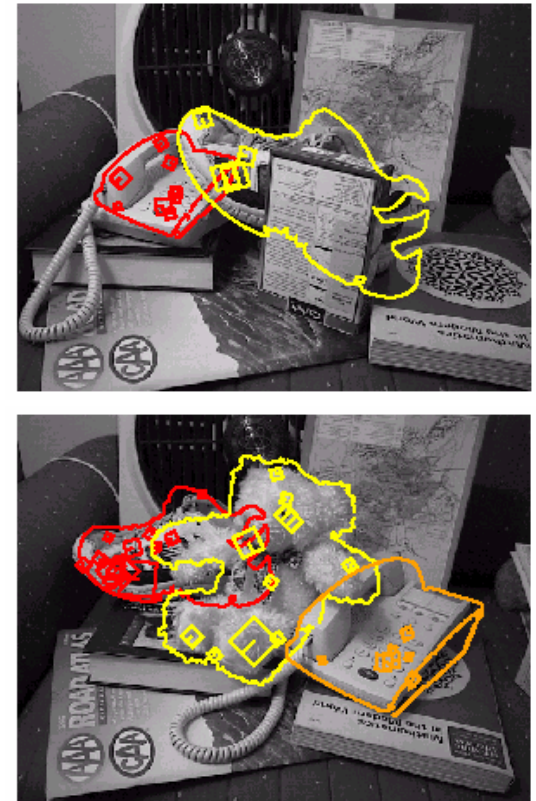
# Results



Background subtraction  
for model boundaries



Objects recognized,



Recognition in spite  
of occlusion

# Difficulties of voting

- Noise/clutter can lead to as many votes as true target
- Bin size for the accumulator array must be chosen carefully
- In practice, good idea to make broad bins and spread votes to nearby bins, since verification stage can prune bad vote peaks

# Generalized Hough vs RANSAC

## GHT

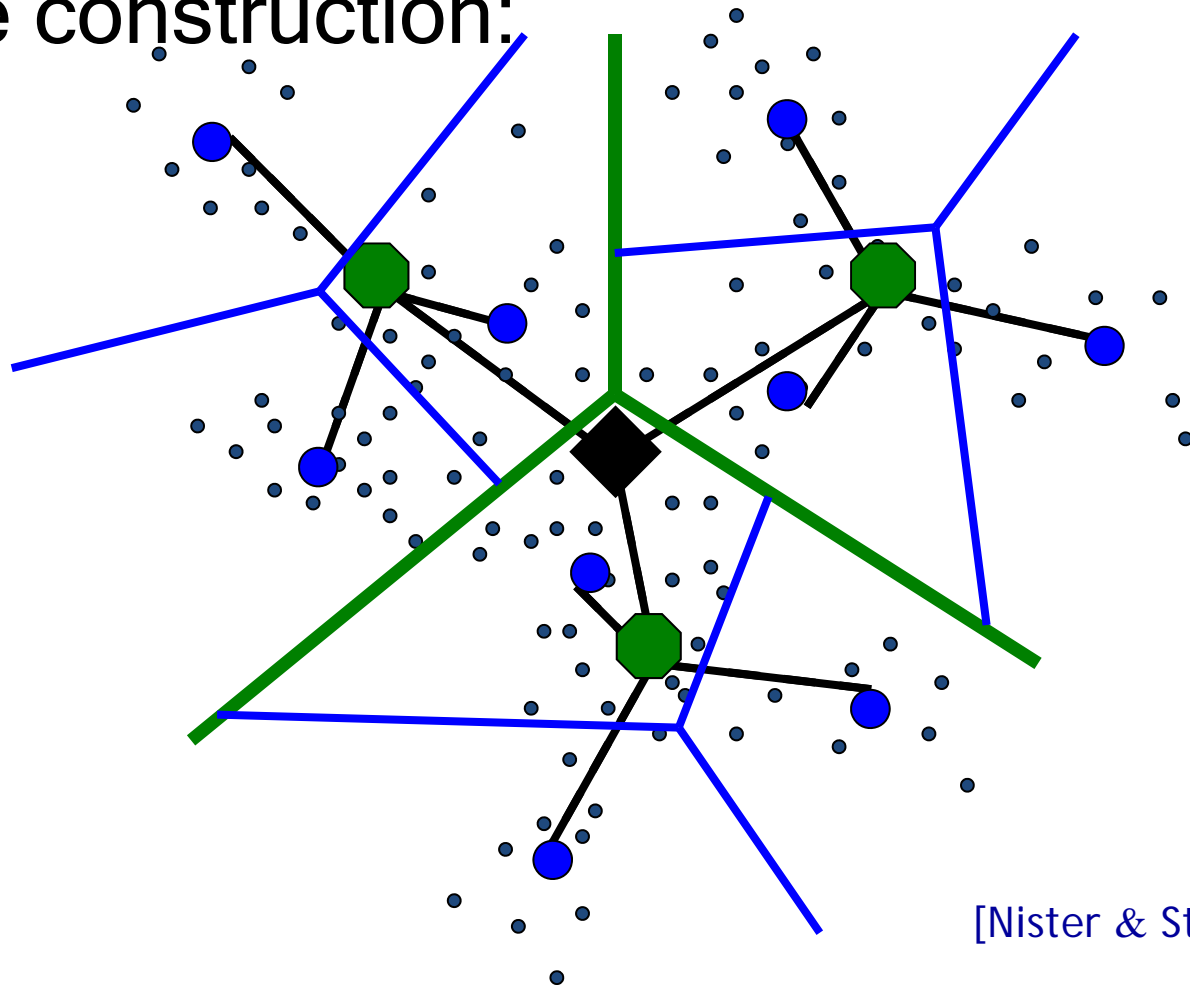
- Single correspondence -> vote for all consistent parameters
- Represents uncertainty in the model parameter space
- Linear complexity in number of correspondences and number of voting cells; beyond 4D vote space impractical
- Can handle high outlier ratio

## RANSAC

- Minimal subset of correspondences to estimate model -> count inliers
- Represents uncertainty in image space
- Must search all data points to check for inliers each iteration
- Scales better to high-d parameter spaces

# Vocabulary Trees: hierarchical clustering for large vocabularies

- Tree construction:

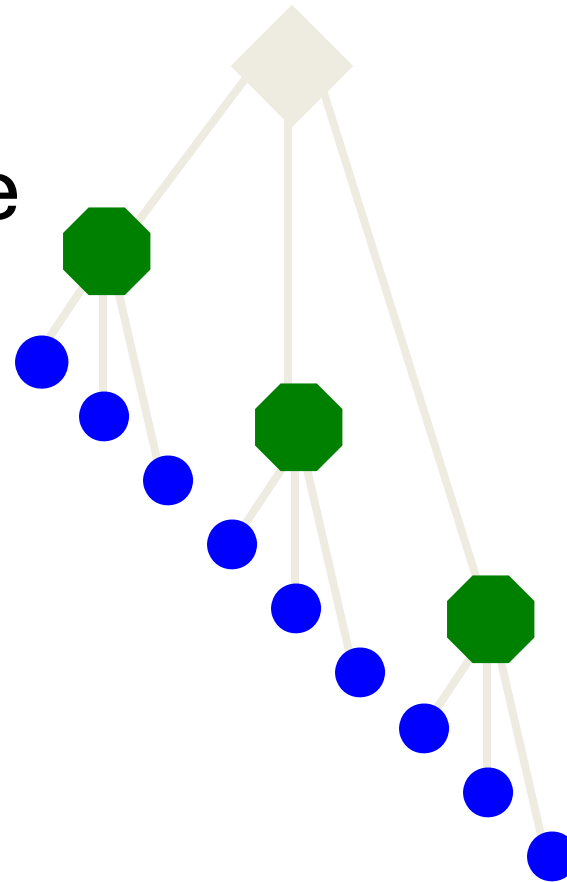


[Nister & Stewenius, CVPR'06]

Slide credit: David Nister

# Vocabulary Tree

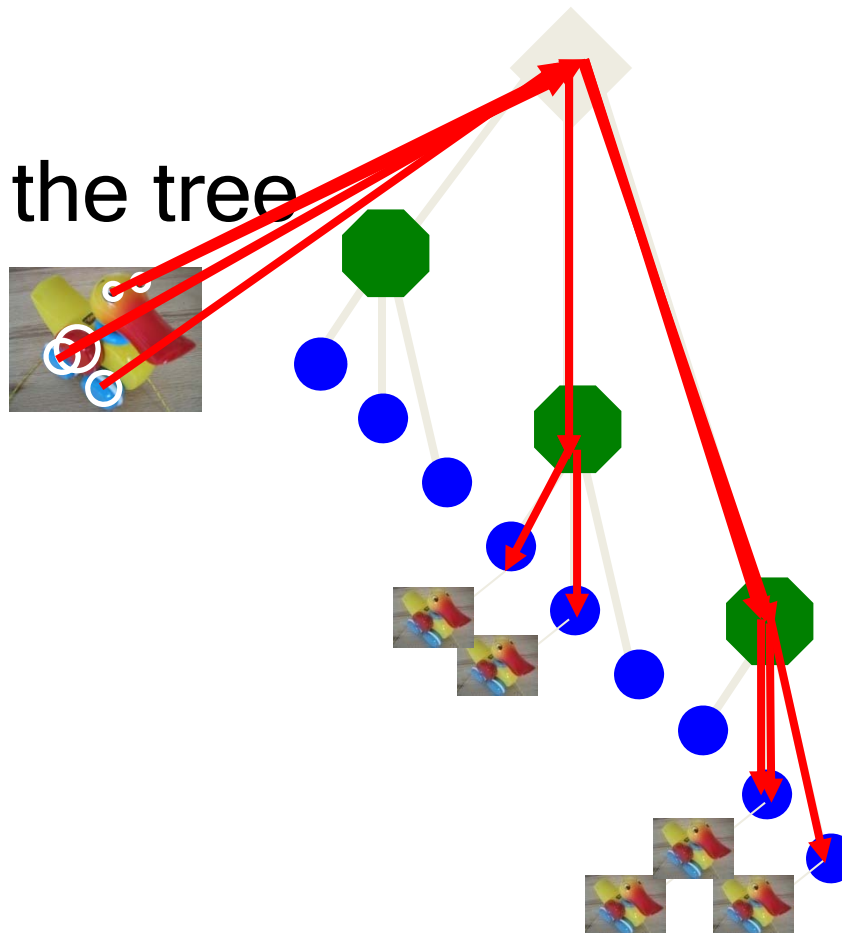
- Training: Filling the tree



[Nister & Stewenius, CVPR'06]

# Vocabulary Tree

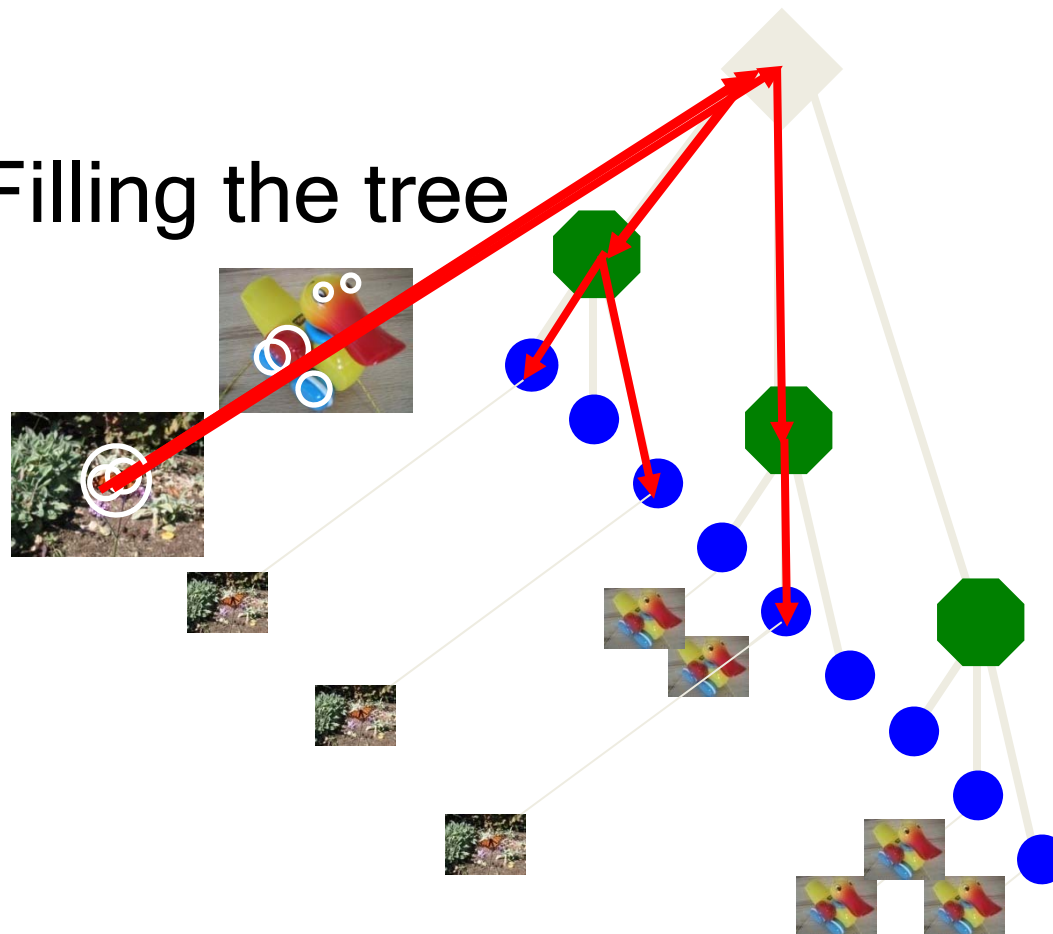
- Training: Filling the tree



[Nister & Stewenius, CVPR'06]

# Vocabulary Tree

- Training: Filling the tree

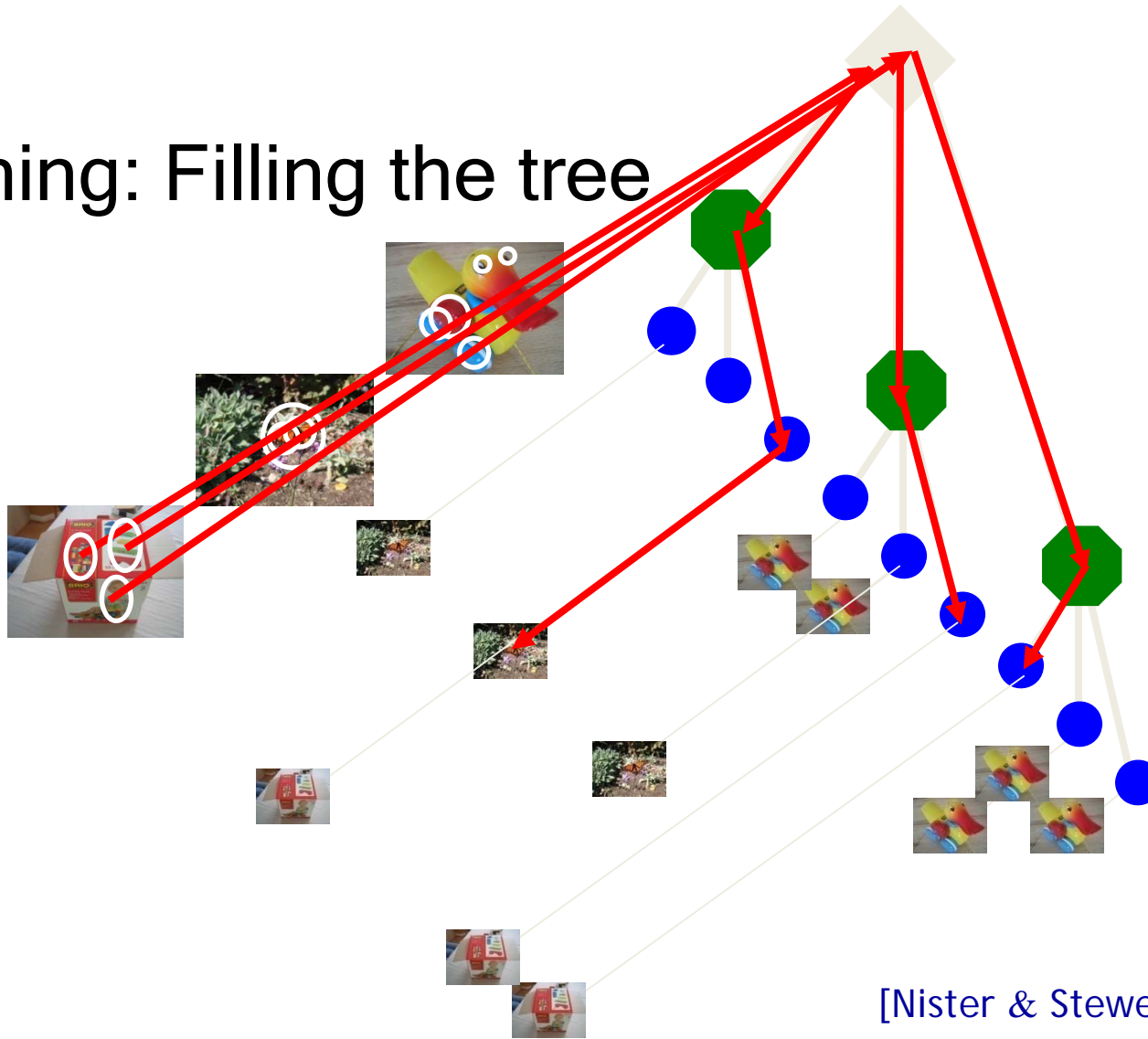


[Nister & Stewenius, CVPR'06]



# Vocabulary Tree

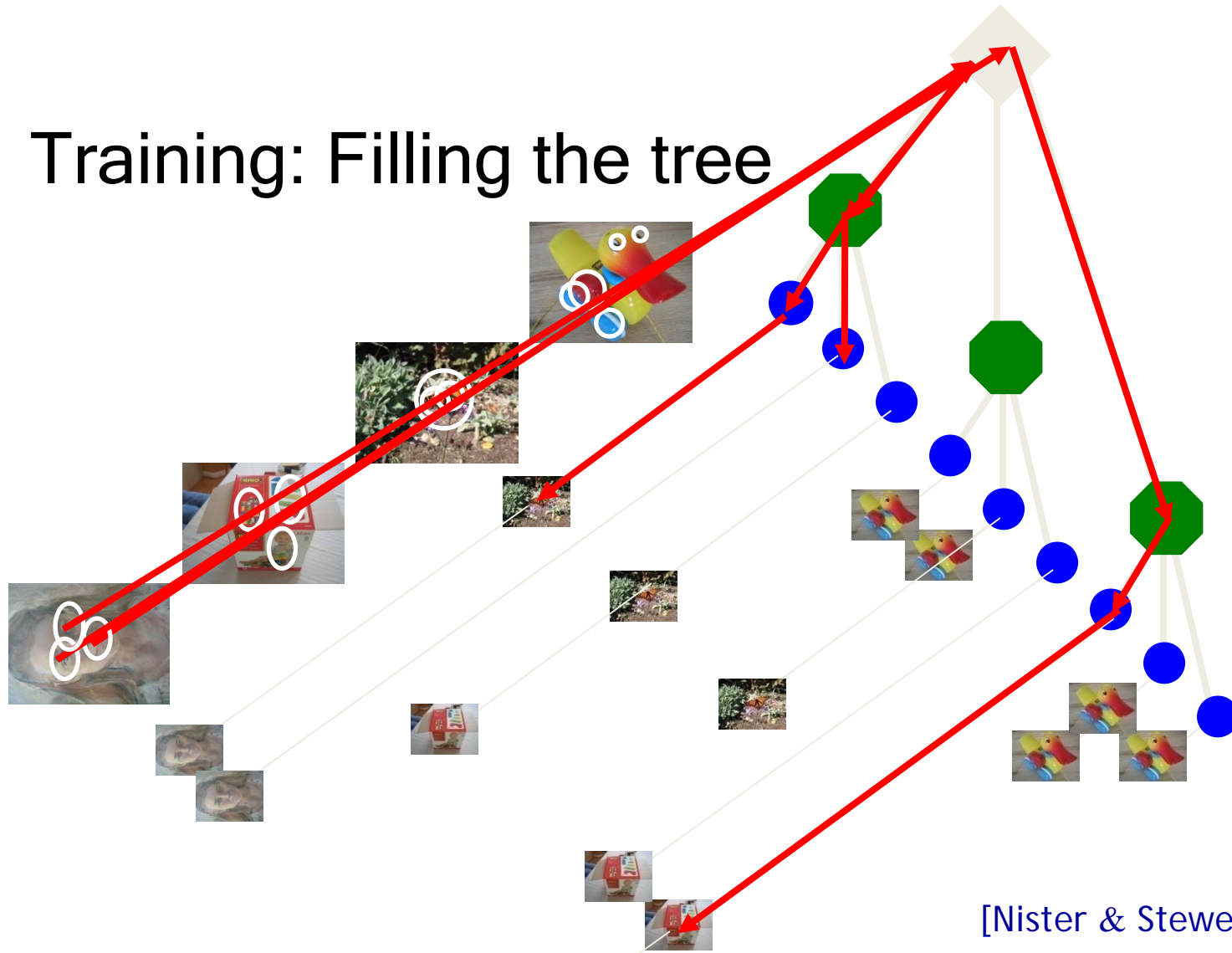
- Training: Filling the tree



[Nister & Stewenius, CVPR'06]

# Vocabulary Tree

- Training: Filling the tree



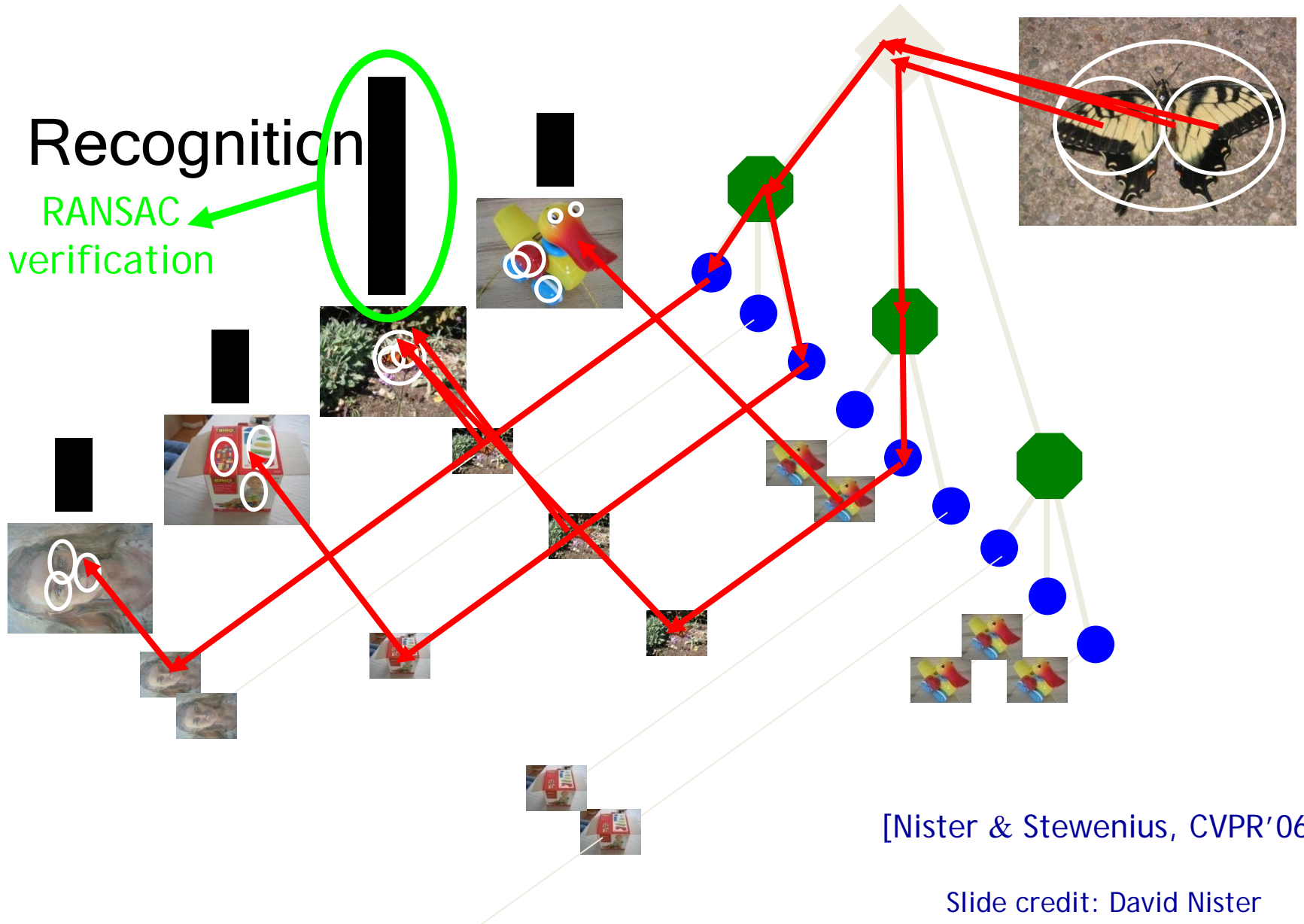
[Nister & Stewenius, CVPR'06]

What is the computational advantage of the hierarchical representation bag of words, vs. a flat vocabulary?

# Vocabulary Tree

- Recognition

RANSAC  
verification



[Nister & Stewenius, CVPR'06]

Slide credit: David Nister

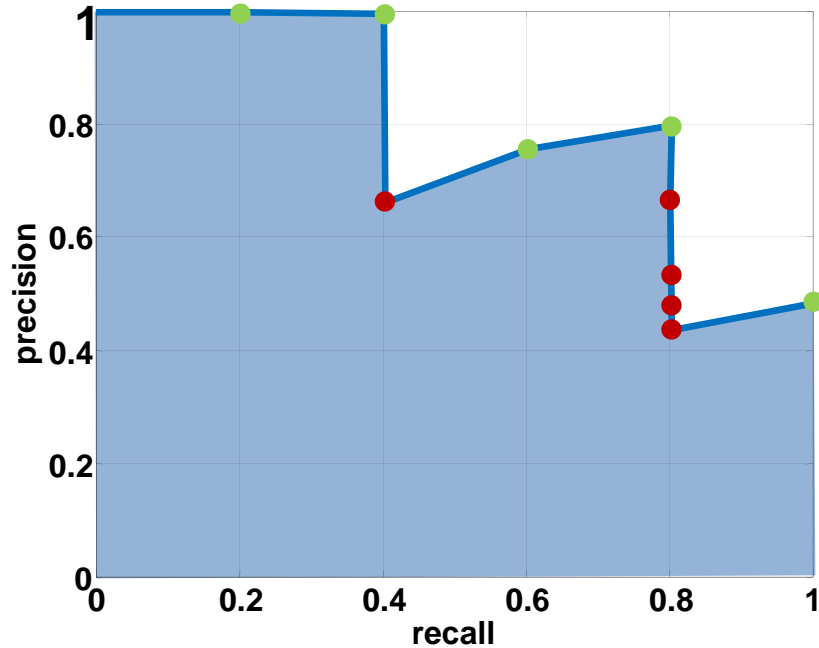
# Scoring retrieval quality



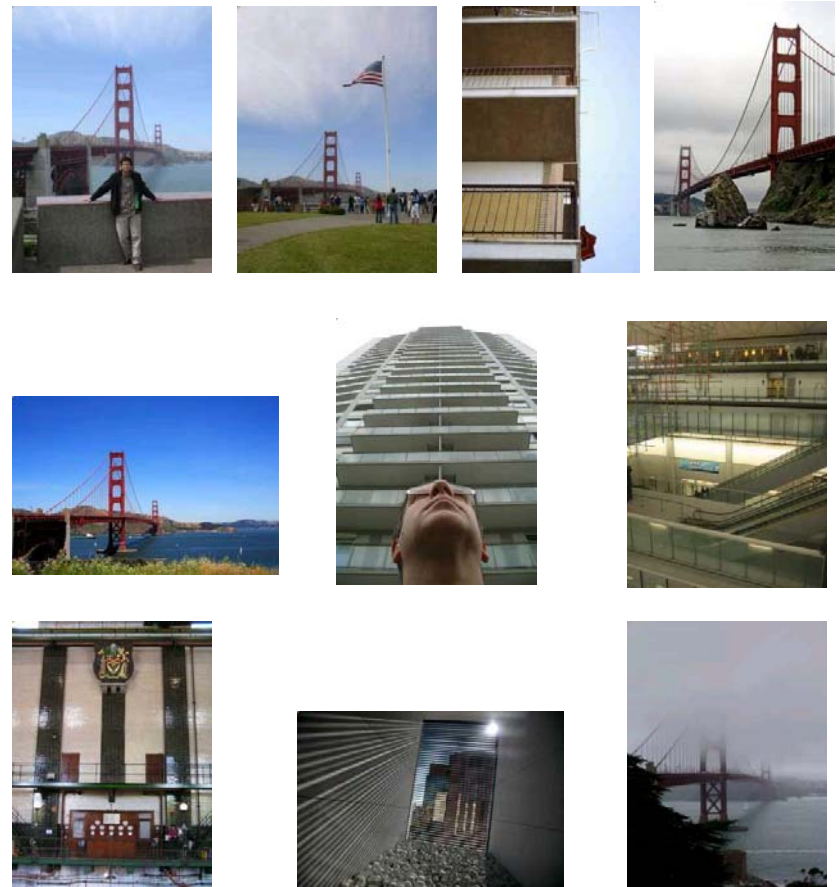
Query

Database size: 10 images  
Relevant (total): 5 images

$\text{precision} = \frac{\# \text{relevant}}{\# \text{returned}}$   
 $\text{recall} = \frac{\# \text{relevant}}{\# \text{total relevant}}$



Results (ordered):

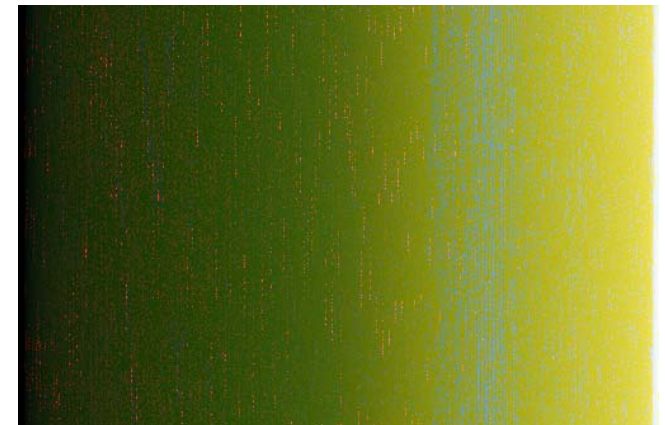
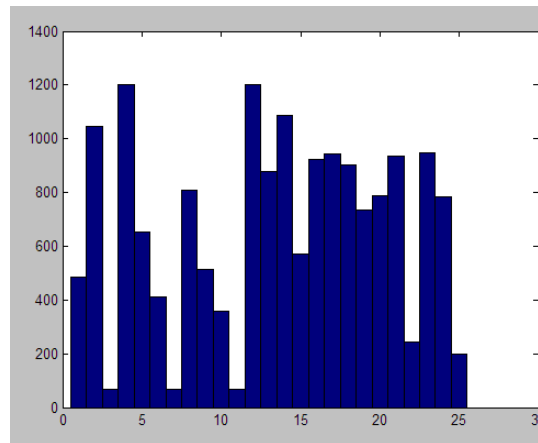


# Bags of words: pros and cons

- + flexible to geometry / deformations / viewpoint
- + compact summary of image content
- + provides vector representation for sets
- + very good results in practice
- basic model ignores geometry - must verify afterwards, or encode via features
- background and foreground mixed when bag covers whole image
- optimal vocabulary formation remains unclear

# Spatial Layout and Context

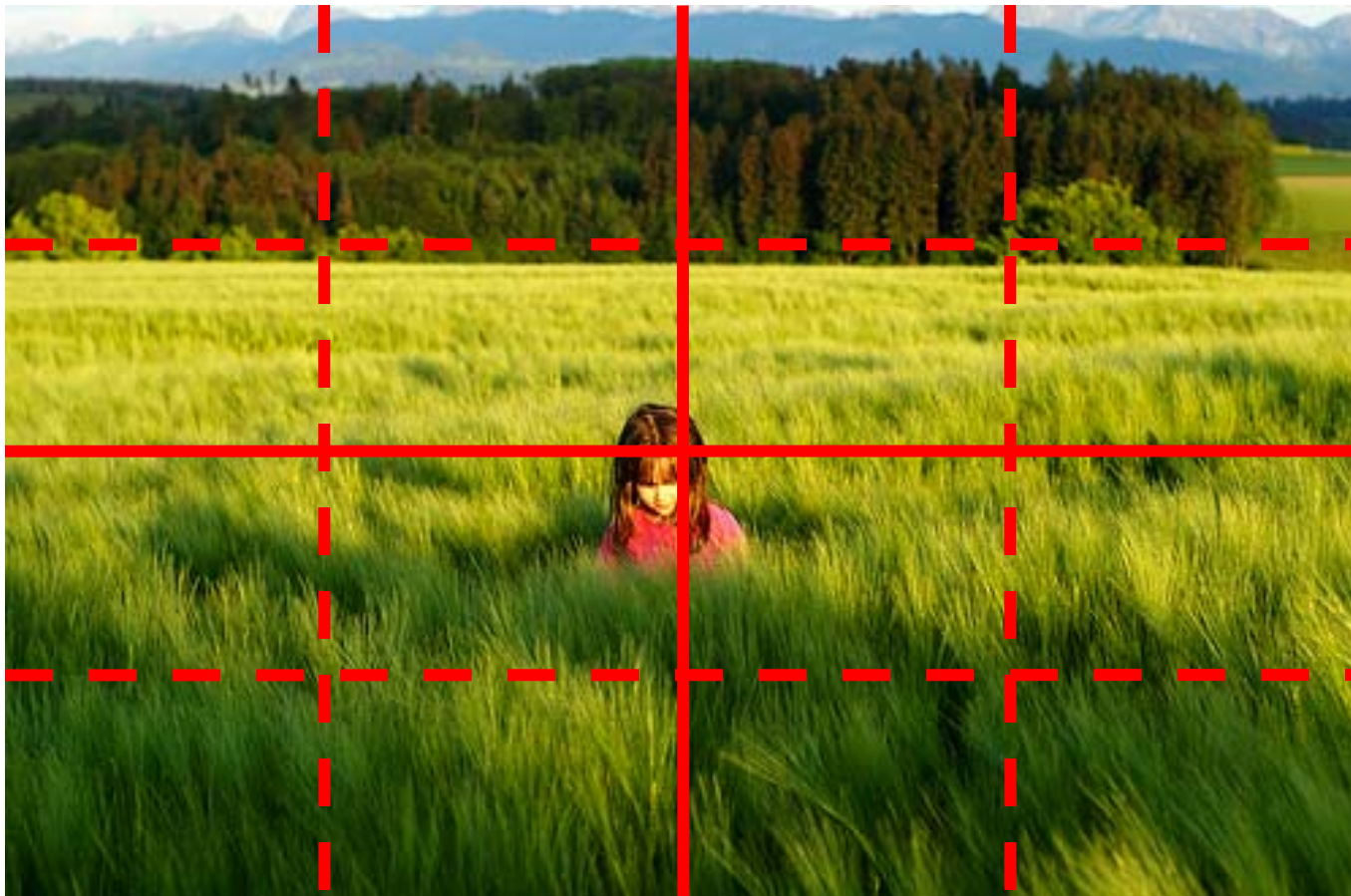
# But what about spatial layout?



All of these images have the same color histogram

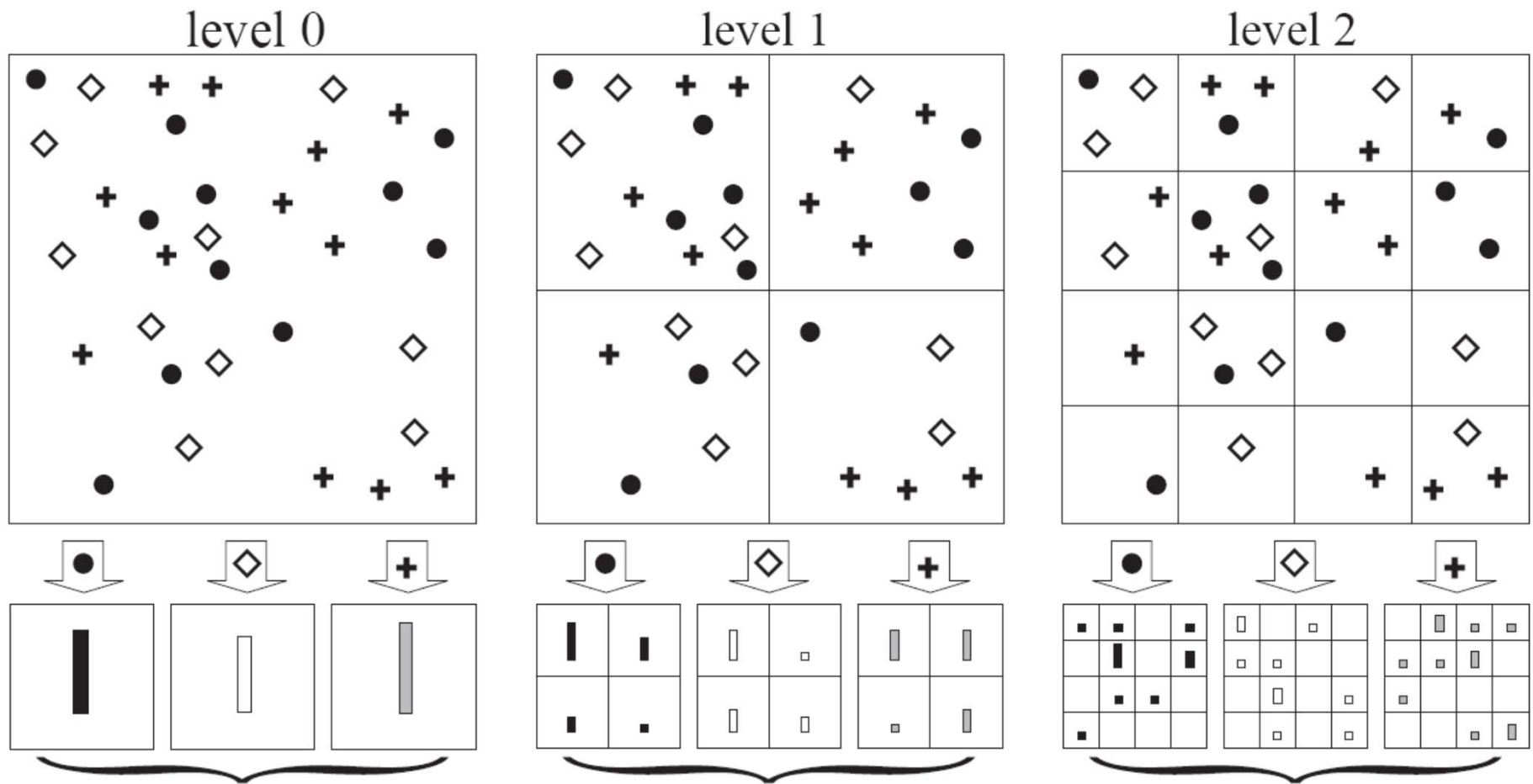


# Spatial pyramid



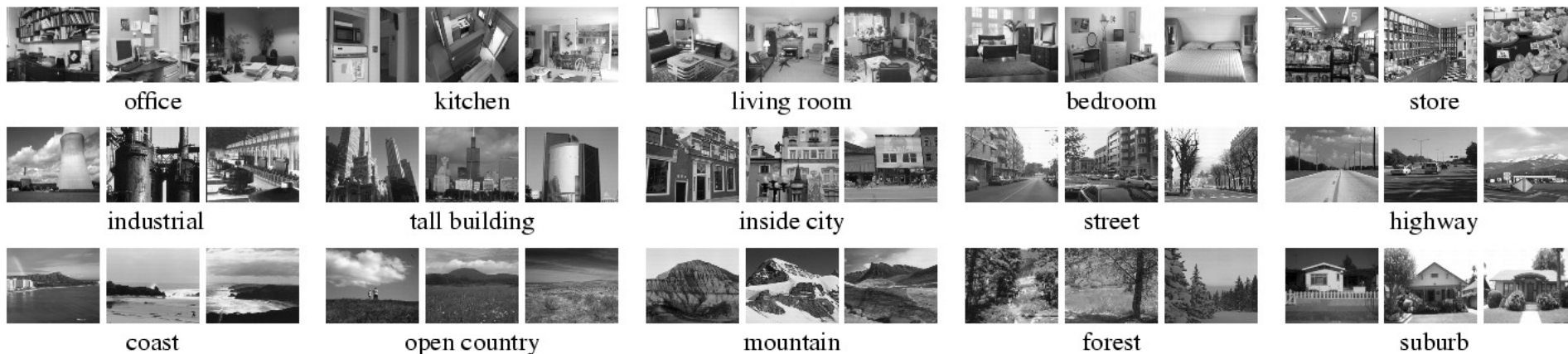
Compute histogram in each spatial bin

# Spatial pyramid



[[Lazebnik et al. CVPR 2006](#)]

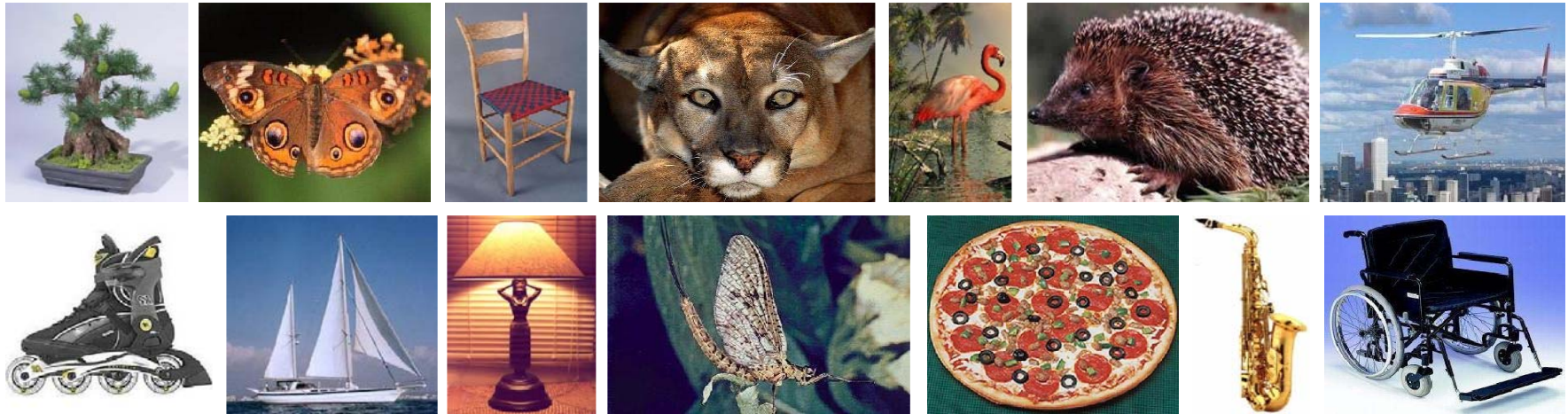
# Results: Scene category dataset



Multi-class classification results  
(100 training images per class)

Level	Weak features (vocabulary size: 16)		Strong features (vocabulary size: 200)	
	Single-level	Pyramid	Single-level	Pyramid
0 (1 × 1)	45.3 ±0.5		72.2 ±0.6	
1 (2 × 2)	53.6 ±0.3	56.2 ±0.6	77.9 ±0.6	79.0 ±0.5
2 (4 × 4)	61.7 ±0.6	64.7 ±0.7	79.4 ±0.3	<b>81.1 ±0.3</b>
3 (8 × 8)	63.3 ±0.8	<b>66.8 ±0.6</b>	77.2 ±0.4	80.7 ±0.3

# Results: Caltech101 dataset

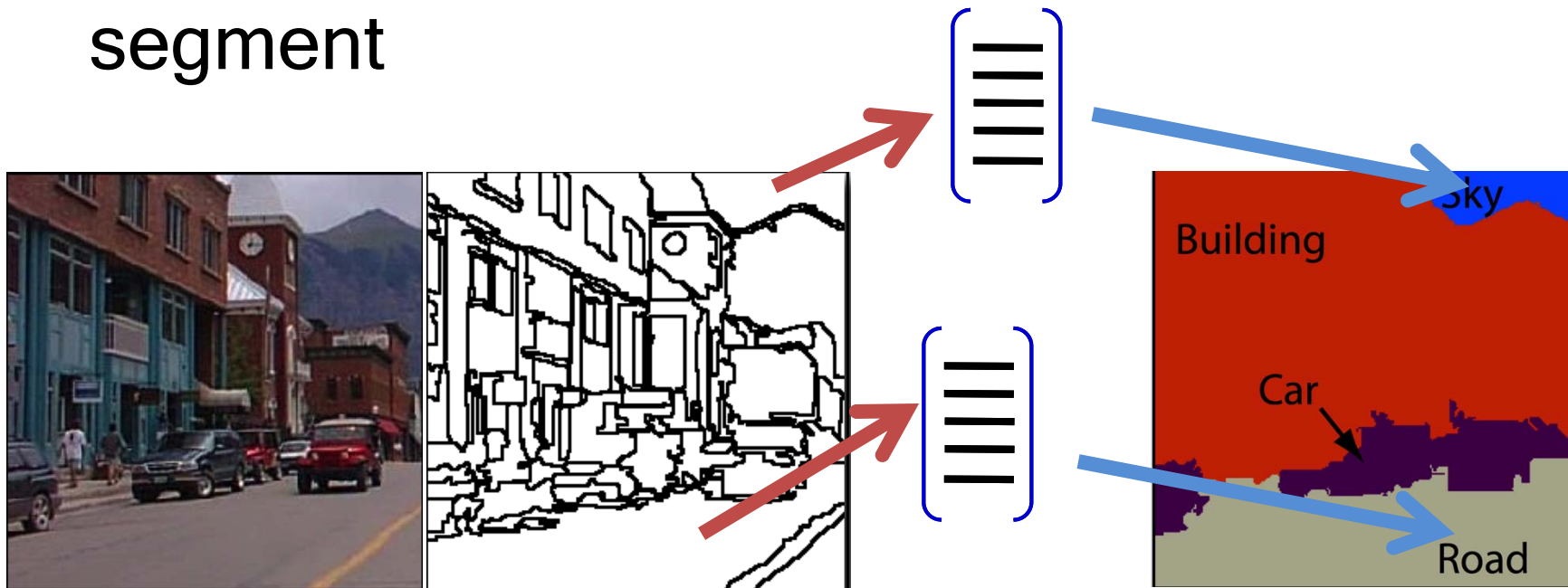


Multi-class classification results (30 training images per class)

	Weak features (16)		Strong features (200)	
Level	Single-level	Pyramid	Single-level	Pyramid
0	15.5 ±0.9		41.2 ±1.2	
1	31.4 ±1.2	32.8 ±1.3	55.9 ±0.9	57.0 ±0.8
2	47.2 ±1.1	49.3 ±1.4	63.6 ±0.9	<b>64.6 ±0.8</b>
3	52.2 ±0.8	<b>54.0 ±1.1</b>	60.3 ±0.9	64.6 ±0.7

# Region representation

- Segment the image into superpixels
- Use features to represent each image segment



# Region representation

- Color, texture, BoW
  - Only computed within the local region
- Shape of regions
- Position in the image

# Working with regions

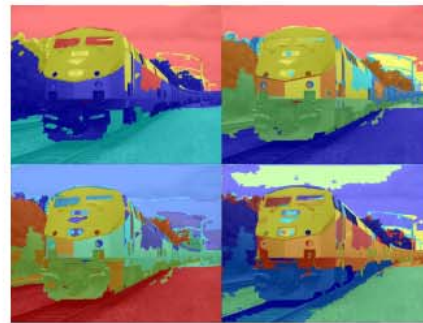
- Spatial support is important - multiple segmentations



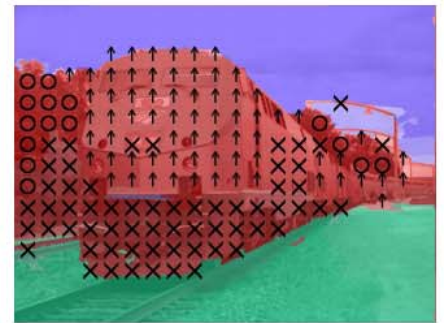
(a) Input



(b) Superpixels



(c) Multiple Hypotheses



(d) Geometric Labels

Geometric context [[Hoiem et al. ICCV 2005](#)]