CS 532: 3D Computer Vision
9th-10th Set of Notes

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Lecture Outline

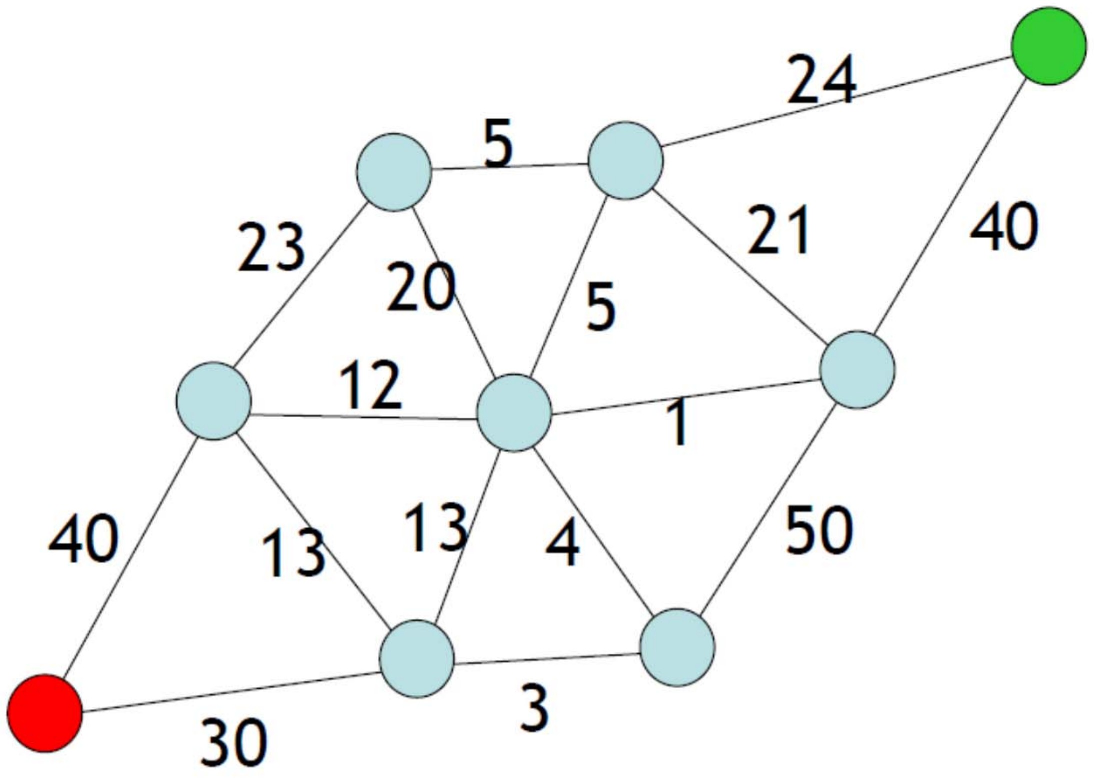
- Multi-view Stereo part II
  - Slides by G. Vogiatzis and L. Zhang
  - Paper by A. Collet et al. (2015)

- Introduction to Computational Geometry
- Convex Hulls
  - David M. Mount, CMSC 754: Computational Geometry lecture notes, Department of Computer Science, University of Maryland, Spring 2012
  - Slides by:
    - B. Gartner, M. Hoffman and E. Welzl (ETH)
    - M. van Kreveld (Utrecht University)
    - P. Indyk and J.C. Yang (MIT)

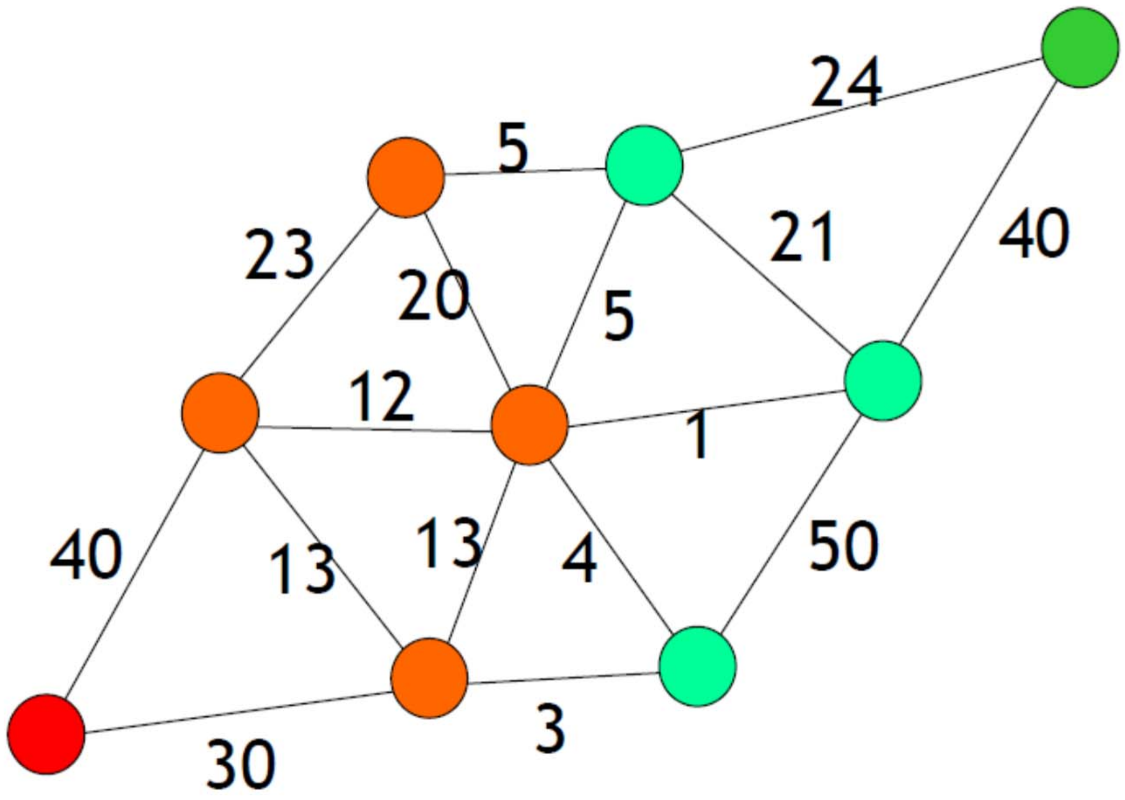
Extracting a Surface from Photo-consistency

- Vogiatzis et al. (PAMI 2007)
- Divide the space in voxels
- Compute the photo-consistency of each voxel
  - By robustly combining all pairwise NCC scores
- Problem: find a minimum cost surface that separates interior from exterior of the object
- Add term that favors large volume, otherwise solution collapses to a point

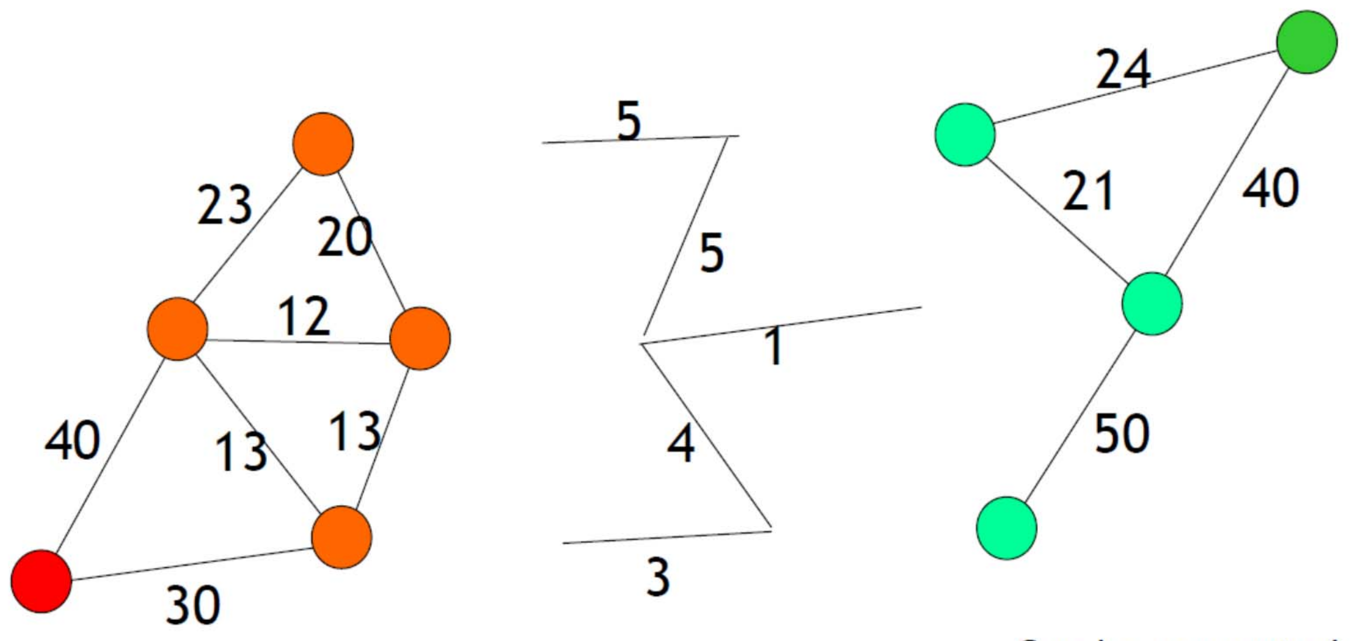
How to Solve?



Graph Cut



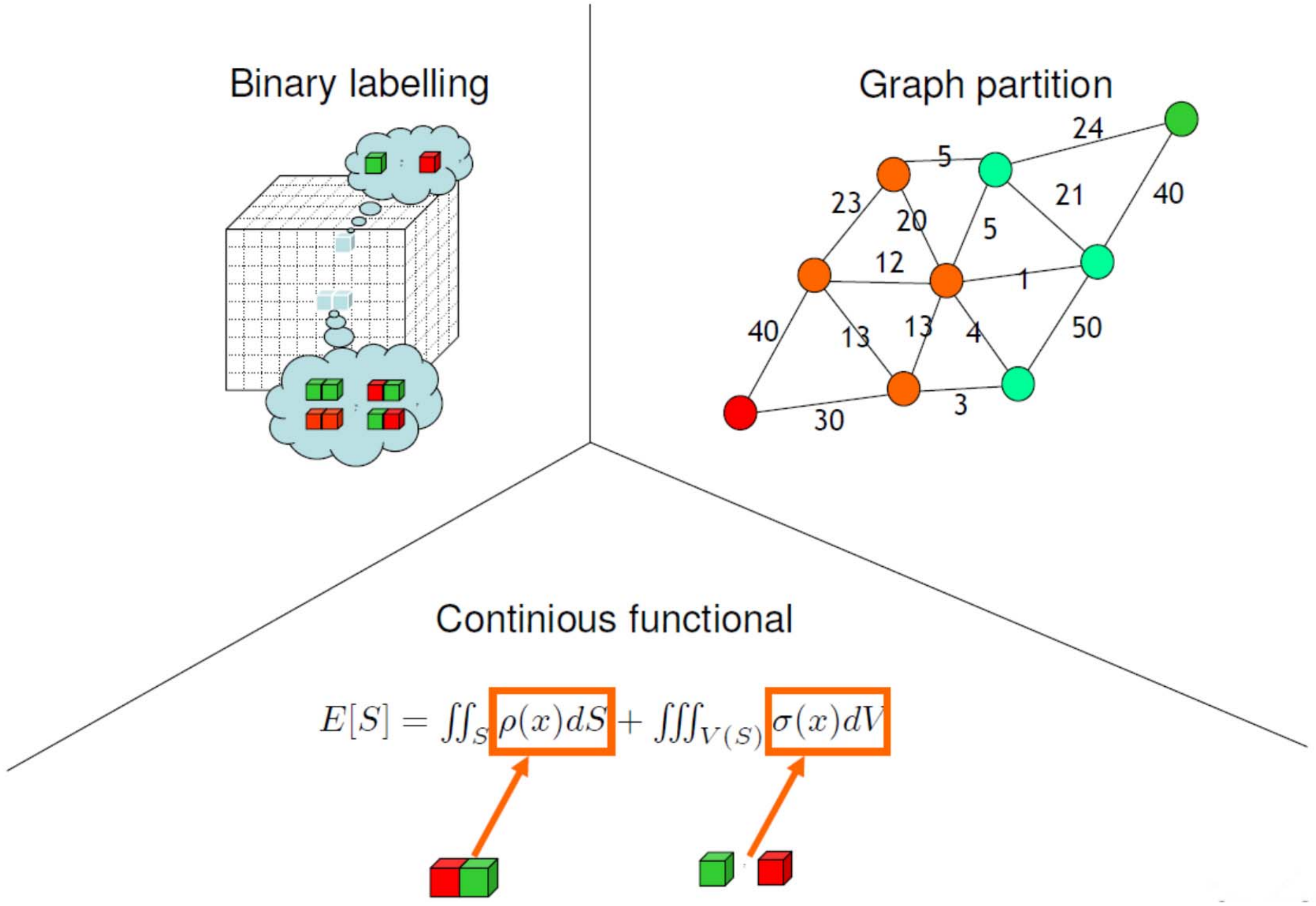
Minimum Cut



5+5+1+4+3=18

Can be computed
in polynomial time
with *Ford-Fulkerson (1956)*
algorithm

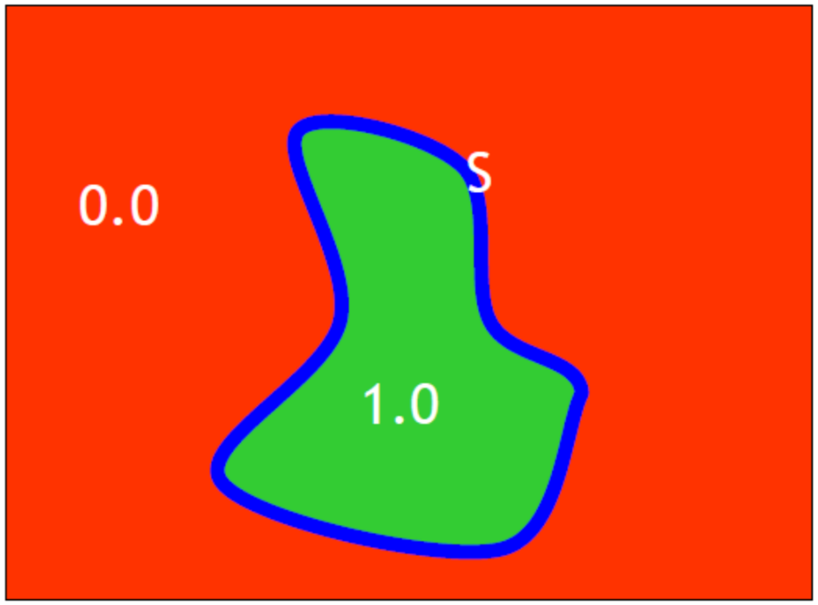
Three Equivalent Representations



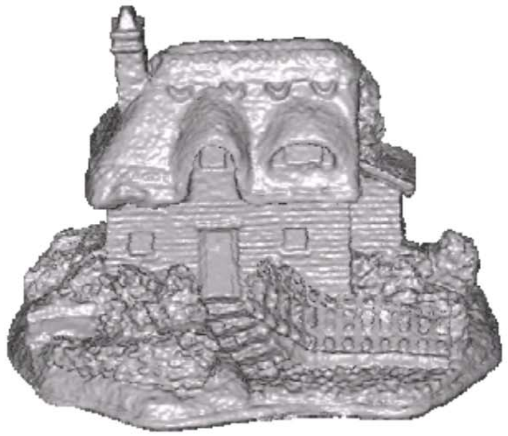
$$E[S] = \iint_S \boxed{\rho(x)dS} + \iiint_{V(S)} \boxed{\sigma(x)dV}$$

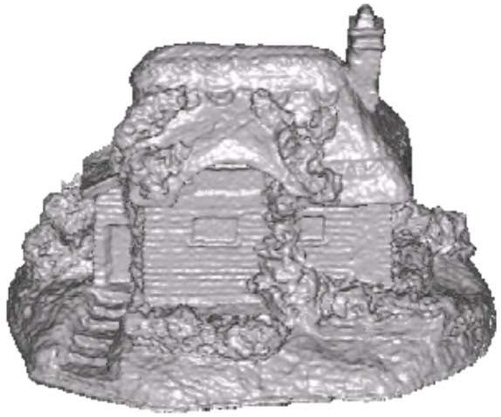
Extracting the Surface

- Marching cubes algorithm can extract isosurfaces
  - Matlab: [tri, pts] = isosurface(V)
  - Where V is a binary volume of 0s and 1s

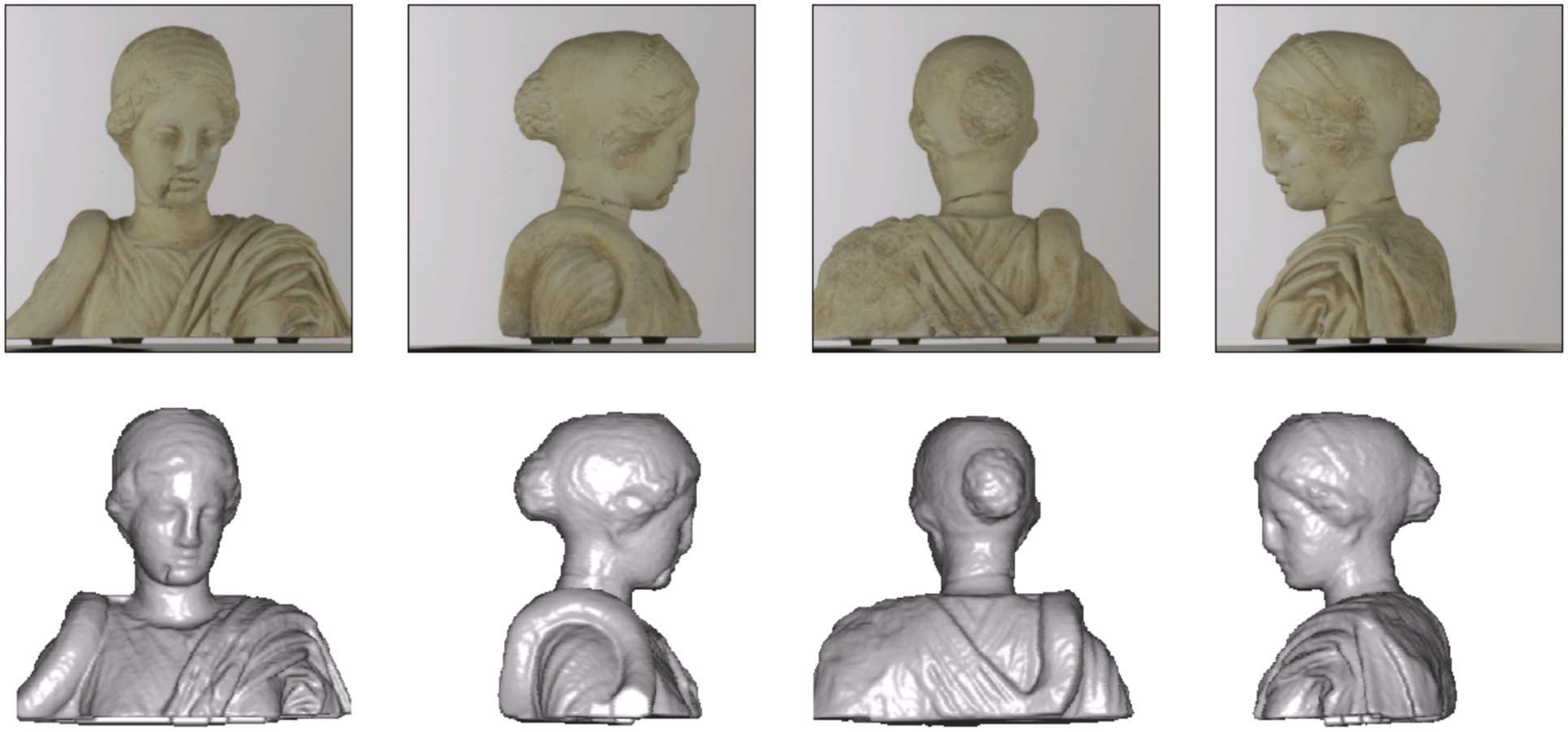


Results



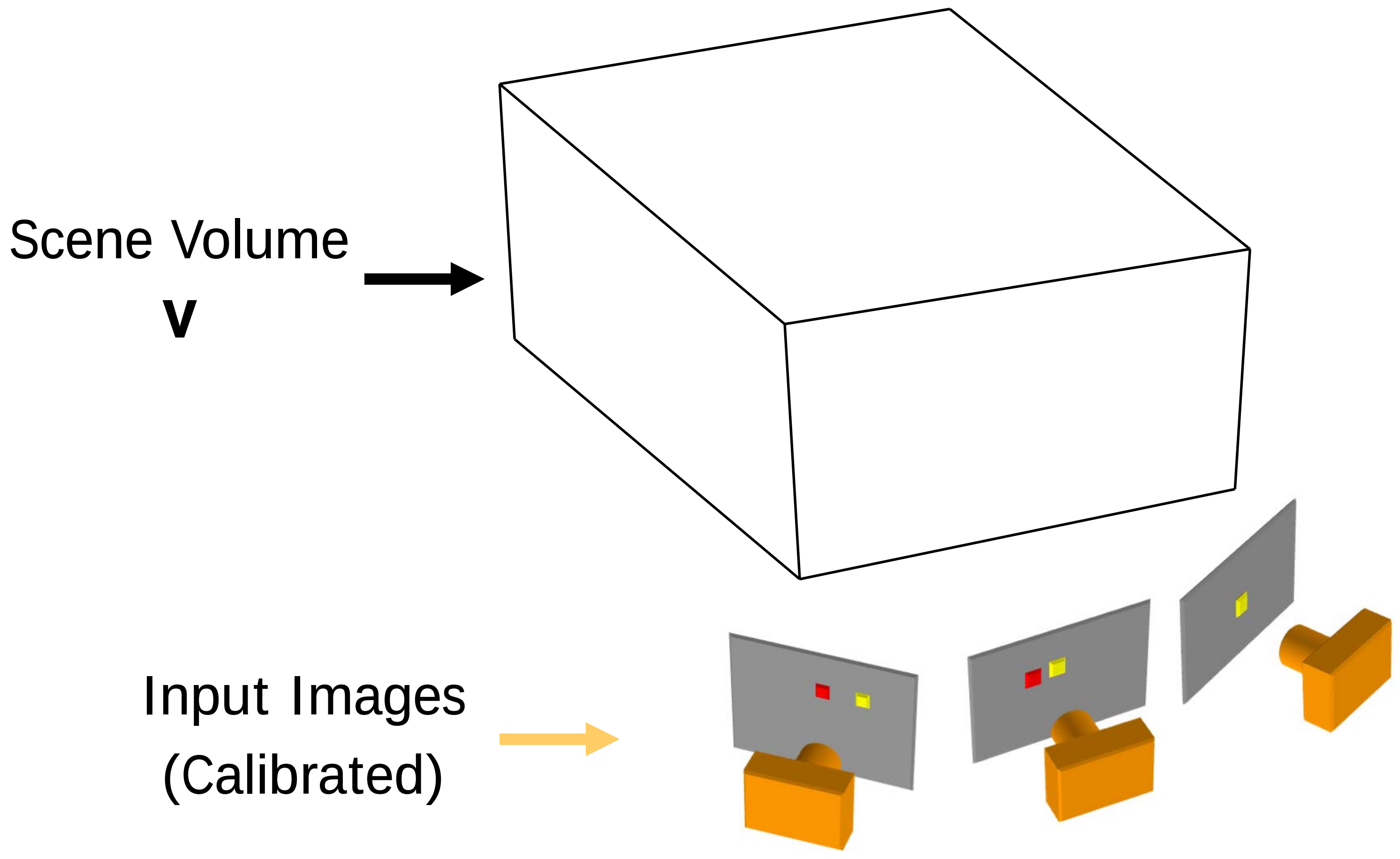


Results



Volumetric Stereo

- Determine occupancy, “color” of points in V
- Slides by L. Zhang

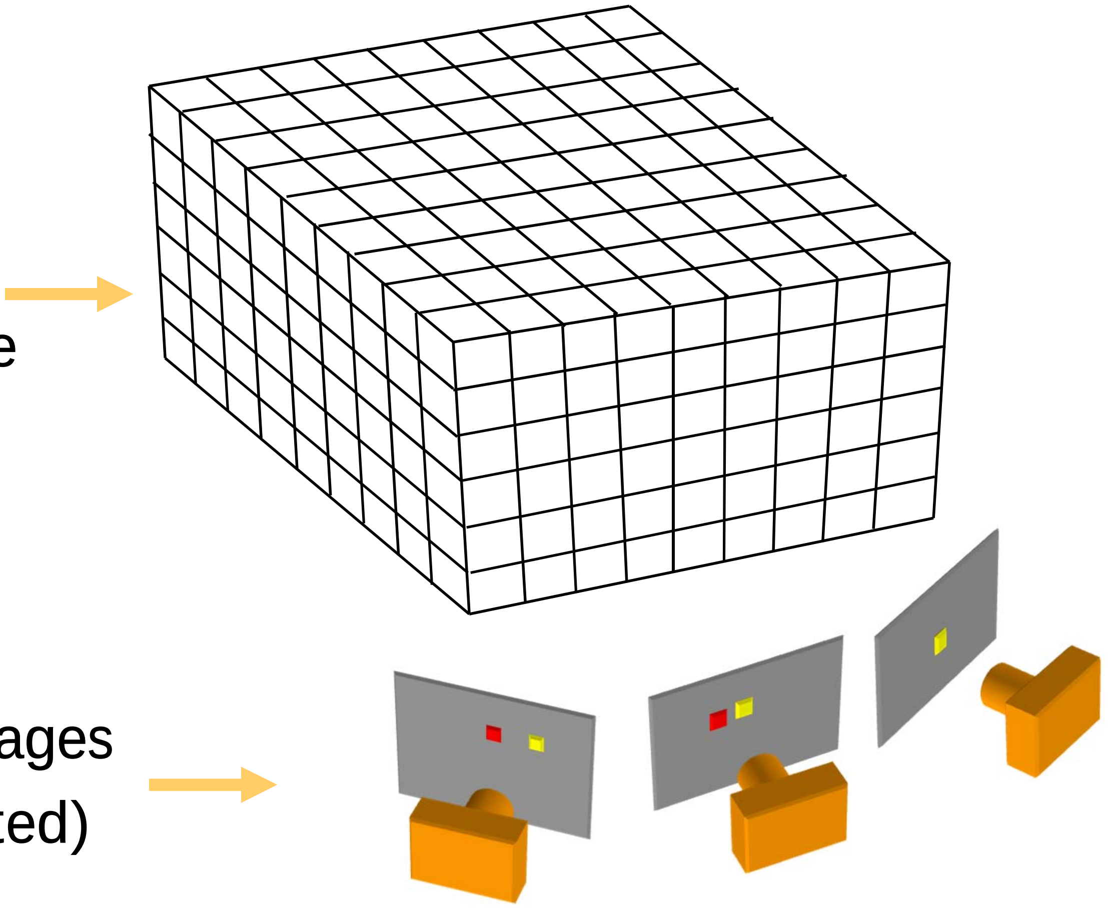


Discrete Formulation: Voxel Coloring

Goal: Assign RGBA values to voxels in V *photo-consistent* with images

Discretized Scene Volume

Input Images (Calibrated)



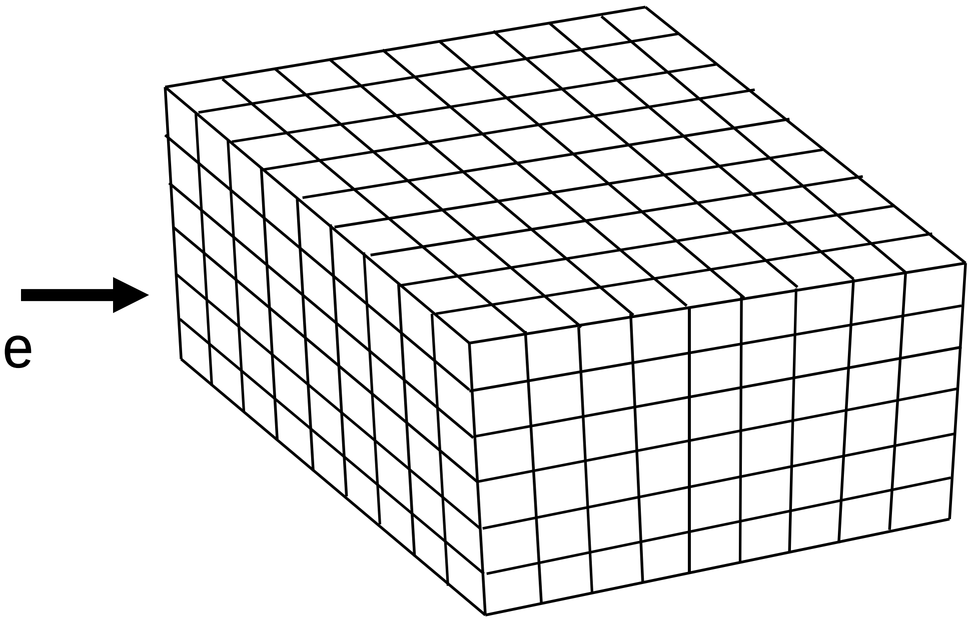
Complexity and Computability

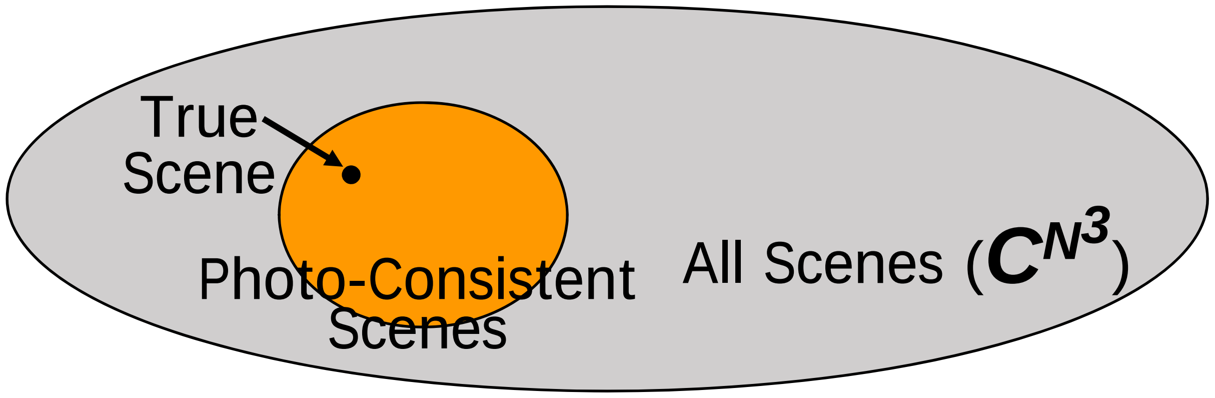
Discretized

Scene Volume

N^3 voxels

C colors

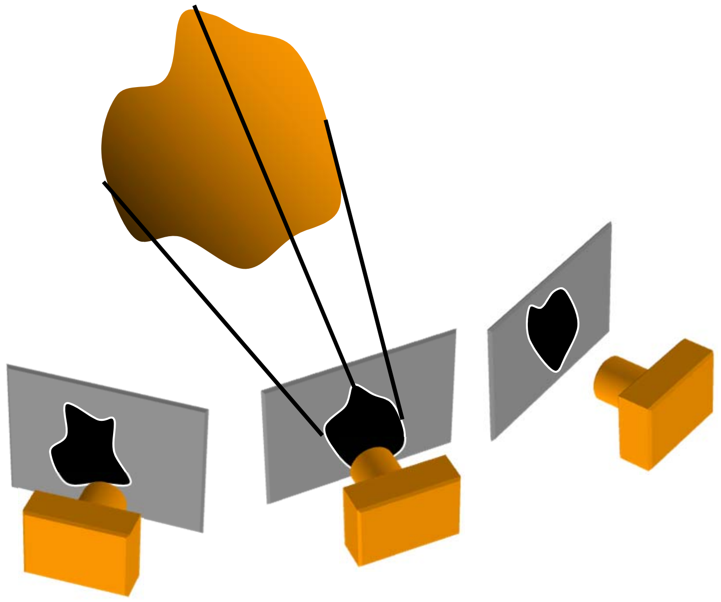




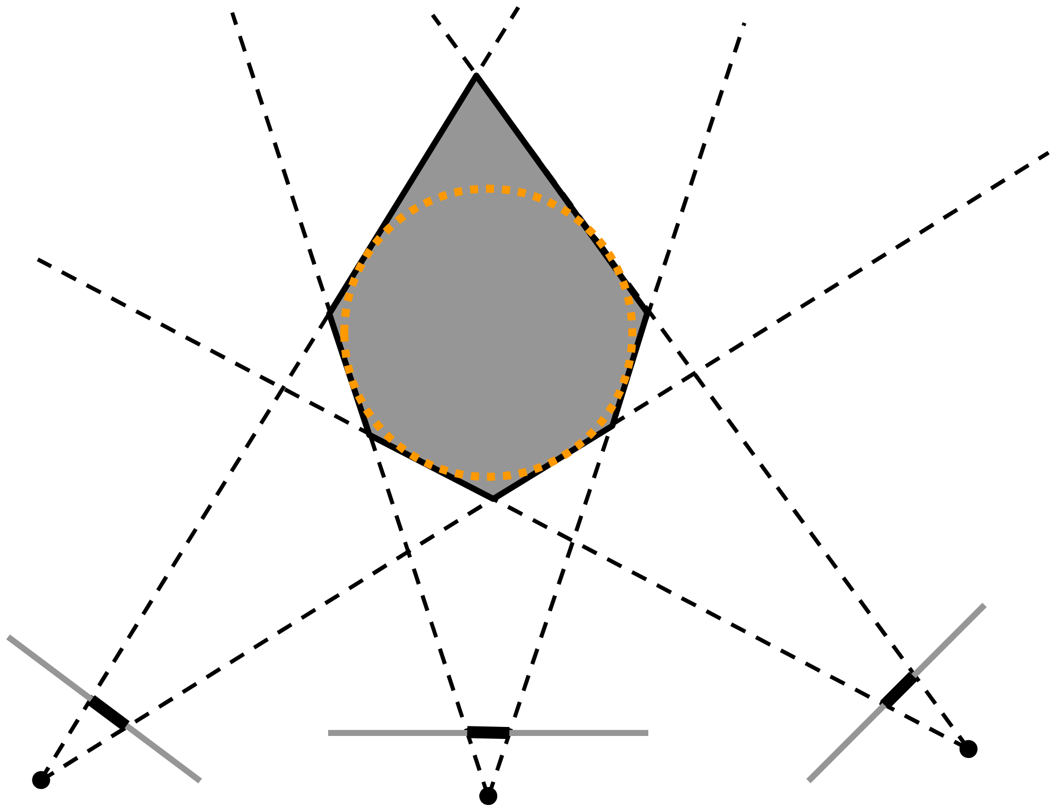
Reconstruction from Silhouettes (C=2)

- Approach:
  - *Back-project* each silhouette
  - Intersect back-projected volumes

Binary Images $\longrightarrow$



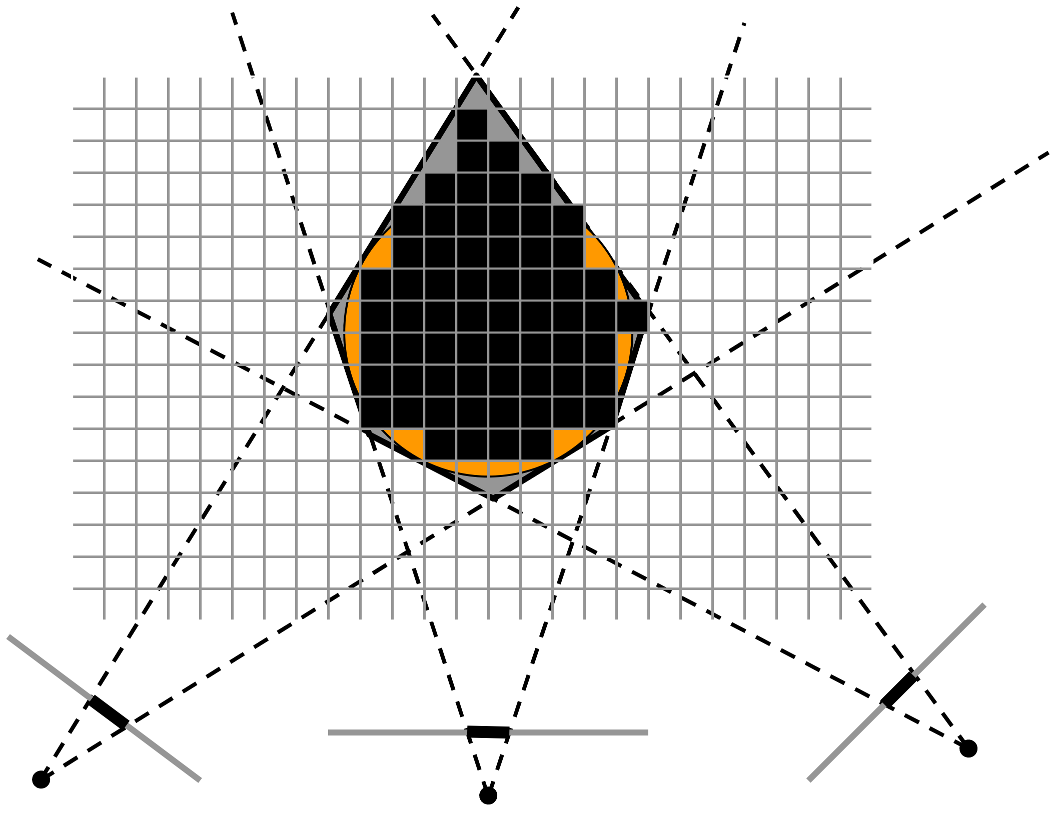
Volume Intersection



Reconstruction Contains the True Scene

- But is generally not the same
- In the limit (all views) we get *visual hull*
    - > Complement of all lines that do not intersect S

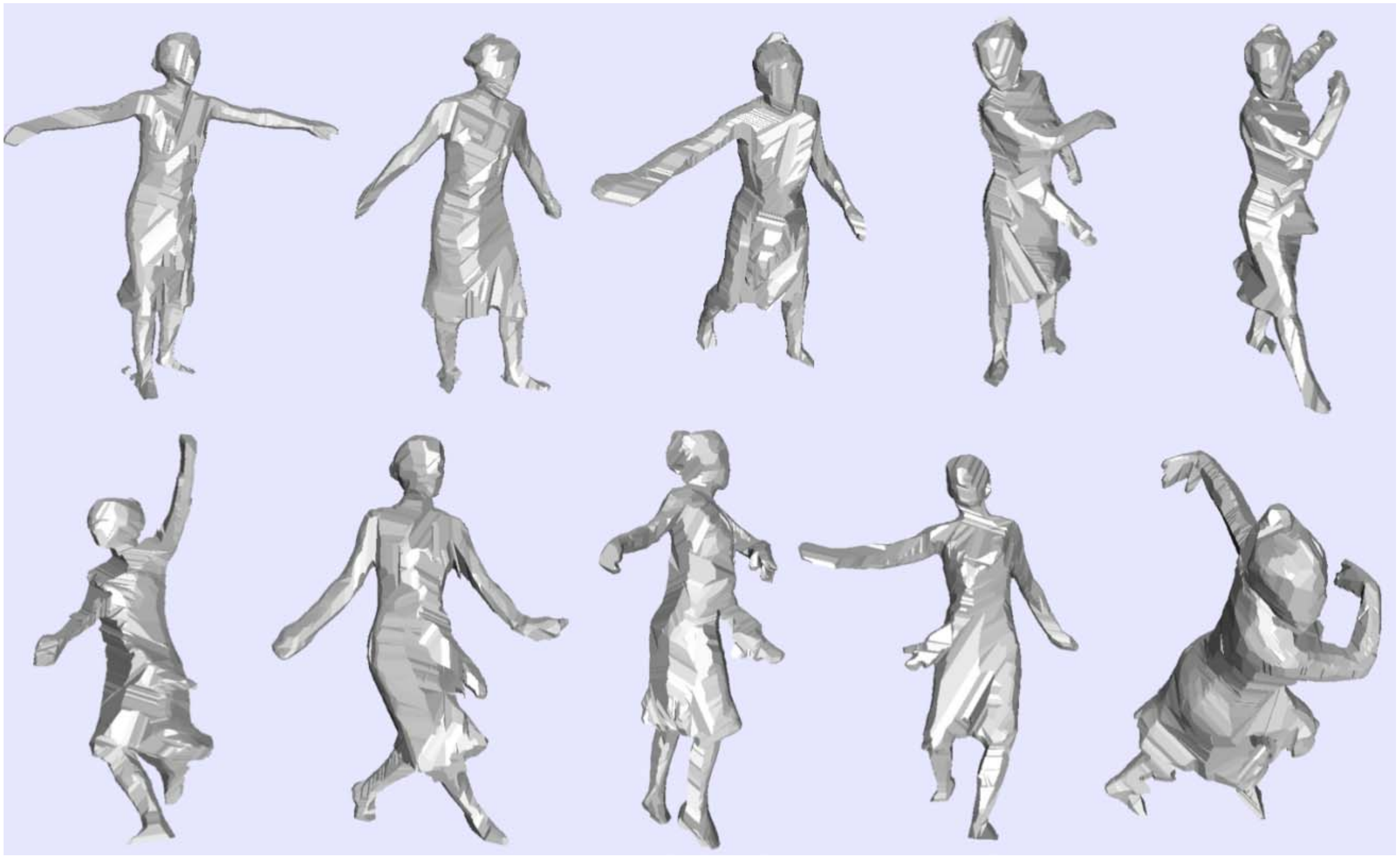
Voxel-based Algorithm



Color voxel black if on silhouette in every image

- O(?), for M images, N^3 voxels
- Don't have to search 2^{N^3} possible scenes!

Results (Franco and Boyer, PAMI 2009)



Properties of Volume Intersection

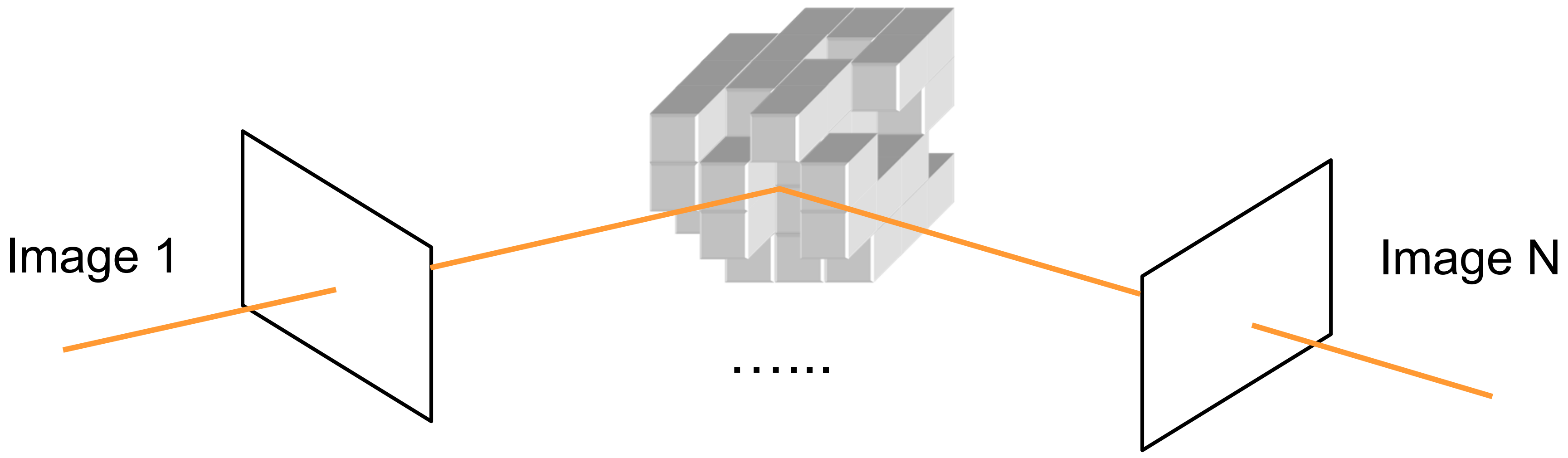
Pros

- Easy to implement, fast
- Accelerated via octrees

Cons

- No concavities
- Reconstruction is not photo-consistent
- Requires identification of silhouettes

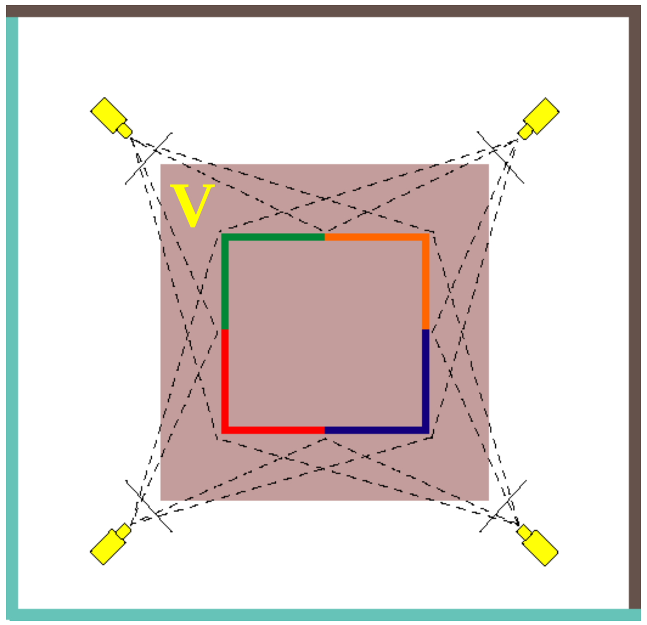
Space Carving



Space Carving Algorithm

- Initialize to a volume V containing the true scene
- Choose a voxel on the current surface
- Project to visible input images
- Carve if not photo-consistent
- Repeat until convergence

Which Shape do You Get?



True Scene

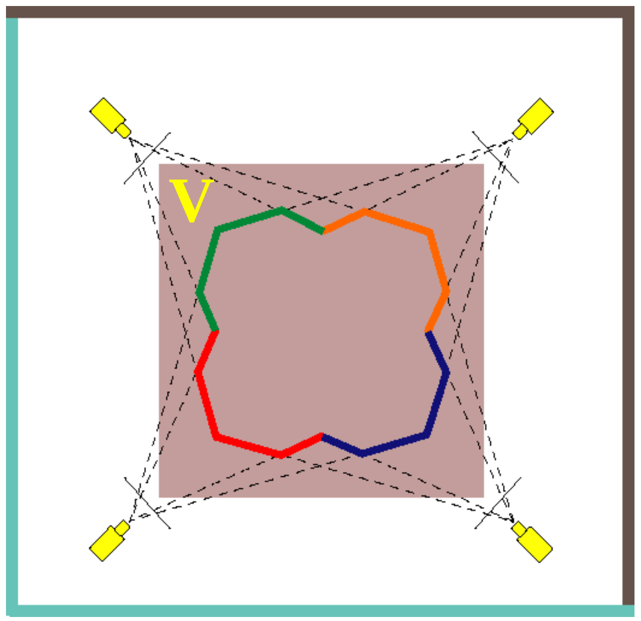
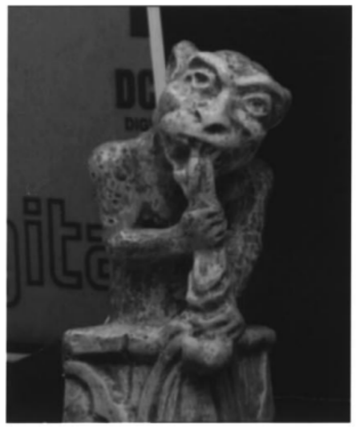


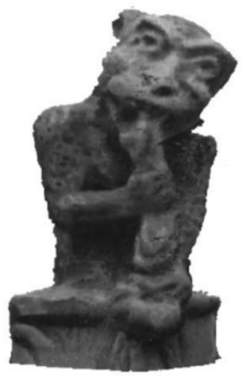
Photo Hull

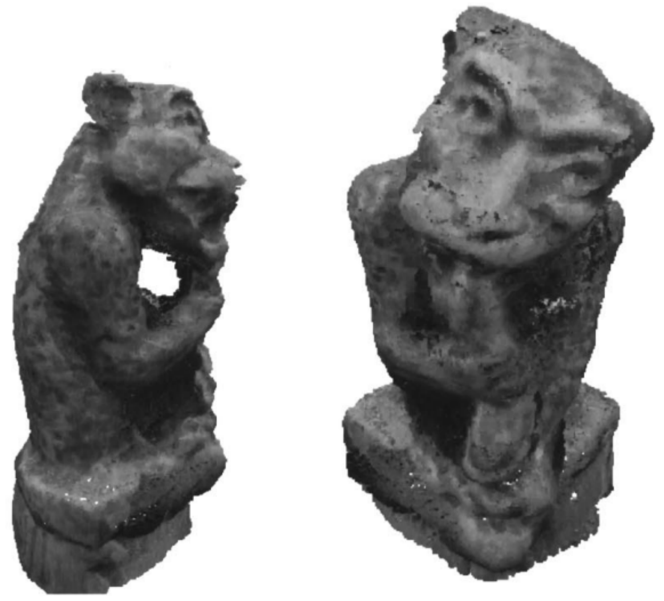
The Photo Hull *is the UNION of all photo-consistent scenes in V*

- It is a photo-consistent scene reconstruction
- Tightest possible bound on the true scene

Results (Kutulakos and Seitz, IJCV 2000)


(a)


(b)

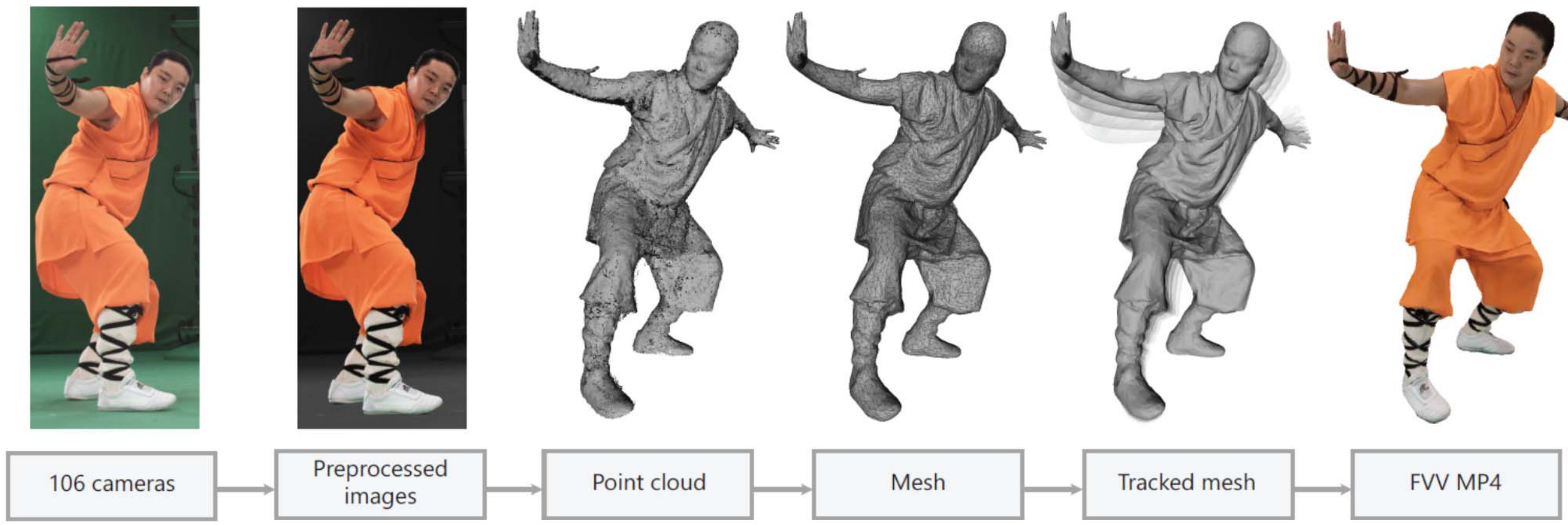


Free-Viewpoint Video

Alvaro Collet, Ming Chuang, Pat Sweeney,
Don Gillett, Dennis Evseev, David Calabrese,
Hugues Hoppe, Adam Kirk, Steve Sullivan
Microsoft Corporation

SIGGRAPH 2015

Pipeline



Capture + Preprocessing

- Calibration
- Bias correction
- Background subtraction

Point Generation

- Depth maps (IR, RGB, SfS)
- Multimodal MVS
- Iterative surface estimation (point-based)

Meshing + Texturing

- Topology denoising
- Island removal
- Hull-constrained PSR
- Occlusion detection
- Texturing

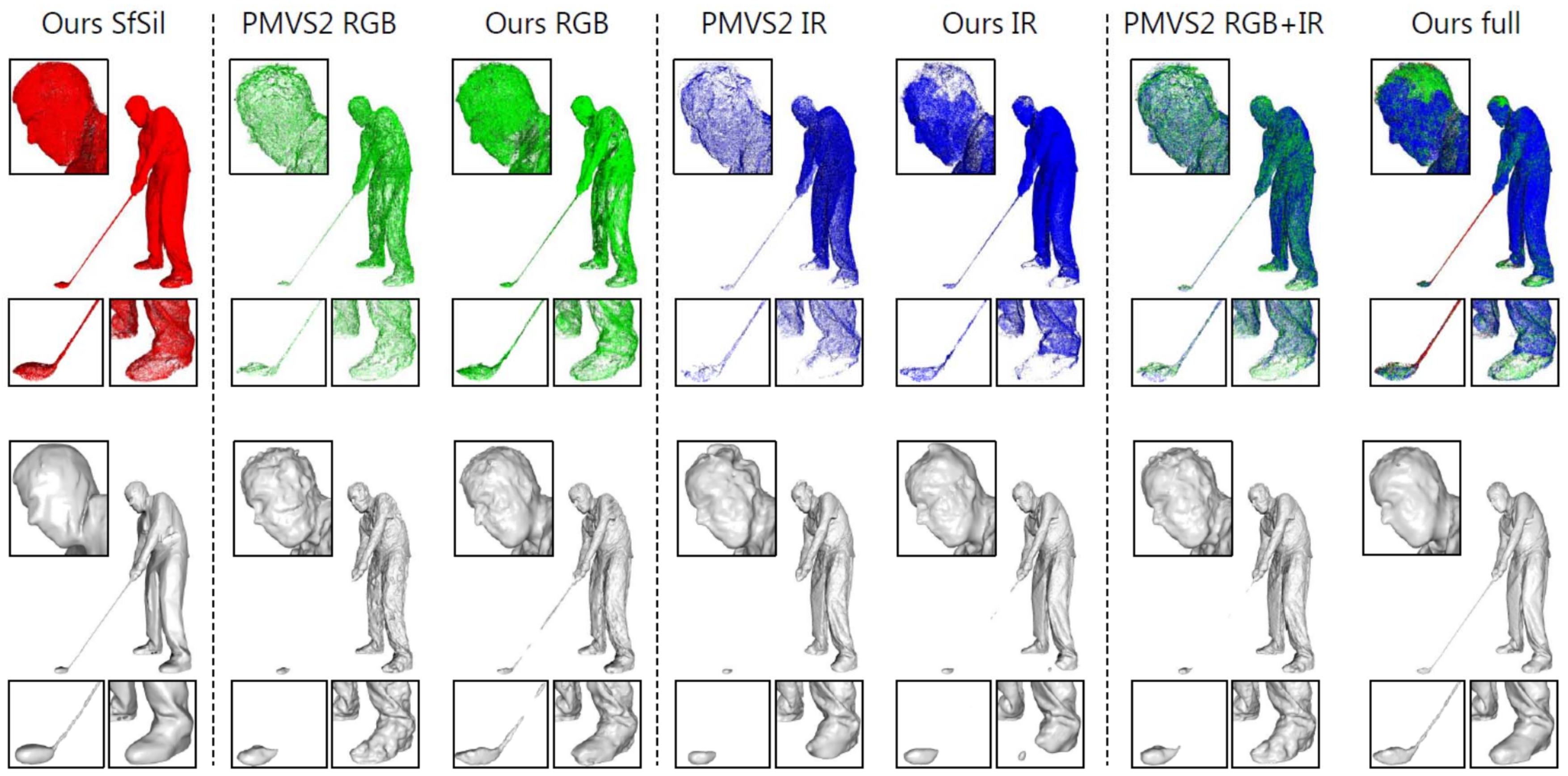
Temporal Mesh Processing

- Mesh tracking
- Importance detection
- Mesh decimation
- Texture atlas generation

Compression + Encoding

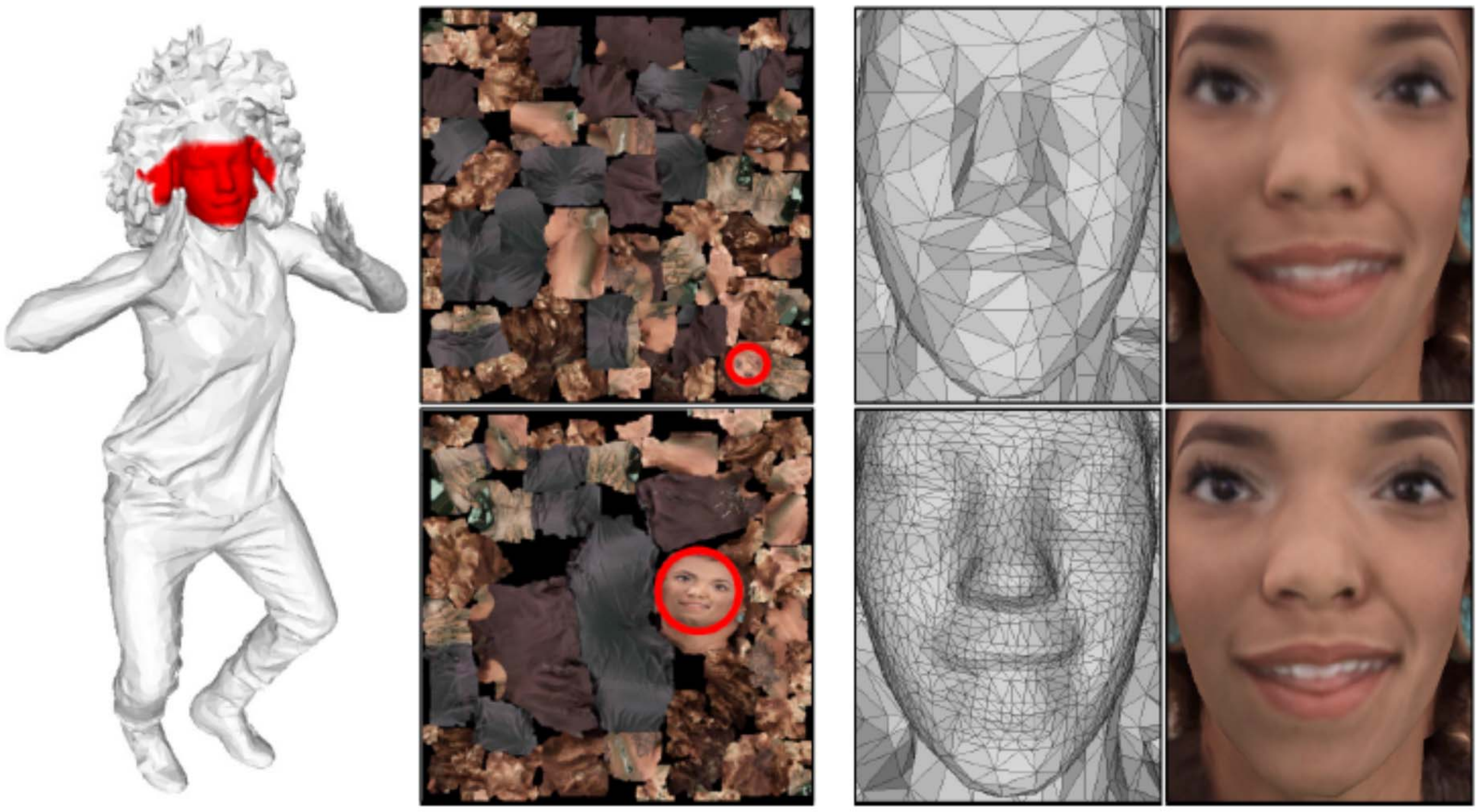
- Video encode H.264
- Audio encode
- Mesh compression

Reconstruction

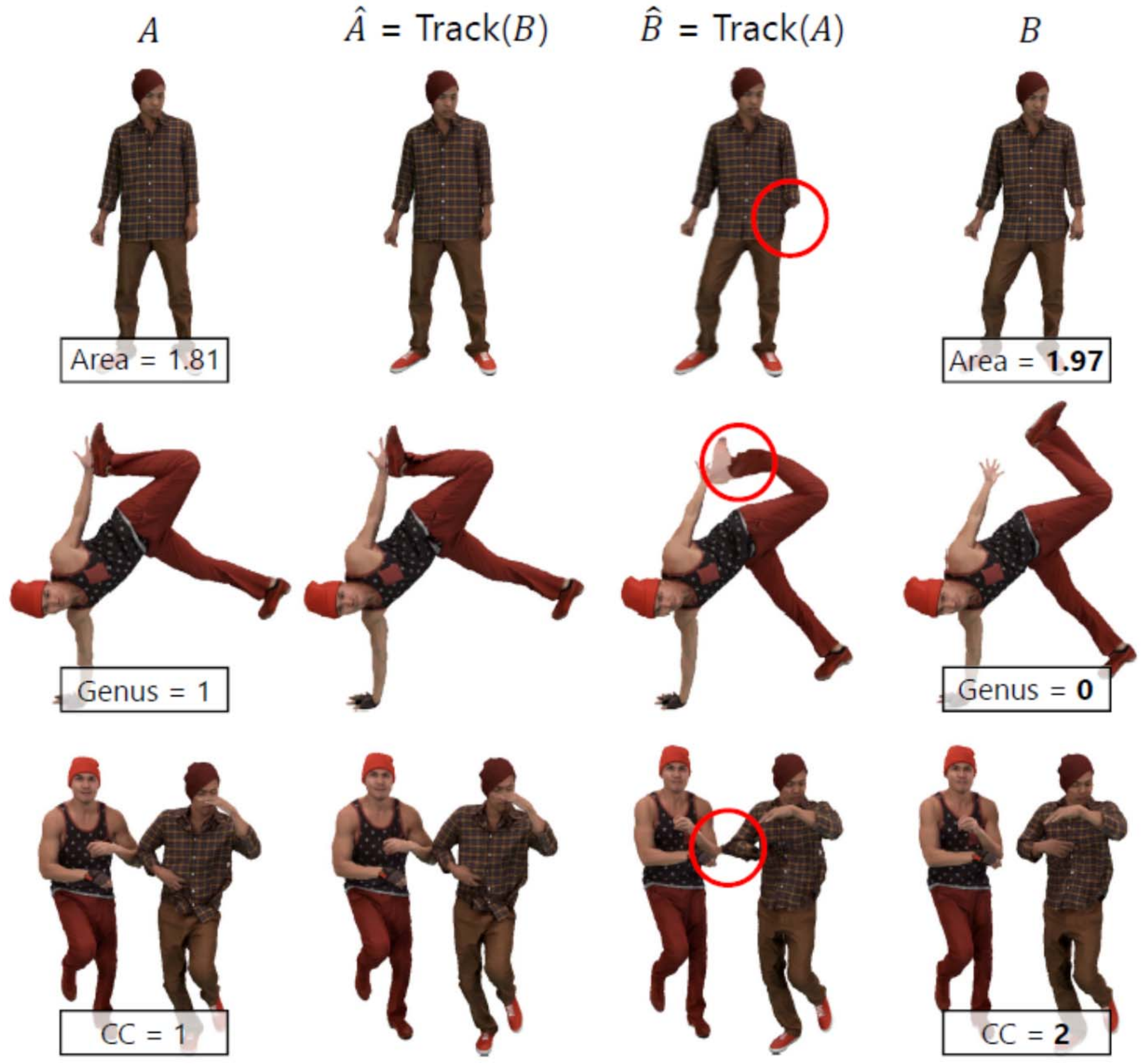


PMVS: system by Furukawa and Ponce, we show last week
PSR: Poisson Surface Reconstruction (last slide last week)

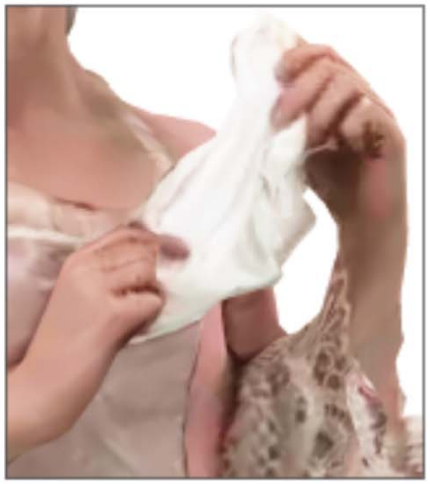
Adaptive Level of Detail



Keyframe-based Mesh Tracking



Synthesized Viewpoints



Synthesized Viewpoints

Computational Geometry

Computational Geometry

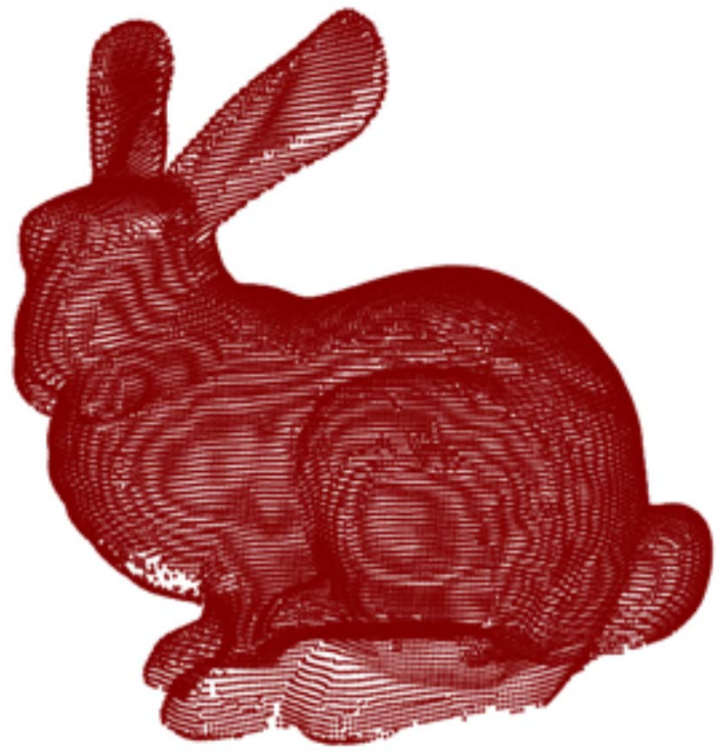
- Subfield of the *Design and Analysis of Algorithms*
- Deals with efficient data structures and algorithms for geometric problems
- Only about 30 years old

Surface Reconstruction

- Digitizing 3D objects, such as the *Stanford Bunny*

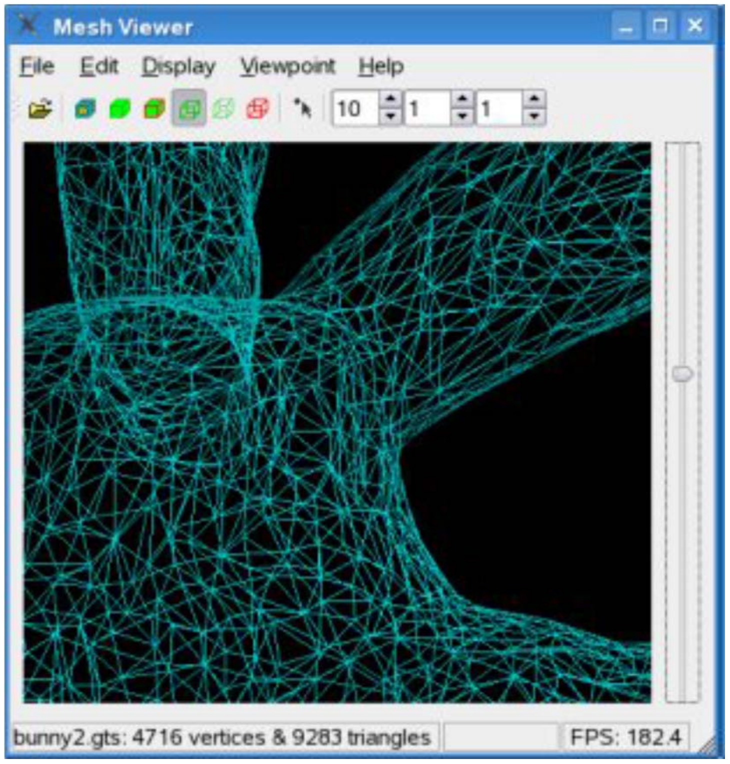
Surface Reconstruction

- Step 1: scan the object with a laser scanner to obtain set of points inR^3



Surface Reconstruction

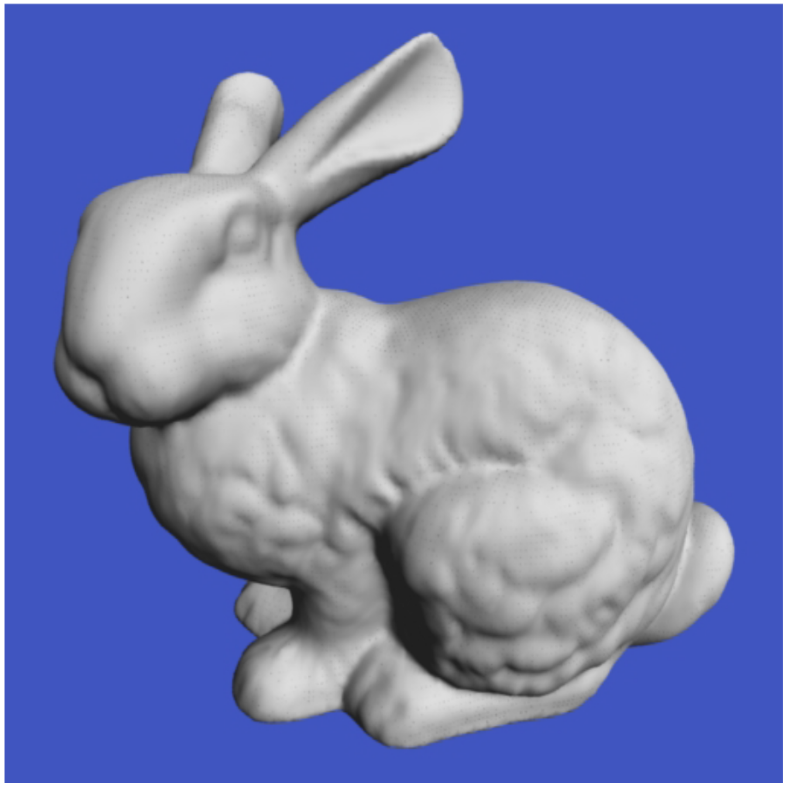
- Step 2: create a triangulation to obtain set of triangles in R^3



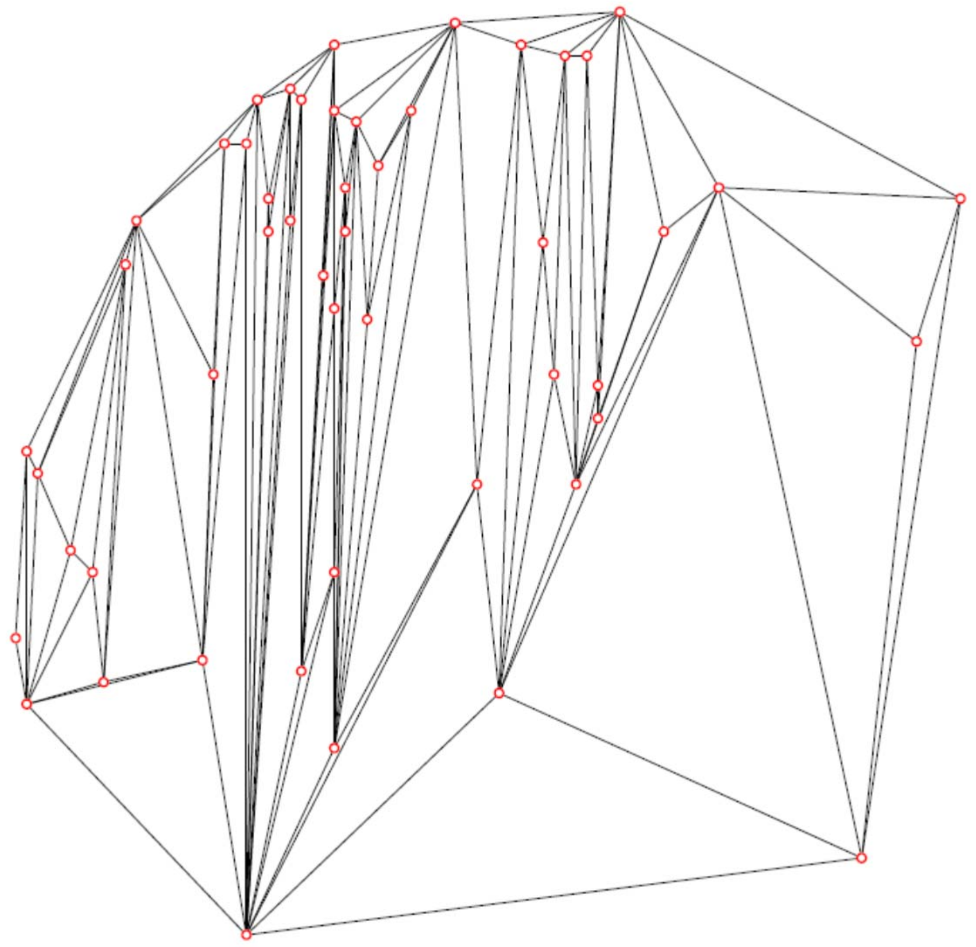
bunny2.gts: 4716 vertices & 9283 triangles
FPS: 182.4

Surface Reconstruction

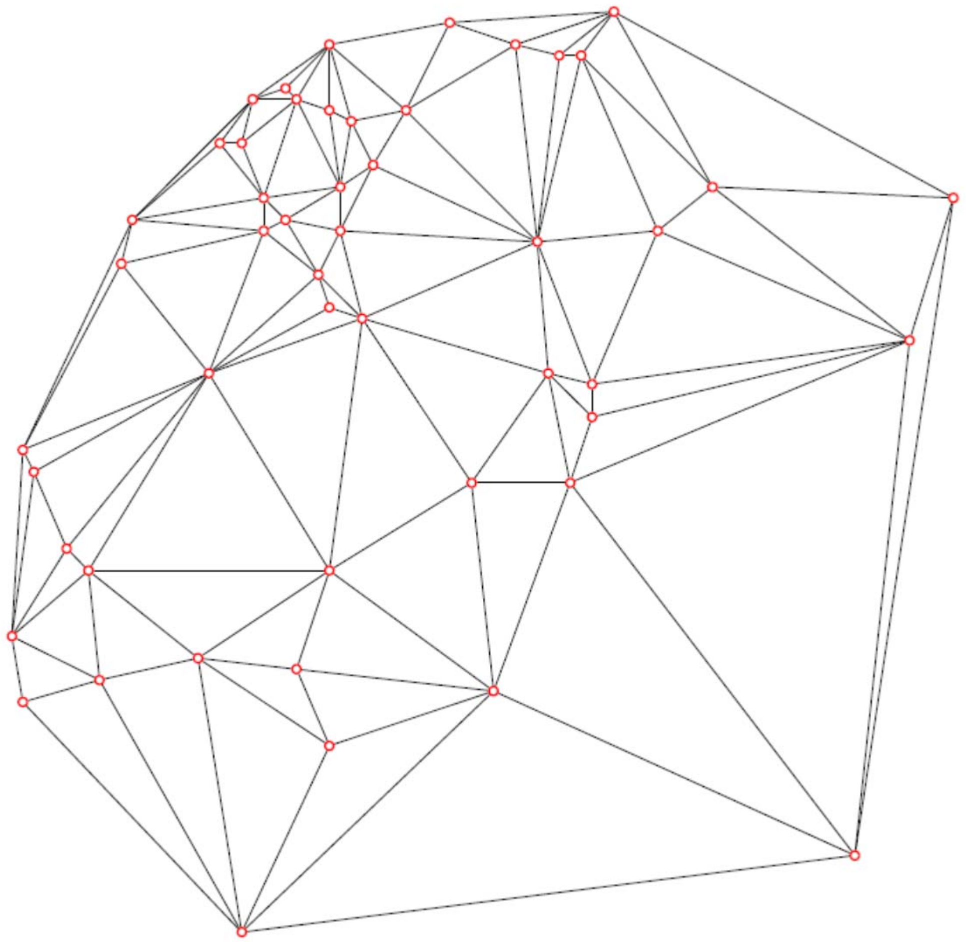
- Step 3: process the triangulation to obtain smooth surface in R^3



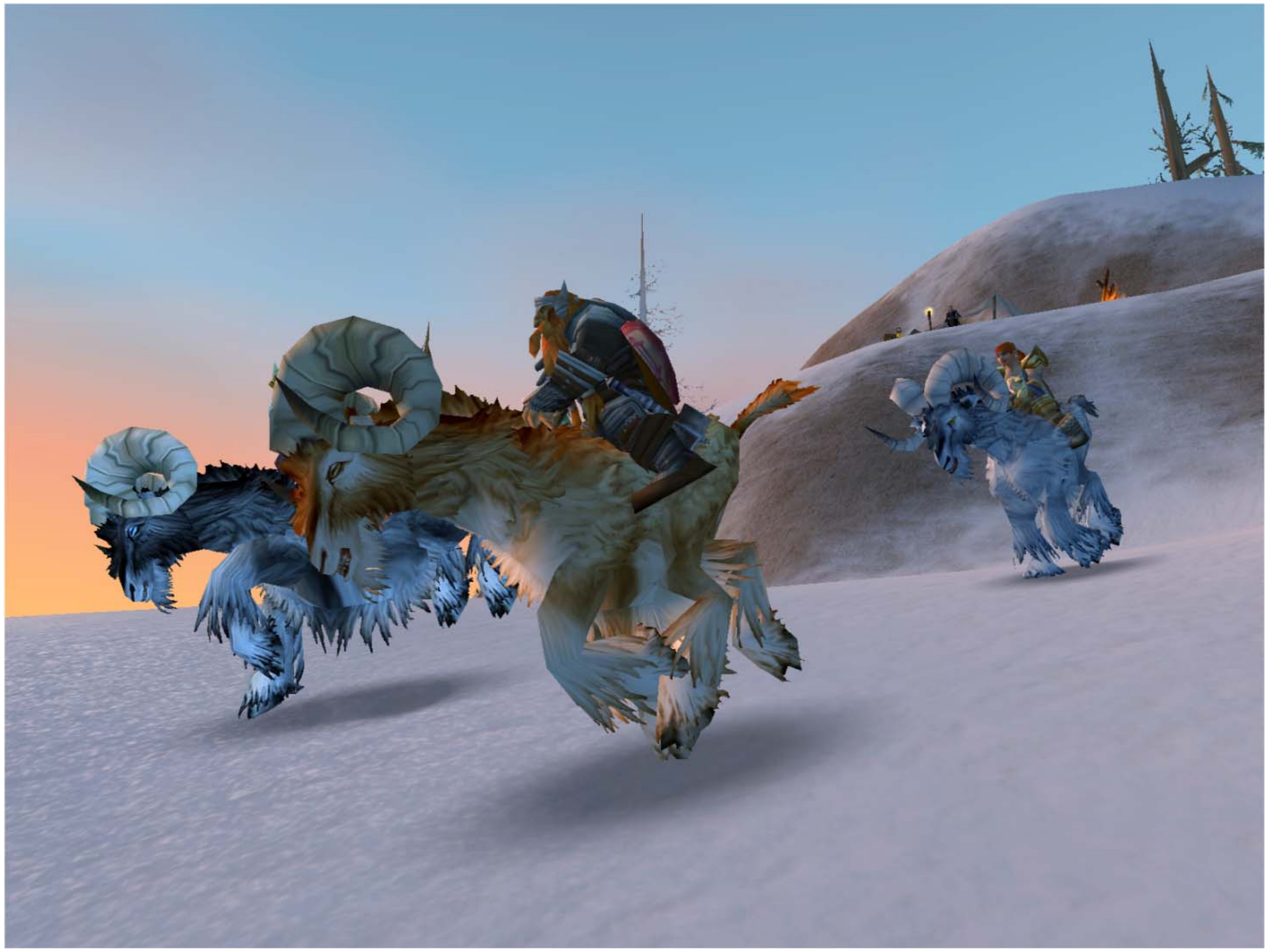
Good and Bad Triangulations in R^2



Good and Bad Triangulations in R^2



Collision Detection

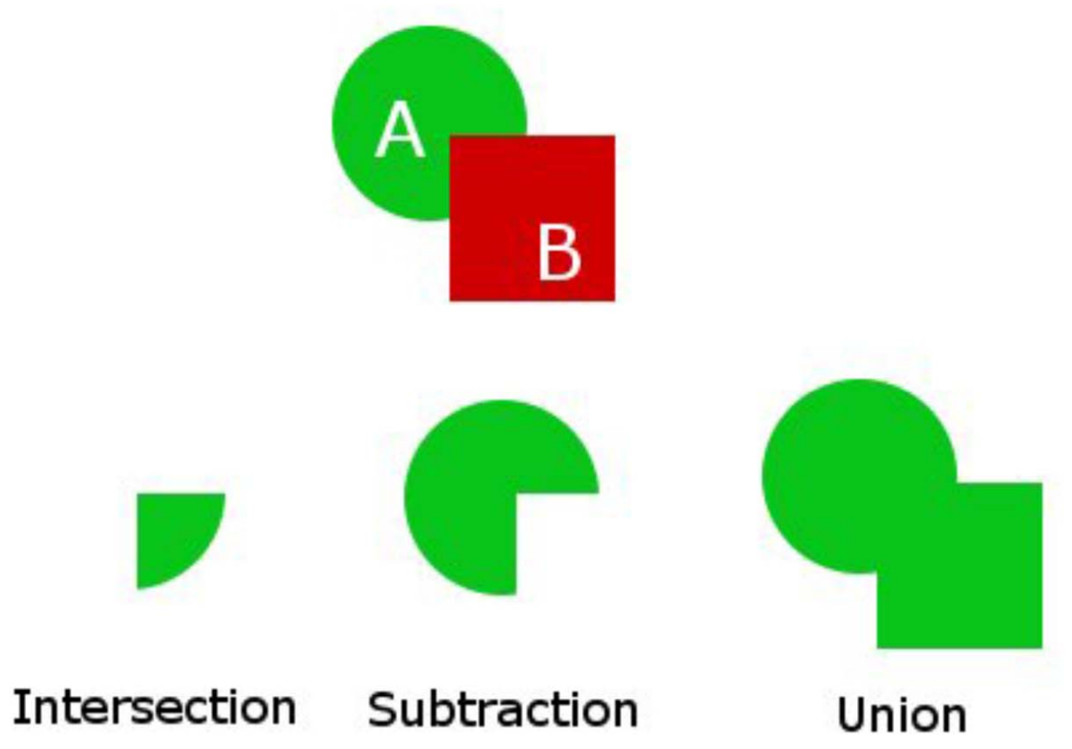


Collision Detection

- Bounding volume heuristic:
  - Approximate the objects by simple ones that enclose them (bounding volumes)
  - popular bounding volumes: boxes, spheres, ellipsoids,...
  - if bounding volumes don’t intersect, the objects don’t intersect, either
  - *only* if bounding volumes intersect, apply more expensive intersection test(s)

Boolean Operations

- Given two shapes, compute their



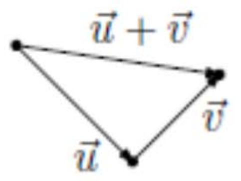
Geometric Basics

Mount – Lecture 21

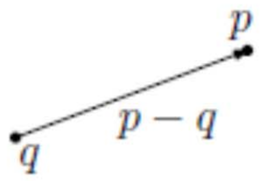
Affine Geometry

- Scalars
- Points
- Free vectors or vectors
  - In contrast to linear algebra where there is no distinction between points and vectors

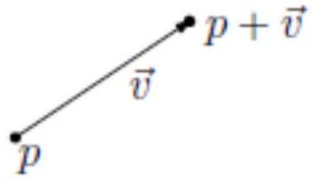
$S \cdot V$	$\rightarrow$	V	scalar-vector multiplication
$V + V$	$\rightarrow$	V	vector addition
$P - P$	$\rightarrow$	V	point subtraction
$P + V$	$\rightarrow$	P	point-vector addition



vector addition



point subtraction



point subtraction

Affine Geometry

- Can easily derive
  - Vector subtraction
  - Scalar-vector division
- Cannot derive
  - Point-scalar multiplication
  - Point addition
- We can define *affine combination*

$$\mathrm{aff}(p_0, p_1; \alpha_0, \alpha_1) = \cancel{\alpha_0 p_0 + \alpha_1 p_1} = p_0 + \alpha_1(p_1 - p_0).$$

Why?

Convex Combination

- If $0 \le a_0, a_1 \le 1$, then the operation is called convex combination
- Set of all convex combinations traces out line segment

$$\operatorname{aff}(p_0, p_1, p_2; \alpha_0, \alpha_1, \alpha_2) = \alpha_0 p_0 + \alpha_1 p_1 + \alpha_2 p_2 = p_0 + \alpha_1 (p_1 - p_0) + \alpha_2 (p_2 - p_0).$$

- Set of all affine combinations of three non-collinear points generates plane
  - *Affine span or affine closure*
- Set of all convex combinations of three non-collinear points generates triangle
  - *Convex closure*

Euclidean Geometry

- Extension of affine geometry that includes inner product
- Maps two real vectors (not points) into real scalar
  - Define length of vector as square root of inner product with itself
  - Normalize vector by dividing with its length

Distance and Angle

- Distance between points: length of the vector between them
- Angle between two non-zero vectors, ranging from 0 to π

$$\operatorname{ang}(\vec{u}, \vec{v}) = \cos^{-1}\left(\frac{\vec{u} \cdot \vec{v}}{||\vec{u}||\,||\vec{v}||}\right) = \cos^{-1}(\hat{u} \cdot \hat{v}).$$

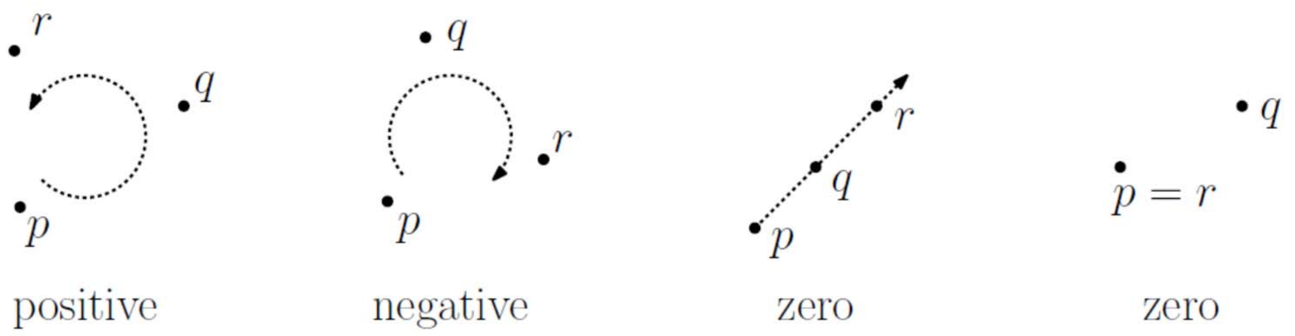
Orientation of Points

- To make discrete decisions, we would like a geometric operation on points that is analogous to the relational operations (<,=,>) with numbers.
- There does not seem to be any natural intrinsic way to compare two points in d-dimensional space,
- but there is a natural relation between ordered (d + 1)-tuples of points in d-space, which extends the notion of binary relations in 1-space:

➢ *orientation*

Orientation of Points

- Given an ordered triple of points <p,q,r>
- Positive orientation if they define counterclockwise triangle
- Negative orientation if they define clockwise triangle
- Zero orientation if they are collinear



Orientation of Points

- Formally the sign of the determinant of the points in homogeneous coordinates

$$\mathrm{Orient}(p,q,r) = \det \begin{pmatrix} 1 & p_x & p_y \\ 1 & q_x & q_y \\ 1 & r_x & r_y \end{pmatrix}$$

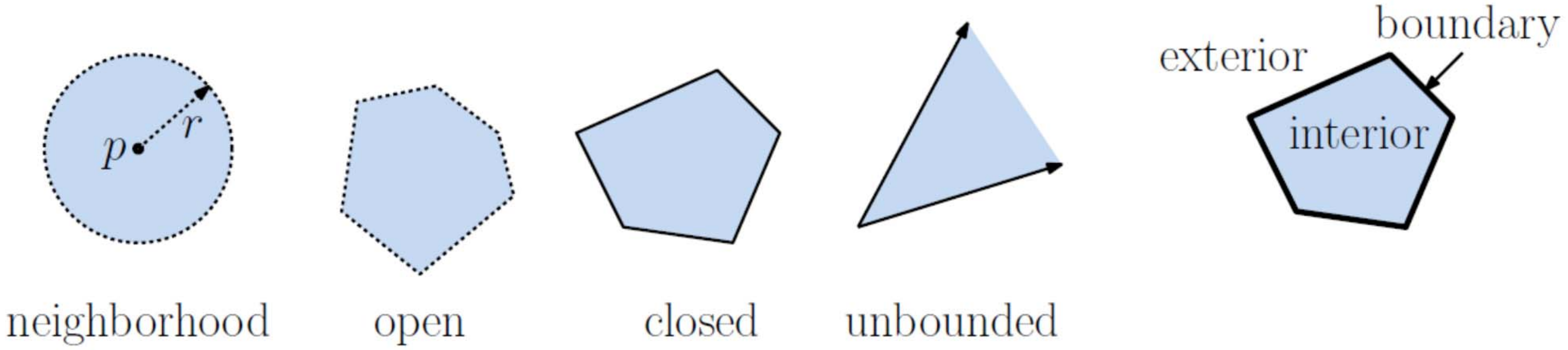
- 1D case: Orient(p,q)=q-p !!
- Orientation is invariant to translation, rotation and scaling by positive scale
- Sign is reversed by reflection e.g. f(x,y)=(-x,y)

Areas and Angles

- The signed area of triangle is ½ of the determinant
  - Can be extended to any dimension
  - Volume is equal to determinant divided by d!
- The sine of the signed angle from vector p-q to r-q
  - Can get cosine from inner product

$$\sin\theta = \frac{\text{Orient}(q, p, r)}{\|p - q\| \cdot \|r - q\|}.$$

Informal Topology



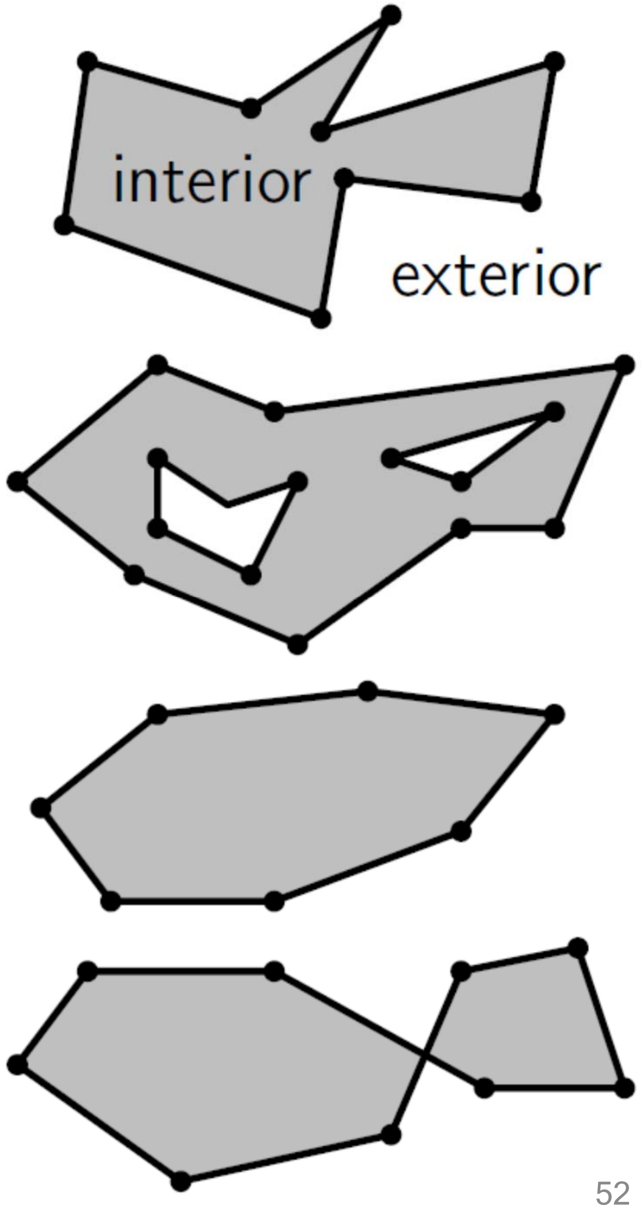
Convex Hulls

D. Mount – Lecture 3

M. Van Kreveld slides

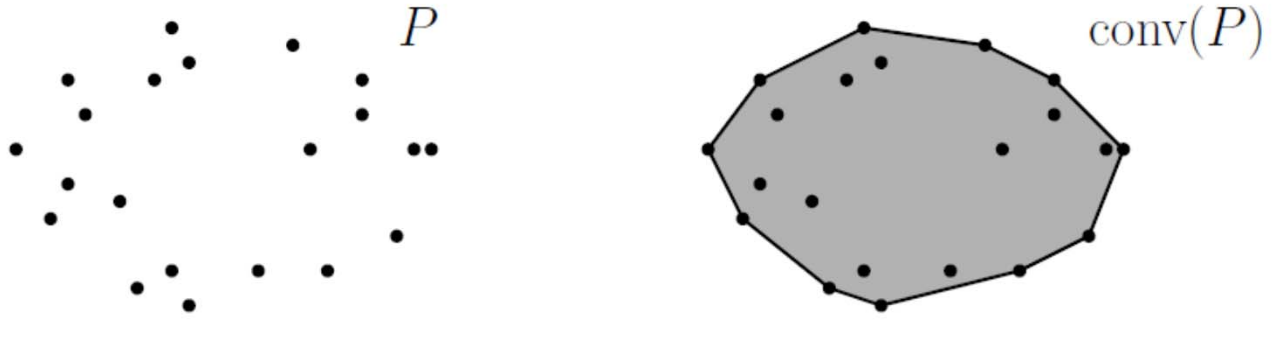
Polygons

- simple polygon
- polygon with holes
- convex polygon
- non-simple polygon



Convex Hulls

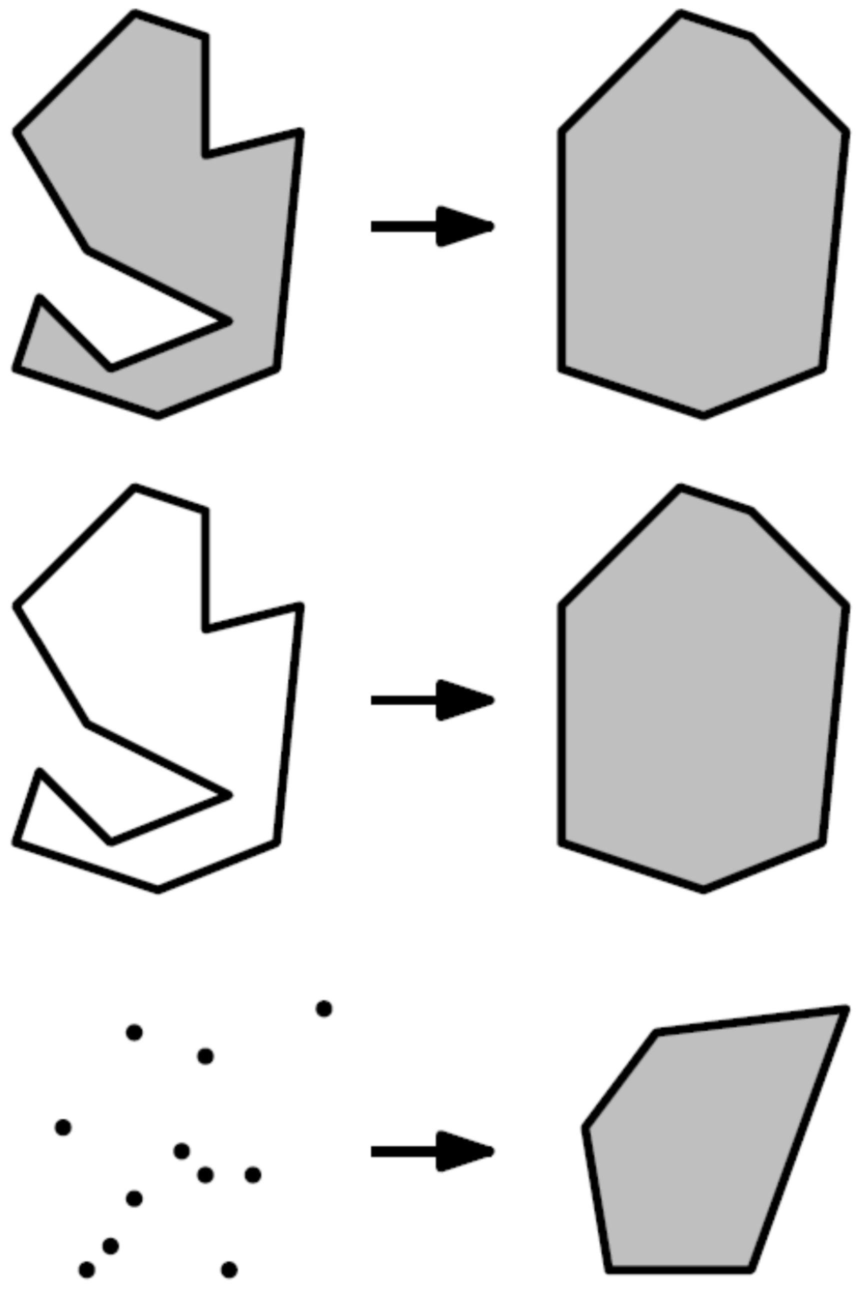
- Simple approximation for set of points
  - Tighter than bounding box, circle or ellipse
- **Convexity:** A set K is *convex* if given any points p, q ∈ K, the line segment pq is entirely contained within K.
- **Convex hull:** The *convex hull* of any set P is the intersection of all convex sets that contains P, or more intuitively, the smallest convex set that contains P.
  - We will denote it by conv(P).



Convex Hull Problem

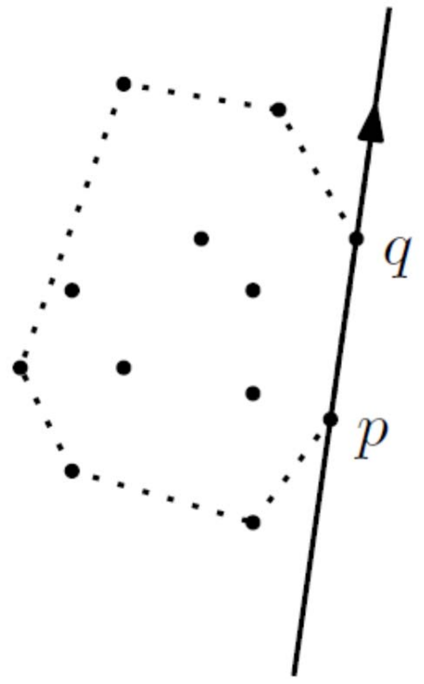
- Given a set of n points P in the plane, output a representation of P's convex hull.
- The simplest representation is a counterclockwise enumeration of the vertices of the convex hull.
  - Although points of P might lie in the interior of an edge of the boundary of the convex hull, such a point is not considered a vertex.
  - We will assume that the points are in *general position*, and in particular, no three are collinear. Then, this issue does not arise.
- Although the output consists only of the boundary of the hull, the convex hull includes both the boundary and interior of the polygon

Illustration



Developing an Algorithm

- Property: The vertices of the convex hull are always points from the input
- Consequently, the edges of the convex hull connect two points of the input
- Property: The supporting line of any convex hull edge has all input points to one side



Slow Algorithm

```
Algorithm SLOWCONVEXHULL($P$)
Input. A set $P$ of points in the plane.
Output. A list $\mathcal{L}$ containing the vertices of $CH(P)$ in clockwise
   order.
1.  $E \leftarrow \emptyset$.
2.  for all ordered pairs $(p,q) \in P \times P$ with $p$ not equal to $q$
3.      do valid $\leftarrow$ true
4.         for all points $r \in P$ not equal to $p$ or $q$
5.             do if $r$ lies left of the directed line from $p$ to $q$
6.                   then valid $\leftarrow$ false
7.         if valid then Add the directed edge $\vec{pq}$ to $E$
8.  From the set $E$ of edges construct a list $L$ of vertices of
    $CH(P)$, sorted in clockwise order.
```

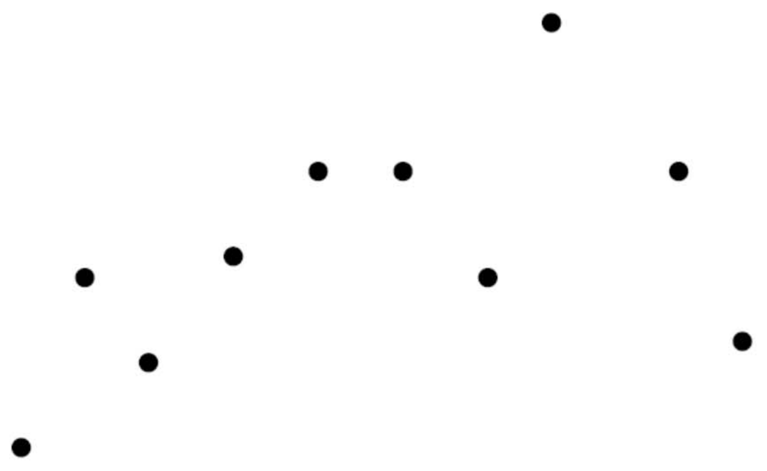
Complexity ?

Incremental Algorithm – Graham's Scan

- Incremental, from left to right
- First compute the upper boundary of the convex hull
  - property: on the upper hull, points appear in x-order
- Main idea: Sort the points from left to right
- Insert the points in this order, and maintain the upper hull
- Then complete the lower hull

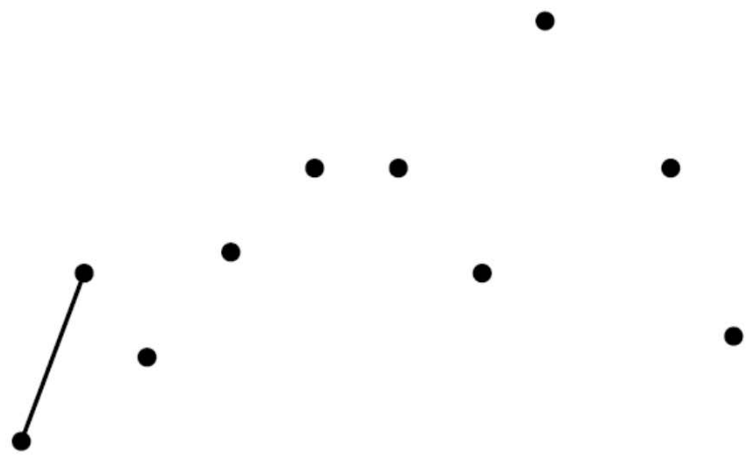
Graham’s Scan

Observation: from left to right, there are only right turns on the upper hull



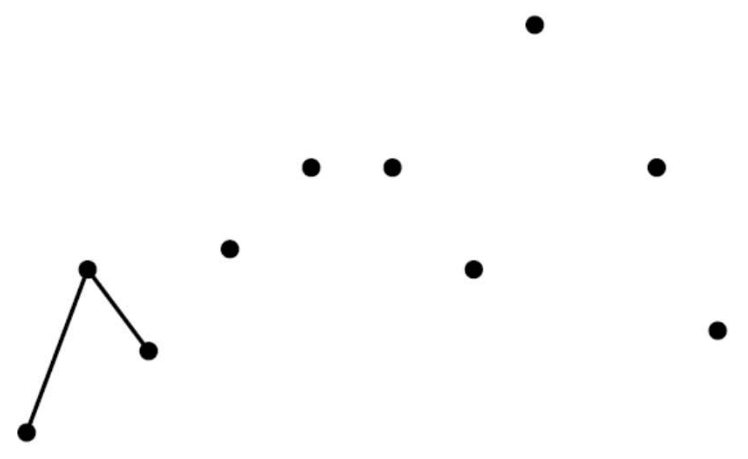
Graham's Scan

Initialize by inserting the leftmost two points



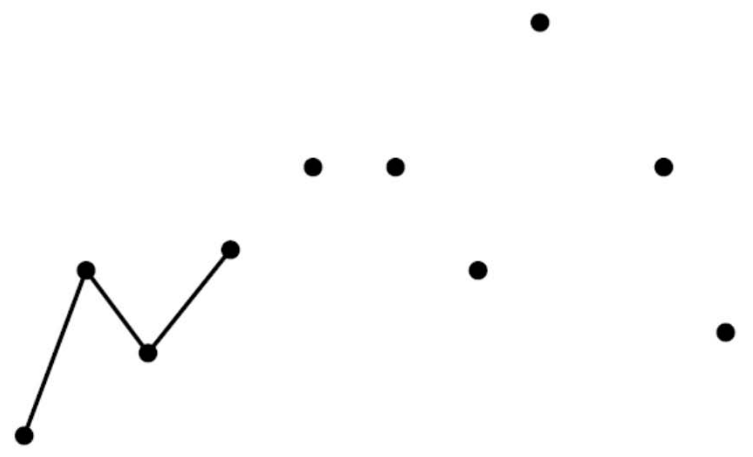
Graham's Scan

If we add the third point there will be a right turn at the previous point, so we add it



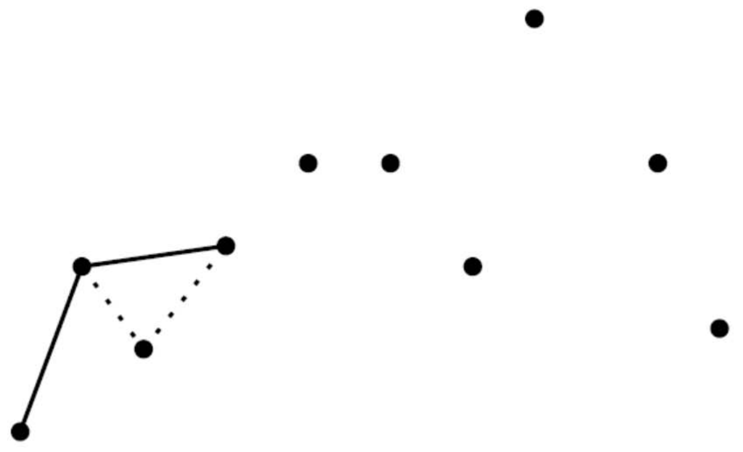
Graham's Scan

If we add the fourth point we get a left turn at the third point



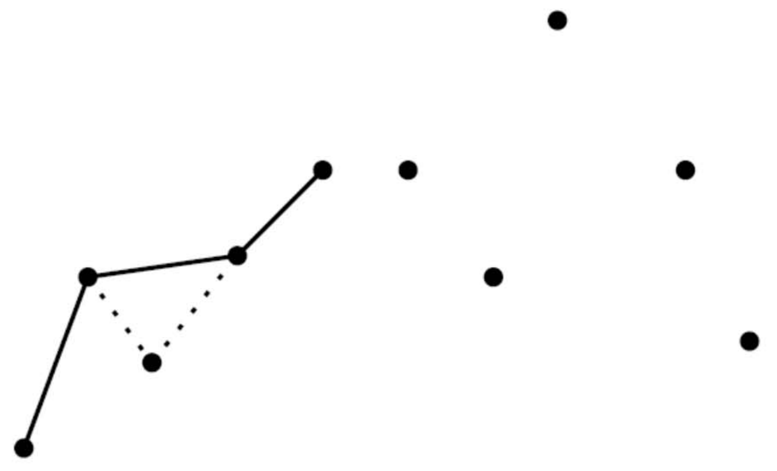
Graham’s Scan

... so we remove the third point from the upper hull when we add the fourth



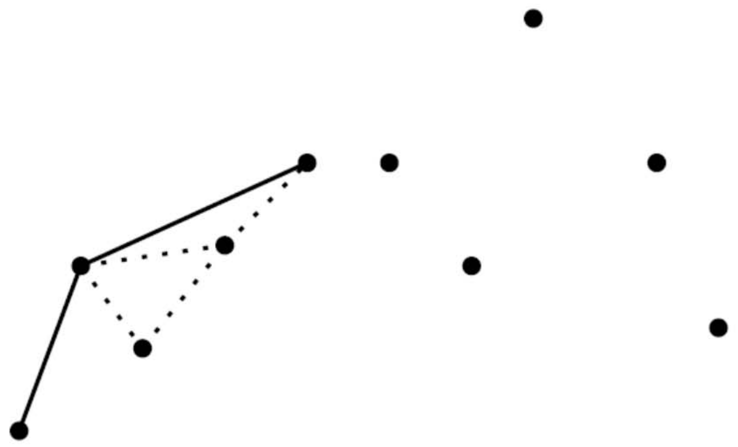
Graham's Scan

If we add the fifth point we get a left turn at the fourth point



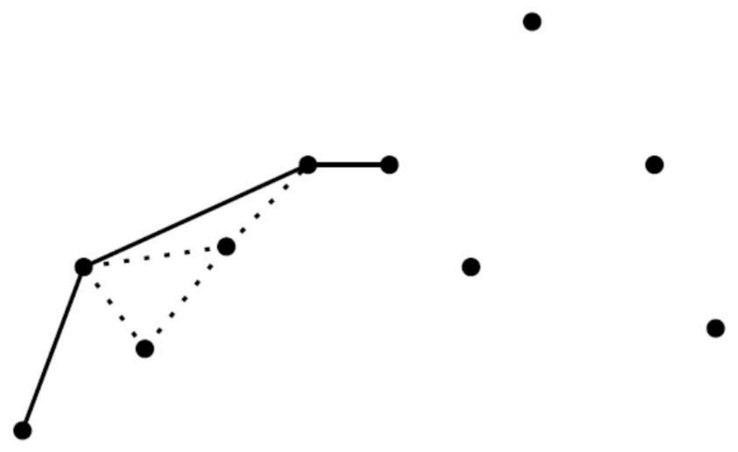
Graham’s Scan

. . . so we remove the fourth point when we add the fifth



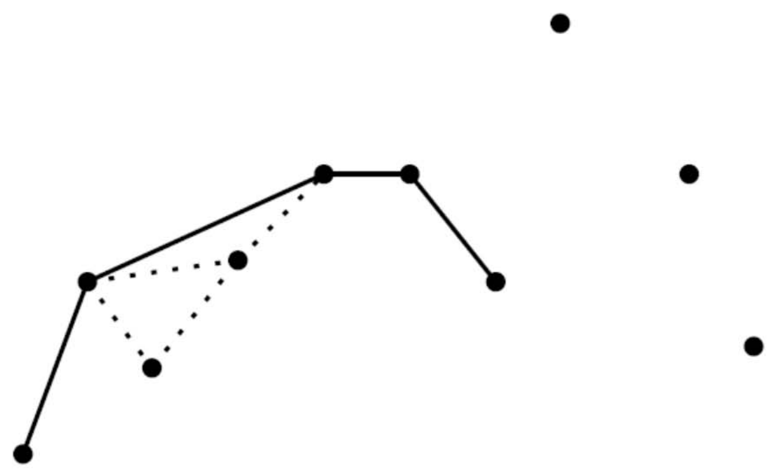
Graham's Scan

If we add the sixth point we get a right turn at the fifth point, so we just add it



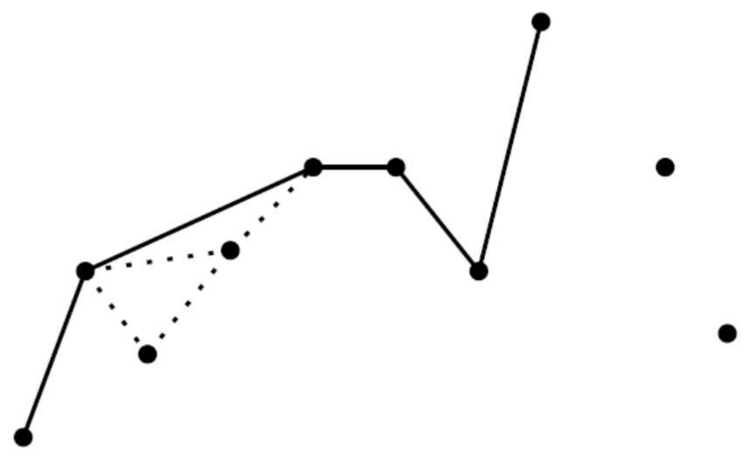
Graham’s Scan

We also just add the seventh point



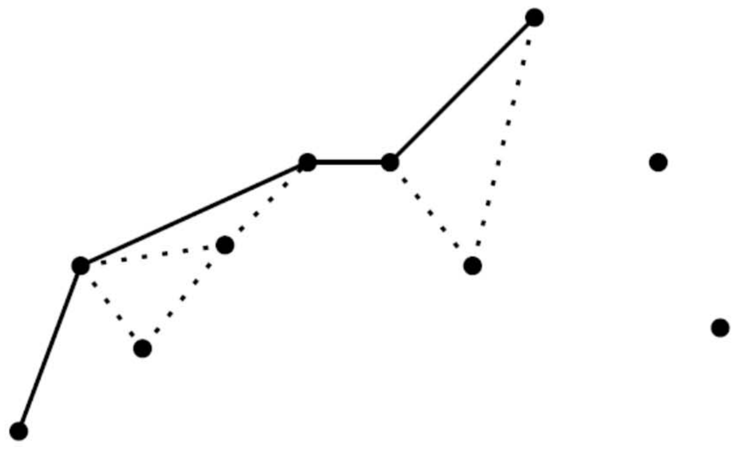
Graham's Scan

When adding the eight point
... we must remove the
seventh point



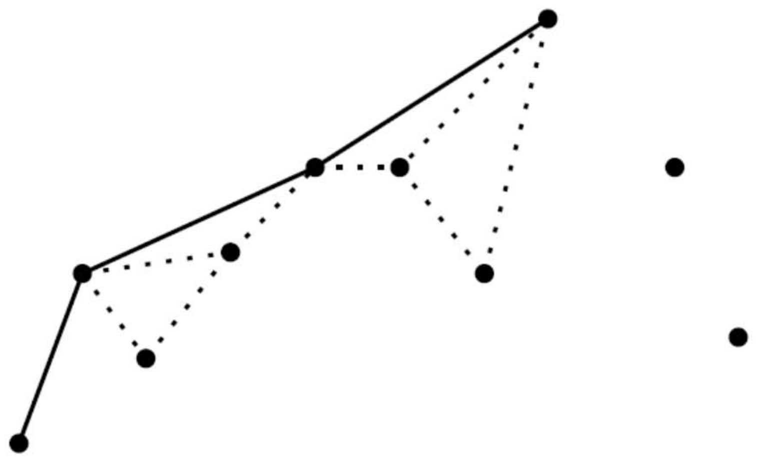
Graham’s Scan

. . . we must remove the seventh point



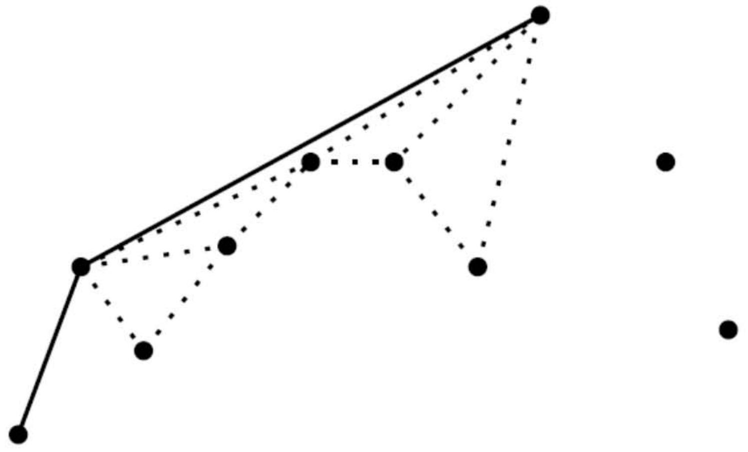
Graham’s Scan

. . . and also the sixth point



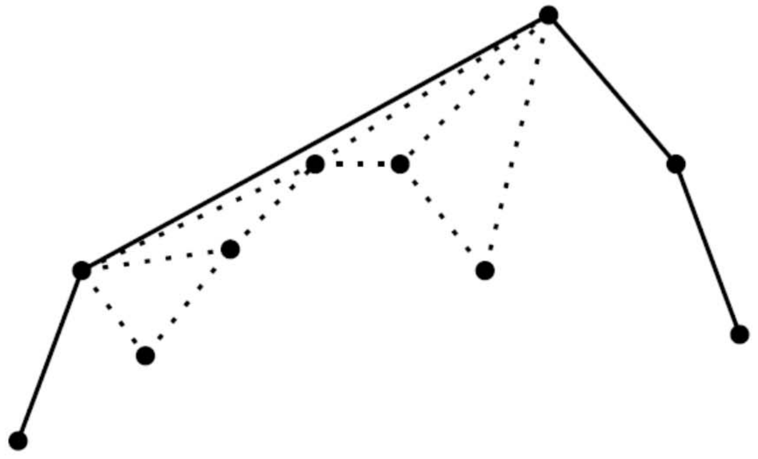
Graham’s Scan

… and also the fifth point



Graham’s Scan

After two more steps we get:



The Algorithm

```
Algorithm ConvexHull($P$)
Input. A set $P$ of points in the plane.
Output. A list containing the vertices of $CH(P)$ in clockwise order.
1.  Sort the points by $x$-coordinate, resulting in a sequence
    $p_1, \dots, p_n$.
2.  Put the points $p_1$ and $p_2$ in a list $L_{\text{upper}}$, with $p_1$ as the first
    point.
3.  for $i \leftarrow 3$ to $n$
4.      do Append $p_i$ to $L_{\text{upper}}$.
5.          while $L_{\text{upper}}$ contains more than two points and the
                 last three points in $L_{\text{upper}}$ do not
                 make a right turn
6.            do Delete the middle of the last three points from
                 $L_{\text{upper}}$.
```

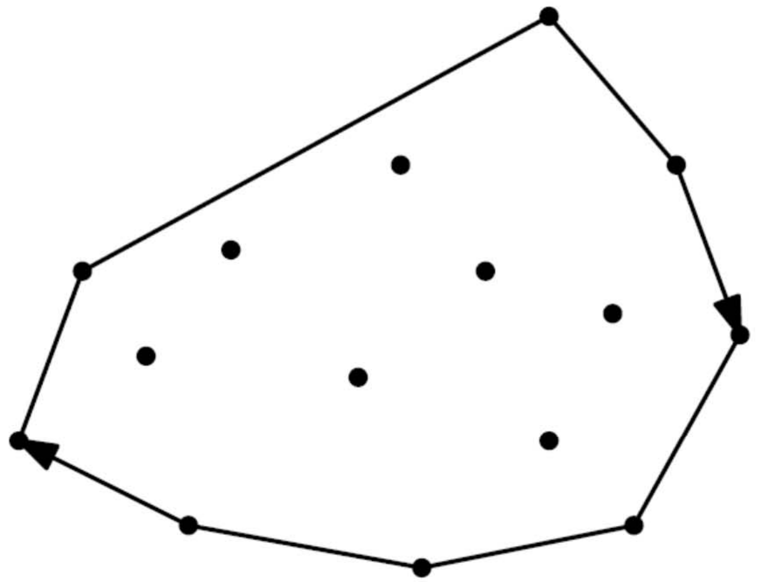
Lower Convex Hull

Then we do the same for the lower convex hull, from right to left

We remove the first and last points of the lower convex hull

... and concatenate the two lists into one

$p_1, p_2, p_{10}, p_{13}, p_{14}$



$p_{14}, p_{12}, p_8, p_4, p_1$

Correctness

- Does the sorted order matter if two or more points have the same x-coordinate?
- What happens if there are three or more collinear points, in particular on the convex hull?

Efficiency

- The sorting step takes O(nlogn) time
- Adding a point takes O(1) time for the adding-part. Removing points takes constant time for each removed point. If due to an addition, k points are removed, the step takes O(1+k) time
- Total time:

$$O(n \log n) + \sum_{i=3}^{n} O(1 + k_i)$$

- if k_i points are removed when adding p_i
- Since k_i = O(n), we get

$$O(n \log n) + \sum_{i=3}^{n} O(n) = O(n^2)$$

Efficiency

- Sometimes there are global arguments why an algorithm is more ecient than it seems, at first
- Global argument: each point can be removed only once from the upper hull
- This gives us the fact:

$$\sum_{i=3}^{n} k_i \leq n$$

- Hence:

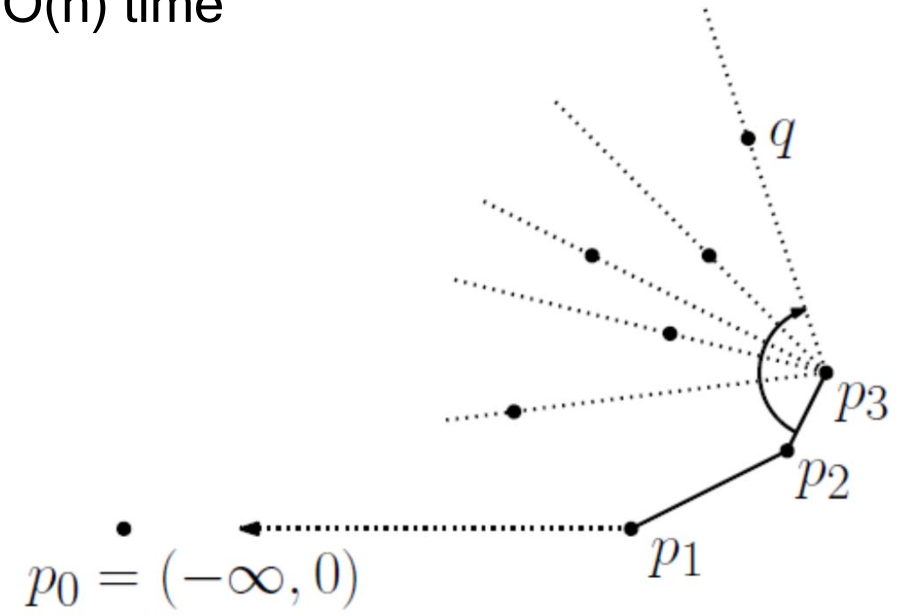
$$O(n \log n) + \sum_{i=3}^{n} O(1 + k_i) = O(n \log n) + O(n) = O(n \log n)$$

Jarvis’s March Algorithm

- Builds the convex hull in O(nh) time
  - h is the number of vertices in the output
- “Gift-wrapping” process
- Analogous to selection sort

Jarvis's March Algorithm

- Start with any one point on the convex hull, e.g. the lowest point
- Then find the "next" edge on the hull in counterclockwise order
  - Assuming that p_k and p_{k-1} were the last two points added to the hull, compute the point q that maximizes the angle $p_{k-1}p_kq$
  - q can be found in O(n) time



Jarvis’s March Algorithm

- If h is o(logn), Jarvis’s march is better than Graham’s algorithm
- Why?

Divide and Conquer

- Analogous to merge sort
- Sort points by x coordinate, then

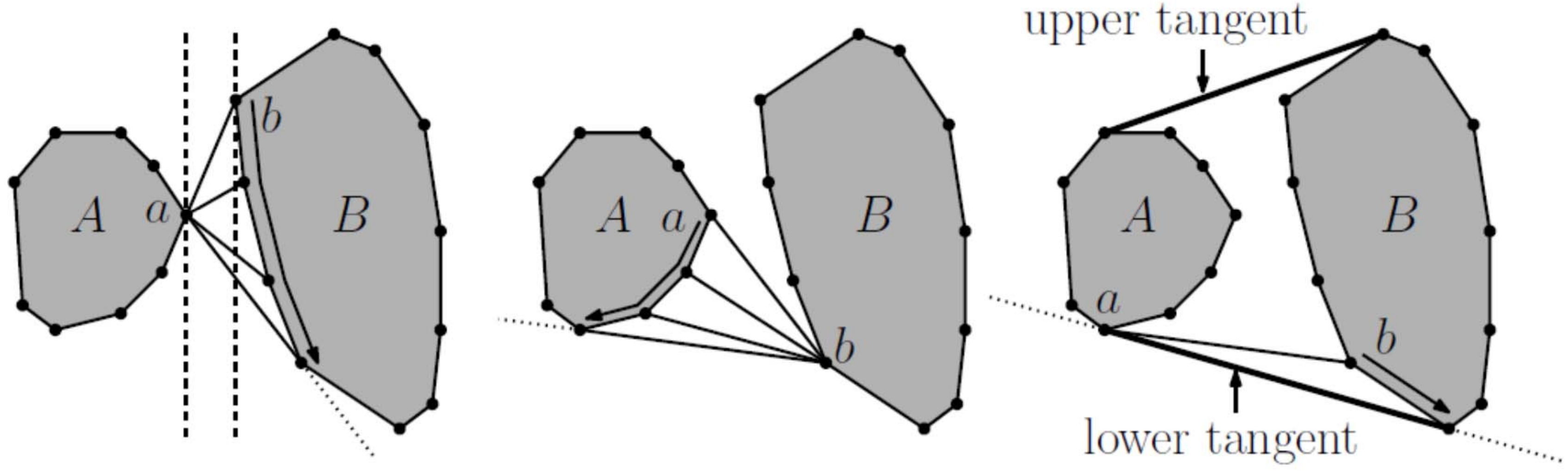
(1) If $|P| \leq 3$, then compute the convex hull by brute force in $O(1)$ time and return.

(2) Otherwise, partition the point set P into two sets A and B, where A consists of half the points with the lowest x-coordinates and B consists of half of the points with the highest x-coordinates.

(3) Recursively compute H_A = conv(A) and H_B = conv(B).

(4) Merge the two hulls into a common convex hull, H, by computing the upper and lower tangents for H_A and H_B and discarding all the points lying between these two tangents.

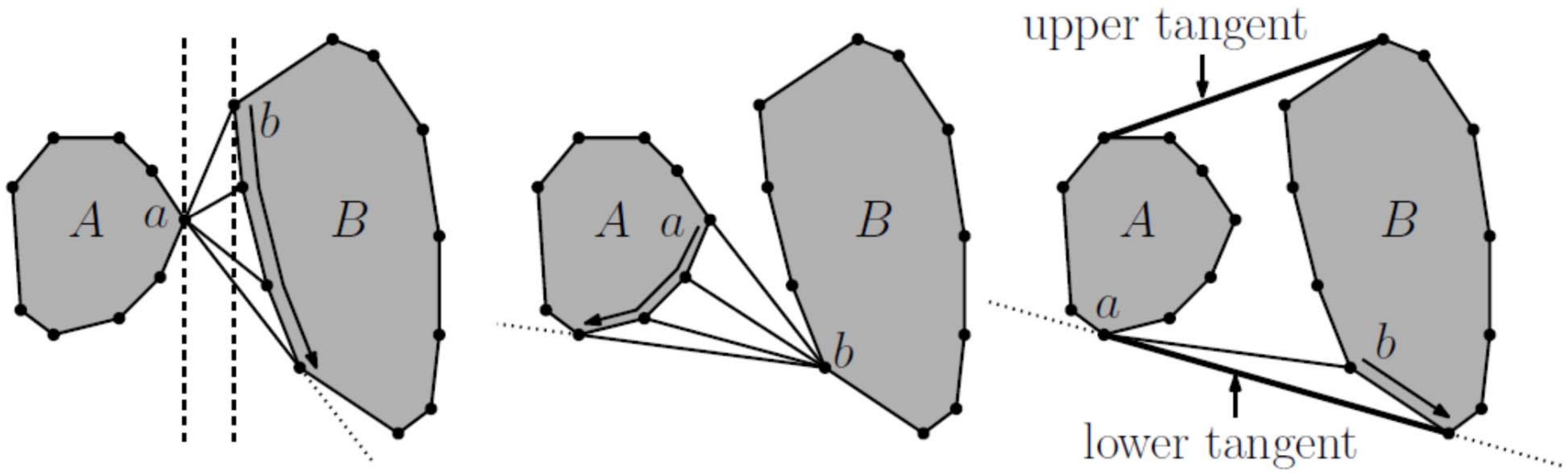
Divide and Conquer



- LowerTangent(H_A,H_B) :

```
(1) Let a be the rightmost point of $H_A$.
(2) Let b be the leftmost point of $H_B$.
(3) While (ab is not a lower tangent for $H_A$ and $H_B$) do
        (a) While (ab is not a lower tangent to $H_A$) do a ← a.pred (move a clockwise).
        (b) While (ab is not a lower tangent to $H_B$) do b ← b.succ (move b counterclockwise).
(4) Return ab.
```

Divide and Conquer



- “ab is not a lower tangent to H_A” is equivalent to Orient(b,a,a.pred)≥0
  - Vertical gap between two partial convex hulls needed for this test to hold
  - Each vertex is visited at most once

Complexity

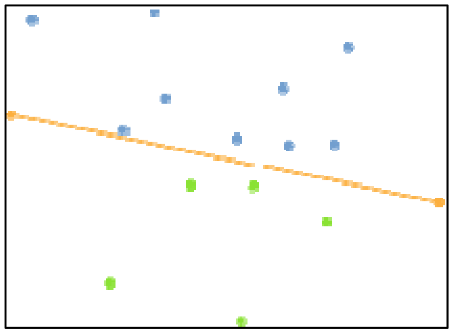
- Partitioning points is O(n) since they are sorted according to x
- Returning final results is O(n)
- Tangent computation is O(n)
- Running time is:

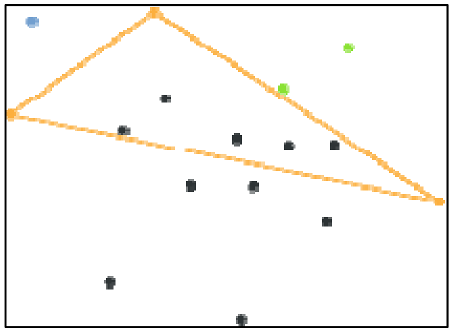
$$T(n) = \begin{cases} 1 & \text{if } n \leq 3 \\ n + 2T(n/2) & \text{otherwise.} \end{cases}$$

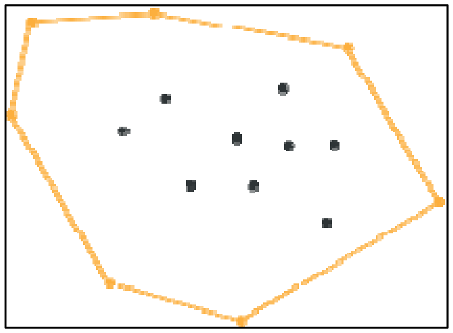
- Therefore, T(n) = O(nlogn)

Quick Hull

1. Find the points with minimum and maximum x coordinates, those are bound to be part of the convex hull.
2. Use the line formed by the two points to divide the set in two subsets of points, which will be processed recursively.
3. Add this line in both directions to the convex hull.
4. For each line, find the point with the maximum positive distance from the line. Form a triangle with this point and the initial endpoints which replaces the line in the convex hull.
5. The points lying inside of the triangle cannot be part of the convex hull and can therefore be ignored in the next steps.
6. Repeat the previous two steps on the two lines formed by the triangle (not the initial line).
7. Repeat until no more points are left. The recursion has come to an end and the points selected constitute the convex hull.





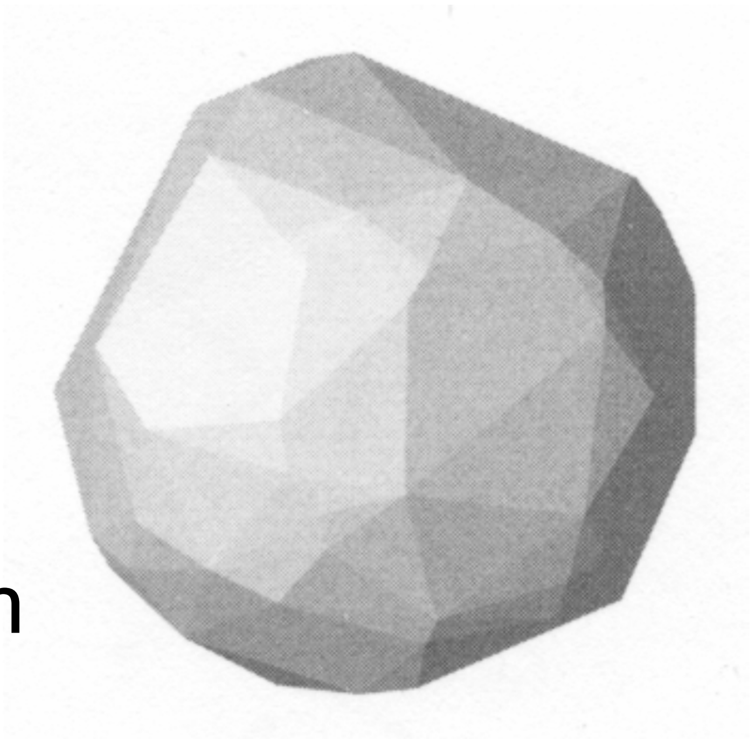


Convex Hulls in 3D

Slides by P. Indyk and J.C. Yang

Problem Statement

- Given P, a set of n vertices in 3D
- Return convex hull of P: CH(P), i.e. the smallest polyhedron such that all elements of P are on or in the interior of CH(P)



Complexity (Spatial)

- Complexity of CH for n points in 3D is O(n)
- ..because the number of edges of a convex polytope with n vertices is at most 3n-6 and the number of facets is at most 2n-4
- ..because the graph defined by vertices and edges of a convex polytope is planar
- Euler’s formula: $n - n_e + n_f = 2$

Randomized Incremental Algorithm

- Initialize the algorithm
- Loop over remaining points
  - Add p_r to the convex hull of P_{r-1} to transform $CH(P_{r-1})$ to $CH(P_r)$
  - [for integer $r \geq 1$, let $P_r := \{p_1, \ldots, p_r\}$]

Initialization

- Need a CH to start with
- Build a tetrahedron using 4 points in P
  - Start with two distinct points in P, say, p_1 and p_2
  - Walk through P to find p_3 that does not lie on the line through p_1 and p_2
  - Find p_4 that does not lie on the plane through p_1, p_2, p_3
  - Special case: No such points exist? Planar case!
- Compute random permutation $p_5,\ldots,p_n$ of the remaining points

Inserting Points into CH

- Add p_r to the convex hull of P_{r-1} to transform $CH(P_{r-1})$ to $CH(P_r)$
- Two Cases:

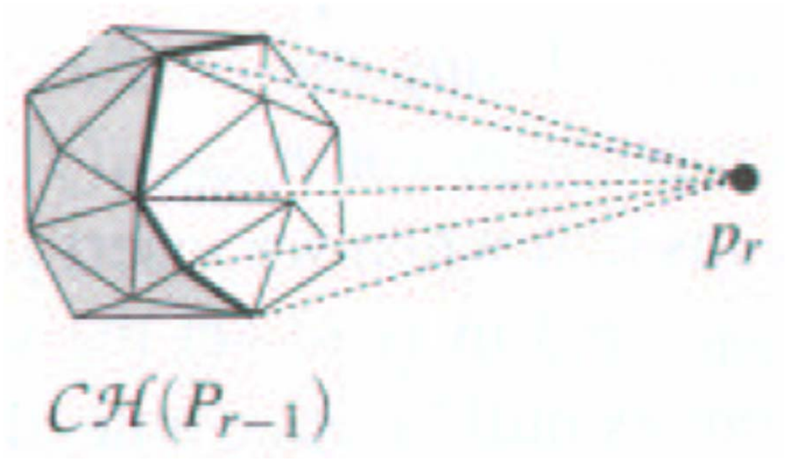
  (1) P_r is inside or on the boundary of $CH(P_{r-1})$

  – Simple: $CH(P_r) = CH(P_{r-1})$

  (2) P_r is outside of $CH(P_{r-1})$ – the hard case

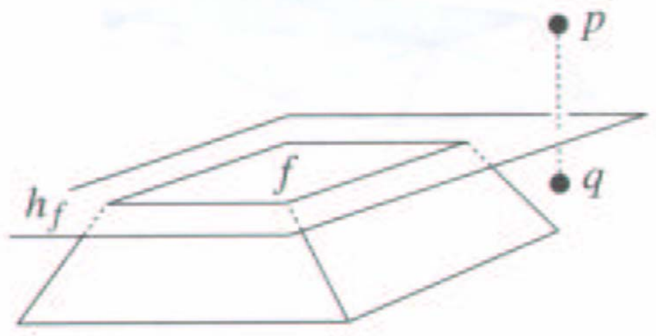
Case 2: P_r outside $CH(P_{r-1})$

- Determine horizon of p_r on $CH(P_{r-1})$
  - Closed curve of edges enclosing the visible region of p_r on $CH(P_{r-1})$



Visibility

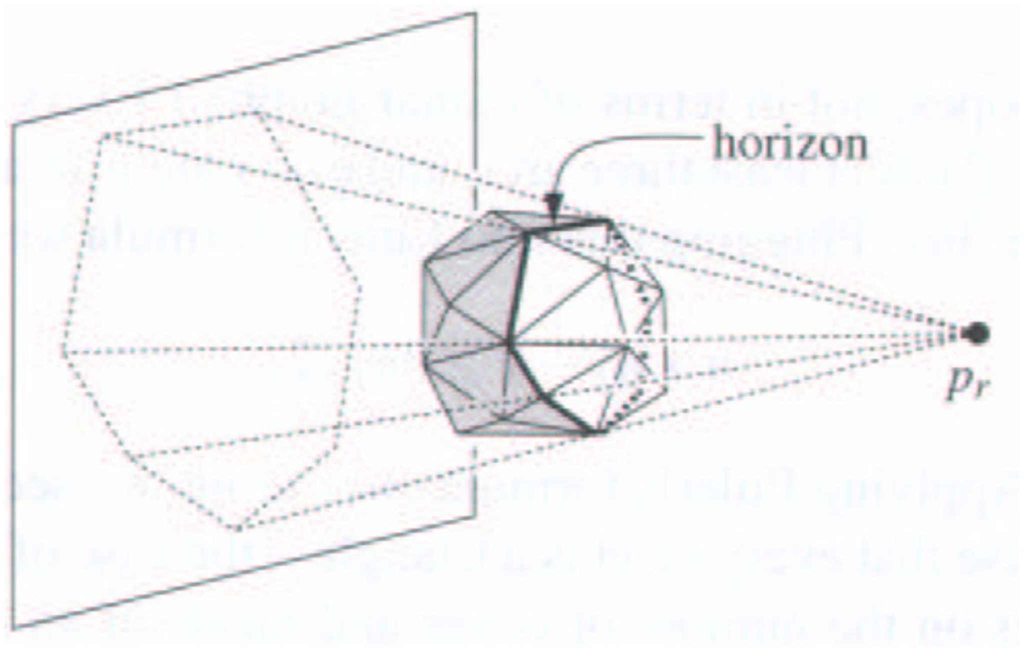
- Consider a plane h_f containing a facet f of $CH(P_{r-1})$
- f is visible from a point p if that point lies in the open half-space on the other side of h_f



f is visible from *p*, but not from *q*

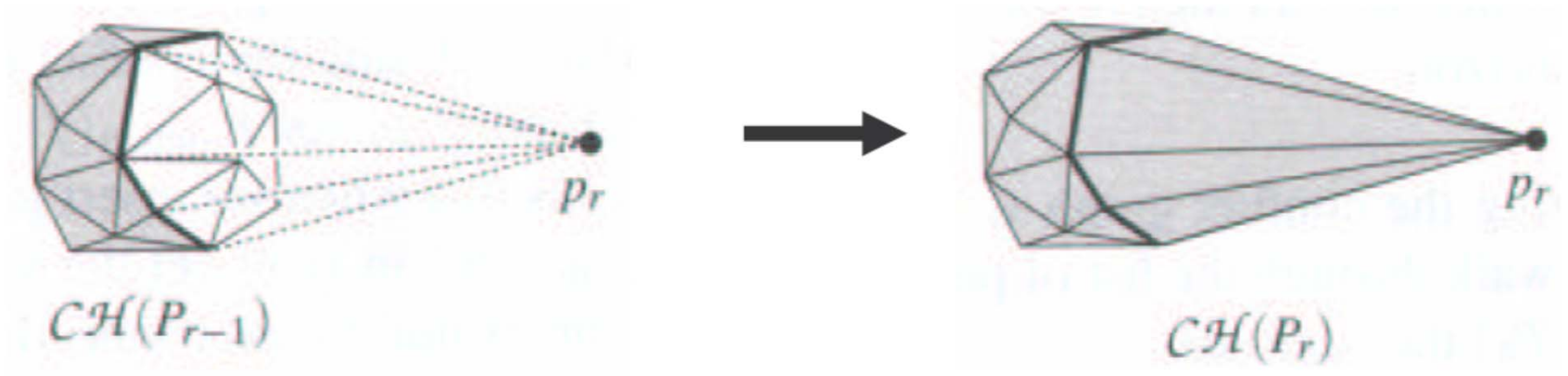
Rethinking the Horizon

- The boundary of polygon obtained from projecting CH(P_{r-1}) onto a plane with p_r as the center of projection



$CH(P_{r-1})$ to $CH(P_r)$

- Remove visible facets from $CH(P_{r-1})$
- Found horizon: Closed curve of edges of $CH(P_{r-1})$
- Form $CH(P_r)$ by connecting each horizon edge to p_r to create a new triangular facet



Algorithm So Far

- Initialization
  - Form tetrahedron CH(P_4) from 4 points in P
  - Compute random permutation of remaining pts
- For each remaining point in P
  - p_r is point to be inserted
  - If p_r is outside CH(P_{r-1}) then
    - Determine visible region
    - Find horizon and remove visible facets
    - Add new facets by connecting each horizon edge to p_r
    - How do we determine the visible region?

How to Find the Visible Region

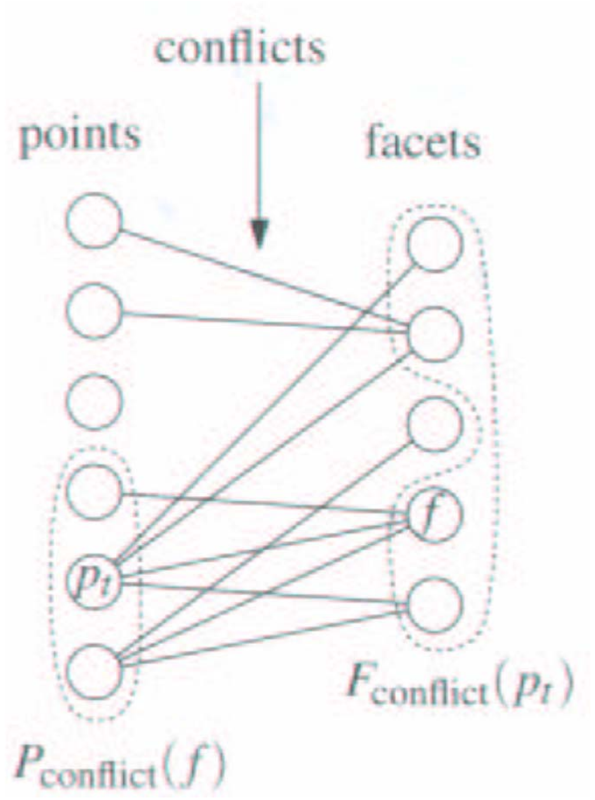
- Naïve approach:
  - Test every facet with respect to p_r
  - $O(n^2)$ work
- Trick is to work ahead:
  - Maintain information to aid in determining visible facets.

Conflict Lists

- For each facet f maintain
  - $P_{conflict}(f)$ subset of $\{p_{r+1}, \ldots, p_n\}$ containing points to be inserted that can see f
- For each p_t, where $t > r$, maintain
  - $F_{conflict}(p_t)$ containing facets of $CH(P_r)$ visible from p_t
- p and f are in conflict because they cannot coexist on the same convex hull

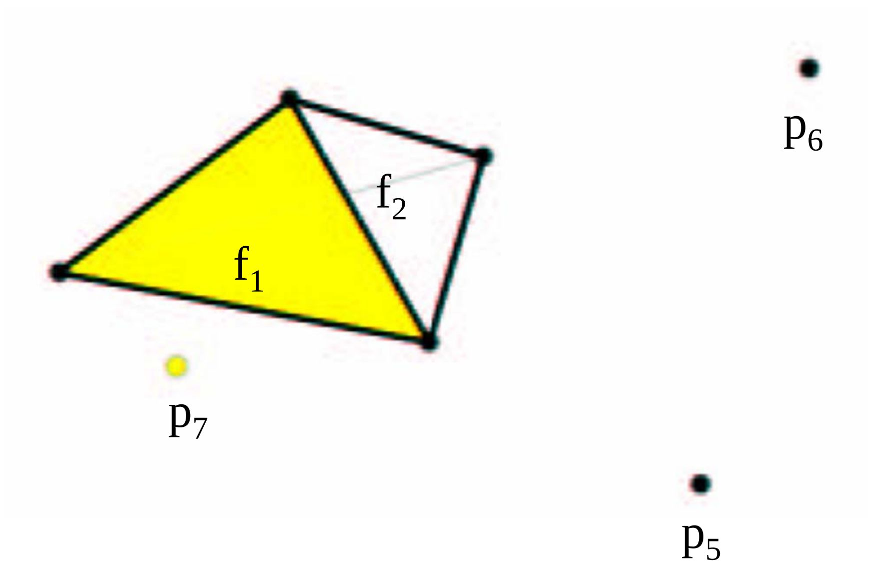
Conflict Graph G

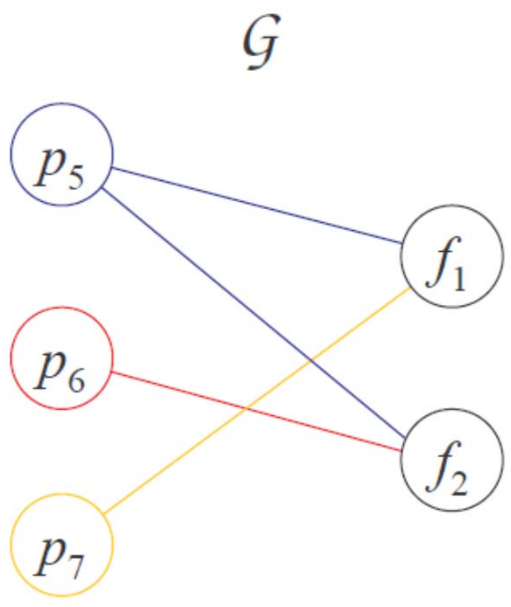
- Bipartite graph
  - points not yet inserted
  - facets on CH(P_r)
- Arc for every point-facet conflict
- Conflict sets for a point or facet can be returned in linear time
- At any step of our algorithm, we know all conflicts between the remaining points and facets on the current CH



Initializing G

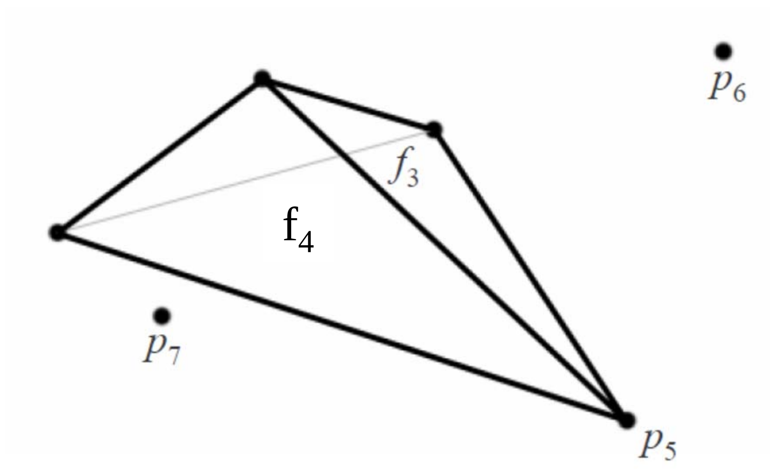
- Initialize G with CH(P_4) in linear time
- Walk through P_{5-n} to determine which facet each point can see

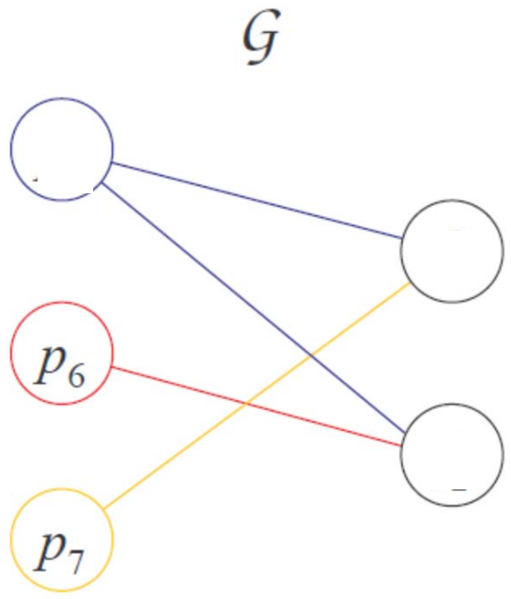




Updating G

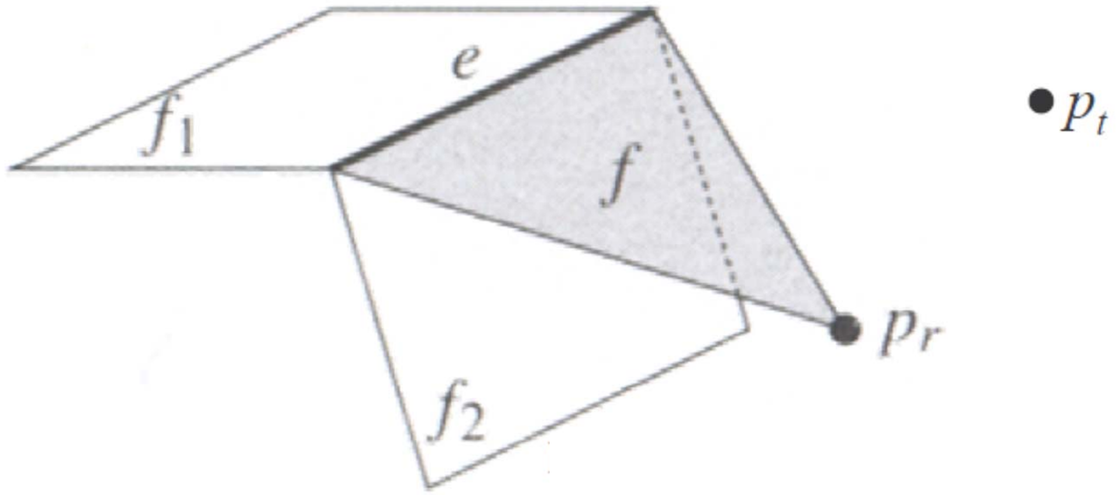
- Discard visible facets from p_r by removing neighbors of p_r in G
- Remove p_r from G
- Determine new conflicts





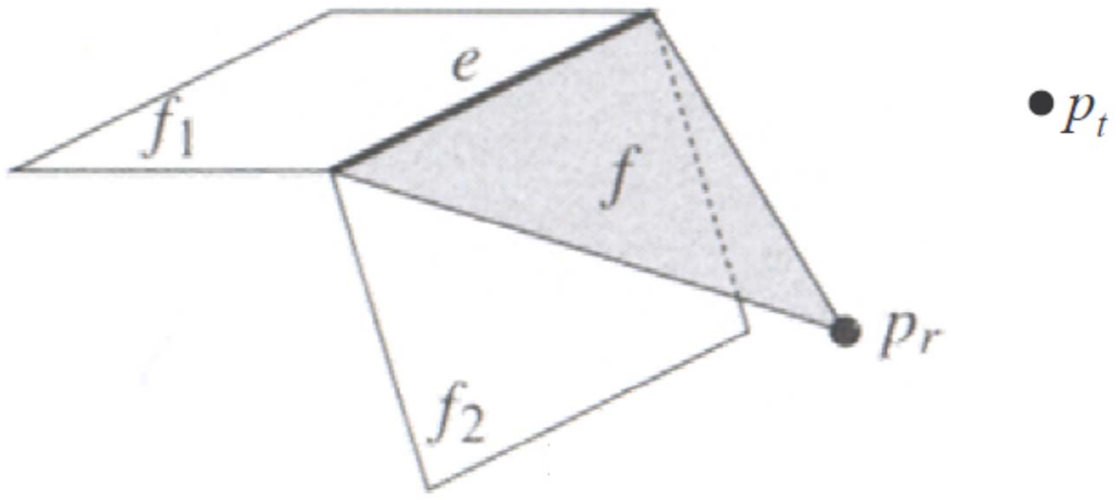
Determining New Conflicts

- If p_t can see new f, it can see edge e of f
- e on horizon of p_r, so e was already in and visible from p_t in $CH(P_{r-1})$
- If p_t sees e, it saw either f_1 or f_2 in $CH(P_{r-1})$
- p_t was in $P_{conflict}(f_1)$ or $P_{conflict}(f_2)$ in $CH(P_{r-1})$



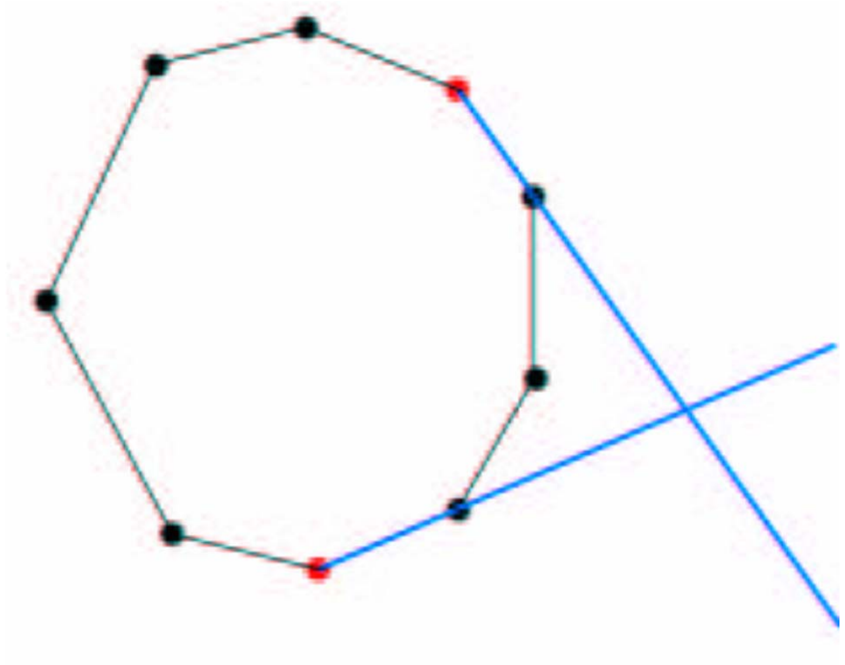
Determining New Conflicts

- Conflict list of f can be found by testing the points in the conflict lists of f_1 and f_2 incident to the horizon edge e in $CH(P_{r-1})$



What About the Other Facets?

- $P_{conflict}(f)$ remains unchanged for any f unaffected by p_r

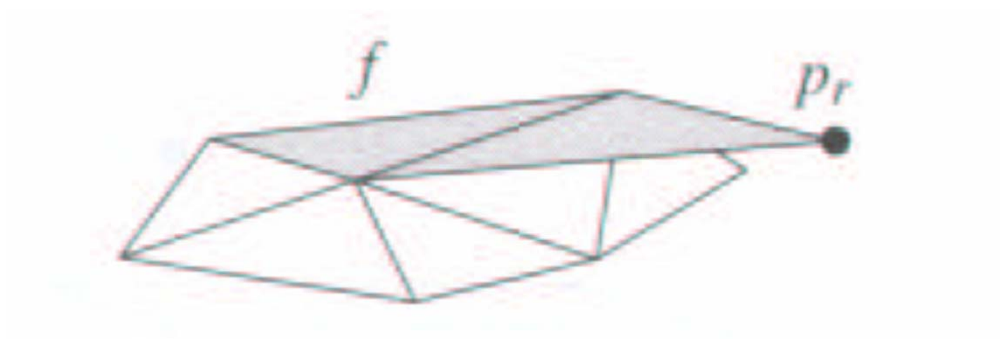


Final Algorithm

- Initialize $CH(P_4)$ and G
- For each remaining point
  - Determine visible facets for p_r by checking G
  - Remove $F_{conflict}(p_r)$ from CH
  - Find horizon and add new facets to CH and G
  - Update G for new facets by testing the points in existing conflict lists for facets in $CH(P_{r-1})$ incident to e on the new facets
  - Delete p_r and $F_{conflict}(p_r)$ from G

Fine Point

- Coplanar facets
  - p_r lies in the plane of a face of $CH(P_{r-1})$



- f is not visible from p_r so we merge created triangles coplanar to f
- New facet has same conflict list as existing facet